

Endeavor

STEM Teaching Certificate Project



Arithmetic Sequences and Series in the Fiber Arts of Knitting and Crocheting

Algebra 1/2

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Art Lesson Enhancement: Describe how the art form enhances a topic that you currently teach or plan to teach. Include a 1 paragraph statement about your personal feelings regarding integrating the arts and the specific art form you chose to use.

Interdisciplinary context: How can this art form (and/or art in general) be used to create interdisciplinary lessons, discussions or activities in your classroom? How can you connect to multiple content areas?

BIG IDEAS

From the Common Core Mathematics Standards

(https://corestandards.org/wp-content/uploads/2023/09/Math_Standards1.pdf)

Mathematical Practices

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

This lesson focuses on #1, 2, 3, 4, 7. Students will

- make sense of a real-word problem
- try to model it algebraically and through the use of simple Excel spreadsheet commands
- work with each other to try to devise a solution to problems that arise.

EDUCATION STANDARDS

Also from the Common Core Mathematics Standards, from the “Functions” unit:

Functions describe situations where one quantity determines another.

- *Because we continually make theories about dependencies between quantities in nature and society, functions are important tools in the construction of mathematical models.*
- *A function can be described in various ways, such as by a graph (e.g., the trace of a seismograph); by a verbal rule, as in, “I’ll give you a state, you give me the capital city;” by an algebraic expression like $f(x) = a + bx$; or by a recursive rule.*
- *A graphing utility or a computer algebra system can be used to experiment with properties of these functions and their graphs and to build computational models of functions, including **recursively defined functions**.*
- *Because functions describe relationships between quantities, they are frequently used in modeling. Sometimes functions are defined by a **recursive process**, which can be displayed effectively using a spreadsheet or other technology.*

Interpreting Functions (F-IF)

F-IF1: Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If f is a function and x is an element of its domain, then $f(x)$ denotes the output of f corresponding to the input x . The graph of f is the graph of the equation $y = f(x)$.

F-IF2: Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.

F-IF3: Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers. For example, the Fibonacci sequence is defined recursively by $f(0) = f(1) = 1$, $f(n+1) = f(n) + f(n-1)$ for $n \geq 1$. Interpret functions that arise in applications in terms of the context

F-IF7b: Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions.

F-IF8: Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.

And

CCSS.Math.Content.7.EE.B.3 - Solve real-life and mathematical problems using numerical and algebraic expressions and equations.

CCSS.Math.Practice.MP1 - Make sense of problems and persevere in solving them.

CCSS.Math.Practice.MP2 - Reason abstractly and quantitatively.

CCSS.Math.Practice.MP3 - Construct viable arguments and critique the reasoning of others.

CCSS.Math.Practice.MP4 - Model with mathematics. MP.4.A - Mathematically proficient students

NGSS Performance Expectation(s)

From

- <https://www.nextgenscience.org/>
- https://www.cde.state.co.us/cdesped/accommodationsmanual_ccss_k12_techsc_ope,
- <https://www.nationalartsstandards.org/sites/default/files/2021-11/Visual%20Arts%20at%20a%20Glance%20rev.pdf>

Science and Engineering Practices	Disciplinary Core Ideas	Crosscutting Concepts:
Developing and using models Asking questions and defining problems Using mathematical and computational thinking	Not applicable	(these are for science, not for math) Interdependence of science, engineering, and technology patterns

Common Core State Standards:

Math: see “Education Standards” above

Art:

Anchor Standard 10: Relate artistic ideas and works with societal, cultural, and historical context to deepen understanding
Enduring Understanding: Media artworks and ideas are better understood and produced by relating them to their purposes

Technology:

Demonstrate proficiency in the use of computers and applications as well as an understanding of the concepts underlying hardware, software and connectivity

RI 7 Use spreadsheets and other applications to make predictions, solve problems, and draw conclusions

MD , SBAC testing skills: Enter/edit data in spreadsheets and perform calculations using formulas

MEASURABLE STUDENT LEARNING OBJECTIVES

Students will be able to:

- Describe the qualities of arithmetic sequences and series
- Write a recursive definition for an arithmetic sequence using appropriate algebraic notation
- Use technology to represent recursive definitions for arithmetic sequences and series
- Determine whether a numerical quantity would appear in a sequence using technology or other methods
- Determine whether a numerical quantity would be represented by a partial sum (series) of the sequence using technology or other methods
- Represent a given arithmetic sequence with a recursive as well as explicit formula
- Represent an arithmetic sequence with a linear function
- Recognize that an arithmetic series can be represented by an exponential function

STEM INTEGRATION

This lesson plan integrates math, technology, history, and art.

Integration of Math, the Art of Handiwork, and History:

At the university level, crocheting was used to illustrate properties of multiple-dimension topology: <https://www.youtube.com/watch?v=w1TBZhd-sN0>

(fast forward to 15:00 for her discussion of math and art)

More on this: <https://pi.math.cornell.edu/~dtaimina/crochet/hplane.htm>

One interdisciplinary connection is that crocheted hyperplanes can model coral reefs: <https://crochetcoralreef.org/exhibitions/>.

And another interdisciplinary connection is how in general, handiwork of all kinds: knitting, crocheting, embroidery, quilting was a way for the artist to record history, and sometimes used as a way to prove literacy!

<https://www.vam.ac.uk/articles/embroidery-a-history-of-needlework-samplers>

(Interesting to note that in Europe and the United States, handiwork was considered a women's pursuit; in Latin America it is often the pursuit of men)

<https://www.travlinmad.com/blog/taquile-peru-colorful-culture-where-men-knit>

Knitting was very important during wartime: Fabric of Britain - 1 Knitting's Golden Age (BBC)

<https://www.youtube.com/watch?v=BdlITPVCCGI&list=PPSV&t=434s>

So at the most basic and most complex of levels, this art form is very mathematical and also very connected to the humanities in every way!

Integration of Math and Technology:

Technology and history of technology: Excel programming is part of the lesson. Spreadsheet functions are often dependent on algebra, and "coding" an Excel cell and copying and pasting a formula is one way to elucidate a recursive or explicit mathematical formula.

Modern knitting machines are "coded" much like regular computers are coded.

"Languages for 3D Industrial Knitting" by Lea Albaugh

<https://www.youtube.com/watch?v=02h74L1PmaU&list=PPSV>

Therefore there is a natural relationship between mathematics and technology in this manner.

MATERIALS NEEDED

- Computer with access to Microsoft excel and/or coding ability
- Knitting materials

ENGAGING CONTEXT/PHENOMENON

One of the NASA resources was from the course “The Arts in STEM: Advancing Meaningful Integration.” The Getty Museum has a collection of videos for educators regarding the integration of art in education.

<https://www.youtube.com/@gettymuseum/search?query=education%20>

Although I did not use any of these videos specifically for this lesson, the videos support the idea that art history can be impactful for STEM education.

Engaging phenomenon is the mathematics of knitting with a link to technology: how to code excel, how knitting machines knit, how knitting machines are coded.

DATA INTEGRATION

This is not a science lesson and therefore NASA data is not appropriate. However, NASA resources through the Art in STEM course are employed.

TEACHER BACKGROUND KNOWLEDGE

Teacher should

- Have requisite background knowledge of sequences or series
- Have requisite background knowledge of the Excel commands in the lesson and a basic understanding of Excel.
- watch the video links provided for the students
- find a colleague who knits or crochets for support in helping students who are interested in knitting or crocheting
- Understand the very basics of knitting:
 - A certain number of stitches is CAST ON. This is your starting number of stitches in row 1.
 - Standard “flat” knitting is done typewriter-style: A row of stitches is created, the work is turned around, and then a new row of stitches is created.
 - For each new row, you can make the same number of new stitches, add more stitches (called “increasing”) and/or subtract the total number of stitches (called “decreasing.”) There are a number of different types of increase and decrease stitches and each of them has a different name, and affect the overall appearance of the work. (to get a better understanding https://youtu.be/H3rl_Js4EN8?si=M3kfLCRrK5im2b7Z)

- Any row may have regular stitches, increase stitches AND decrease stitches.

Also, take the time to visit the youtube video links provided in the 5E lesson plan itself.

DIFFERENTIATION OF INSTRUCTION

Students should be thoughtfully put into small groups of 3 or 4 based on diverse mathematical ability and computer facility.

REAL-WORLD CONNECTIONS FOR STUDENTS

I am a knitter and crocheter, and often I find myself wondering if I have missed a stitch or not. Knitting is weaving with two needles and crocheting is knotting with one hook. In both cases, the working row gets larger and narrower using certain stitches called “increases” and “decreases” performed in various manners.

I have always seen this as a practical application of the Algebra2 topic of arithmetic sequences and series.

The arts provide many opportunities to engage students and improve their lives. Not only is there a lot of cultural and historical significance in handicraft, in many places in the world those handicraft arts are essential parts of the culture today. It is a talent and skill that is passed down from generation to generation, building community and keeping traditions alive. In addition, it is a great way to relieve stress, teach and practice patience.

Many handicrafts have strong and essential mathematical components. Patterns, symmetry, proportion, transformations...they're everywhere. Quilters need to think about transformations while maximizing the amount of cutouts one can make in a given yardage of fabric so as not to be wasteful. In knitting and crocheting there are repeating patterns that are written like algorithms; patterns follow specific counts and ratios, which is where this lesson idea was born.

A diverse student population brings a wide range of experiences with handicrafts. Even though it's become a bit of a lost art in America, most students have had *some* exposure to it, whether it's something they've seen a parent or grandparent do, or a tradition that's specific to their culture and that they're aware of.

INTEGRATION POSSIBLE MISCONCEPTIONS

Some students will have experience knitting, or experience sitting with a family member and watching them knit. This will make those students the lesson “experts.”

Arithmetic SEQUENCES are LISTS of numbers that are created by recursively adding the same constant (called a common difference) to the previous number.

An arithmetic SERIES is a SUM of all of the terms of an arithmetic sequence.

Students often confuse the two concepts and terminology because (a) they are similar and (b) often students don't see the importance of using proper language to explain mathematical concepts.

The difference between the two can be easily distinguished with knitting: a sequence counts the number of stitches in each row; a series counts the sum total of all of the stitches in all of the rows.

The topic of sequences/series is also generally a student's first experience with recursion; a concept essential for success in computer programming. The recursion notation is also very confusing to students even though the idea of conjuring up a recursion rule is much more natural than conjuring up an explicit rule.

LESSON PROCEDURE

5E	Details of 5E Lesson Implementation
<p><u>Engage</u></p>	<p>Procedure: (put students in small groups). Ask the students if anyone knits, crochets, or if they have family members that do this. Ask them what they know about knitting, how it is done.</p> <p>Post this question on the board and give students time to answer the questions and ask questions of the teacher to get as far as they can get on their own. Let the majority of the students get to question 3 before stopping them, and let them know that it's OK if they don't finish. Ask them what questions they still have remaining about the problem.</p> <p>Introduce hook:</p> <p>The 1st row of the sweater I'm knitting has 50 stitches. Every even row, I just knit the same number of stitches as the previous row. Every odd row after the 1st, I have to increase the number of stitches by 8.</p> <ol style="list-style-type: none"> 1. How many stitches are in each of the first 10 rows? 2. How many stitches in total are in the first 10 rows? 3. If there is a row with 118 stitches, have I made an error? How did you figure this out? 4. What if I wanted to know If there is a row with 2000 stitches, have I made an error?



(Image from <https://nimble-needles.com>)

Modifications: Maybe regroup students to have a “knitting expert” in each, if there are enough students in the class who seem to have knowledge of knitting or crocheting.

Standards Addressed

CCSS.Math.Practice.MP1 Make sense of problems and persevere in solving them.

CCSS.Math.Practice.MP2 Reason abstractly and quantitatively.

Formative/Summative Assessments: Formative, the teacher walks around to assess and give hints without giving too much away.

Resources: (image above)

Explore

Procedure: Once the students understand the problem at hand, send the student groups to a computer to get them started on Excel. The instructions are on the lesson, and they will need help to execute.

The lesson: The lesson itself is given on this Excel Spreadsheet.

<https://docs.google.com/spreadsheets/d/1IZdtDUn7c-xg6V5ZtxHP7bt1XRx4ihagwss7Qbg9MH0/edit?usp=sharing>

The students are guided through the technology to help them understand and represent the problem, and to find solutions without any understanding of the formulae for n th terms of arithmetic sequences or partial sums of the related series.

	<p><i>This lesson is an introduction to the idea of arithmetic sequences and the relationship between recursive definitions of arithmetic sequences and explicit definitions of linear functions.</i></p> <p>Modifications: If there are students who are familiar with Excel, this is a good time to re-distribute students in the group as necessary.</p> <p>Standards Addressed</p> <p>CCSS.Math.Practice.MP4 Model with mathematics.</p> <p>CCSS.Math.Practice.MP5 Use appropriate tools strategically.</p> <p>Demonstrate proficiency in the use of computers and applications as well as an understanding of the concepts underlying hardware, software and connectivity</p> <p>RI 7 Use spreadsheets and other applications to make predictions, solve problems, and draw conclusions</p> <p>MD , SBAC testing skills: Enter/edit data in spreadsheets and perform calculations using formulas</p> <p>CCSS.Math.Content.7.EE.B.3</p> <p>Solve real-life and mathematical problems using numerical and algebraic expressions and equations.</p> <p>Formative/Summative Assessments</p> <p>Formative: teacher engagement while students work in small groups</p>
<p><u>Explain</u></p>	<p>Procedure: The Excel Spreadsheet is a collection of sheets. Assign each group a sheet to explain their answers to. If the sheet allows for different interpretations of the questions, allow for and encourage other groups to share what they think. Remind the students, if necessary, that their critiques of each other’s ideas should be gentle and model that if necessary.</p> <p>Modifications: Some students are more vocal and confident than others. Try your best to make each student lend their voice, and if they are confident, allow them to speak in pairs.</p> <p>Standards Addressed</p> <p>CCSS.Math.Content.7.EE.B.3</p> <p>Solve real-life and mathematical problems using numerical and algebraic expressions and equations.</p>

	<p>CCSS.Math.Practice.MP3 Construct viable arguments and critique the reasoning of others.</p>
<p><u>Elaborate</u></p>	<p>Procedure:</p> <p>Students will select one of the following related videos to watch and report on. They can choose to work alone or in pairs. Whichever resource they choose, they must give a written report of their findings that addresses what they learned, what they want to know more about or what they didn't understand, and the relationship to mathematics/art/history.</p> <p>Modifications</p> <p>Some of the videos are more challenging than others, and this will allow for modification due to student ability and interest.</p> <p>Standards Addressed</p> <p>Anchor Standard 10: Relate artistic ideas and works with societal, cultural, and historical context to deepen understanding Enduring Understanding: Media artworks and ideas are better understood and produced by relating them to their purposes</p> <p>Formative/Summative Assessments</p> <p>A rubric for this will be given in the "evaluate" piece.</p> <p>Resources / Options for Choice</p> <p>1. "Languages for 3D Industrial Knitting" by Lea Albaugh</p> <p>https://www.youtube.com/watch?v=02h74L1PmaU&list=PPSV</p> <p>Industrial computer-controlled knitting machines are churning out seamless three-dimensional fabric objects as you read this -- think gloves, sweaters, sportswear, furnishings, architectural fabrics, and plushies. But right now the programming languages for these machines are terrible. With a knitting machine, complex structures are formed with a very limited set of low-level machine operations. Thinking on the level of those machine operations becomes untenable pretty quickly when you start trying to make real objects. How can we ensure knitability? Can we encode constraints in an understandable way? Can we optimize production speed? We'll look at several ways to describe knit structures and generate machine instructions for their fabrication, including both the delightfully arcane language that ships with the machine and the language from our SIGGRAPH 2016 technical paper, "A Compiler for 3D Machine Knitting."</p>

	<p>2. “A Brief History of the Knitting Machine Machine Knitting a Tuck Stitch Sweater”</p> <p>https://www.youtube.com/watch?v=99V_hZ-yOr0&list=PPSV&t=17s</p> <p>Follow along as I machine knit a tuck stitch pattern sweater while I talk about getting into machine knitting and a brief history of the knitting machine.</p> <p>3. Fabric of Britain - 1 Knitting's Golden Age (BBC)</p> <p>https://www.youtube.com/watch?v=BdIITPVCCGI&list=PPSV&t=434s</p> <p>3-part BBC documentary series. How knitting rose from basic craft to the height of popular fashion in the 20th century. It's a craft that has given us scratchy jumpers, sexy bathing costumes and the infamous poodle loo cover, has sustained Britain through the hardships of war and shown a mother's love to generations of little ones. Today, knitwear has become a staple of every wardrobe thanks to a prince's golfing taste, The Beatles and 80s breakfast television. Warm-hearted and surprising, this is the story of the people's craft, and a very British one at that.</p>
<p><u>Evaluate</u></p>	<p>Procedure: (What happens during this phase? What is the teacher doing? What is the student doing?)</p> <p>Modifications (What student needs must be addressed? How can you make each experience accessible for ALL learners?)</p> <p>Standards Addressed</p> <p>All Functions standards listed in “Education standards” section</p> <p>Formative/Summative Assessments (How will you assess in each phase?)</p> <p>Assessment:</p> <p>Recall the original problem:</p> <p>The 1st row of the sweater I’m knitting has 50 stitches. Every even row, I just knit the same number of stitches as the previous row. Every odd row after the 1st, I have to increase the number of stitches by 8.</p> <ol style="list-style-type: none"> 1. Write the first 8 terms of the sequence. 2. Explain why this sequence is NOT arithmetic. 3. Explain what a series is in terms of the problem given. 4. Copy the spreadsheet given to you. Invent a knitting problem like this and alter the spreadsheet and its formulae to represent the new situation.

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| | <ol style="list-style-type: none">5. Write THREE problems regarding your new situation, and use the spreadsheet to answer those questions.6. Determine the sum of the first 50 pairs of rows of your knitting project. Show on the spreadsheet where that answer lies. |
|--|---|

Possible extensions:

https://stitchmastery.com/?doing_wp_cron=1743962592.2133409976959228515625

RESOURCES

<https://www.nextgenscience.org/>

<https://www.nationalartsstandards.org/sites/default/files/2021-11/Visual%20Arts%20at%20a%20Glance%20rev.pdf>

https://www.cde.state.co.us/cdesped/accommodationsmanual_ccss_k12_techscope

<https://www.getty.edu/museum/>

<https://www.youtube.com/@gettymuseum/search?query=education%20>

<https://www.youtube.com/watch?v=w1TBZhd-sN0>

<https://pi.math.cornell.edu/~dtaimina/crochet/hplane.htm>

[https://crochetcoralreef.org/exhibitions/.](https://crochetcoralreef.org/exhibitions/)

<https://www.vam.ac.uk/articles/embroidery-a-history-of-needlework-samplers>

<https://www.travlinmad.com/blog/taquile-peru-colorful-culture-where-men-knit>

<https://www.youtube.com/watch?v=BdlITPVCcGI&list=PPSV&t=434s>

<https://www.youtube.com/watch?v=02h74L1PmaU&list=PPSV>

<https://nimble-needles.com>