

5E Integrated STEM Lesson Plan

Lesson Title: “Method of Joints” Analysis for Bridge Truss System.

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Topic:

- Apply knowledge of free-body diagrams, force vectors, moment (torque), and trigonometry to conduct a method of joints approach to bridge truss analysis.

Targeted Grade Level: Grades 10 through 12.

Time Needed: 270 minutes (three 90-minute class periods)

Subject Integration: This lesson will integrate all four areas of STEM: engineering, math, science, and technology.

Justification: Students will use the engineering design process to build a bridge truss out of balsa wood, test the amount of weight that the truss can support, compete to support more weight than classmates, then reflect on the design process. After testing the trusses, students will analyze the forces acting on the bridge truss using a mathematical approach called the “method of joints”. The analysis requires an understanding of vector forces (physics). The lesson also includes the use of software called Bridge Designer so students are learning how to use computers to analyze structural forces acting on a bridge system. All four areas of STEM are addressed with logical progression from start (building a series of triangles) to finish (structural analysis with math and computers).

Standards:

<u>NGSS Performance Expectations:</u>		
HS-ETS1-1.	Analyze a major global challenge to specify qualitative and quantitative criteria and constraints for solutions that account for societal needs and wants.	
HS-ETS1-2.	Design a solution to a complex real-world problem by breaking it down into smaller, more manageable problems that can be solved through engineering.	
HS-ETS1-3.	Evaluate a solution to a complex real-world problem based on prioritized criteria and trade-offs that account for a range of constraints, including cost, safety, reliability, and aesthetics as well as possible social, cultural, and environmental impacts.	
HS-ETS1-4.	Use a computer simulation to model the impact of proposed solutions to a complex real-world problem with numerous criteria and constraints on interactions within and between systems relevant to the problem.	
Science and Engineering Practices	Disciplinary Core Ideas	Crosscutting Concepts:
Asking Questions and Defining Problems Asking questions and defining problems in 9–12 builds on K–8 experiences and progresses to formulating, refining, and evaluating empirically testable questions and design problems using models and simulations. Analyze complex real-world problems by specifying criteria and constraints for successful solutions. (HS-ETS1-1)	ETS1.A: Defining and Delimiting Engineering Problems Criteria and constraints also include satisfying any requirements set by society, such as taking issues of risk mitigation into account, and they should be quantified to the extent possible and stated in such a way that one can tell if a given design meets them. (HS-ETS1-1) Humanity faces major global challenges today, such as the need	Systems and System Models Models (e.g., physical, mathematical, computer models) can be used to simulate systems and interactions—including energy, matter, and information flows— within and between systems at different scales. (HS-ETS1-4) Connections to Engineering, Technology, and Applications of Science

Using Mathematics and Computational Thinking Mathematical and computational thinking in 9-12 builds on K-8 experiences and progresses to using algebraic thinking and analysis, a range of linear and nonlinear functions including trigonometric functions, exponentials and logarithms, and computational tools for statistical analysis to analyze, represent, and model data. Simple computational simulations are created and used based on mathematical models of basic assumptions. Use mathematical models and/or computer simulations to predict the effects of a design solution on systems and/or the interactions between systems. (HS-ETS1-4) Constructing Explanations and Designing Solutions Constructing explanations and designing solutions in 9–12 builds on K–8 experiences and progresses to	for supplies of clean water and food or for energy sources that minimize pollution, which can be addressed through engineering. These global challenges also may have manifestations in local communities. (HS-ETS1-1) ETS1.B: Developing Possible Solutions When evaluating solutions, it is important to take into account a range of constraints, including cost, safety, reliability, and aesthetics, and to consider social, cultural, and environmental impacts. (HS-ETS1-3) Both physical models and computers can be used in various ways to aid in the engineering design process. Computers are useful for a variety of purposes, such as running simulations to test different ways of solving a problem or to see which one is most efficient or economical; and in making a persuasive presentation to a client about how a	Influence of Science, Engineering, and Technology on Society and the Natural World New technologies can have deep impacts on society and the environment, including some that were not anticipated. Analysis of costs and benefits is a critical aspect of decisions about technology. (HS-ETS1-1) (HS-ETS1-3)
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explanations and designs that are supported by multiple and independent student-generated sources of evidence consistent with scientific ideas, principles and theories. Design a solution to a complex real-world problem, based on scientific knowledge, student-generated sources of evidence, prioritized criteria, and tradeoff considerations. (HS-ETS1-2) Evaluate a solution to a complex real-world problem, based on scientific knowledge, student-generated sources of evidence, prioritized criteria, and tradeoff considerations. (HS-ETS1-3)	given design will meet his or her needs. (HS-ETS1-4) ETS1.C: Optimizing the Design Solution Criteria may need to be broken down into simpler ones that can be approached systematically, and decisions about the priority of certain criteria over others (trade-offs) may be needed. (HS-ETS1-2)	
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Common Core State Standards:

Math: CCSS.MATH.CONTENT.HSG.SRT.D.11

(+) Understand and apply the Law of Sines and the Law of Cosines to find unknown measurements in right and non-right triangles (e.g., surveying problems, resultant forces).

ELA:

CCSS.ELA-LITERACY.W.9-10.1.A

Introduce precise claim(s), distinguish the claim(s) from alternate or opposing claims, and create an organization that establishes clear relationships among claim(s), counterclaims, reasons, and evidence.

CCSS.ELA-LITERACY.W.9-10.1.B

Develop claim(s) and counterclaims fairly, supplying evidence for each while pointing out the strengths and limitations of both in a manner that anticipates the audience's knowledge level and concerns.

CCSS.ELA-LITERACY.W.9-10.1.C

Use words, phrases, and clauses to link the major sections of the text, create cohesion, and clarify the relationships between claim(s) and reasons, between reasons and evidence, and between claim(s) and counterclaims.

CCSS.ELA-LITERACY.W.9-10.1.D

Establish and maintain a formal style and objective tone while attending to the norms and conventions of the discipline in which they are writing.

CCSS.ELA-LITERACY.W.9-10.1.E

Provide a concluding statement or section that follows from and supports the argument presented.

Missouri Learning Standards:

Physical Science:

9-12.PS2.A.1: Analyze data to support and verify the concepts expressed by Newton's 2nd law of motion, as it describes the mathematical relationship among the net force on a macroscopic object, its mass, and its acceleration.

9-12.PS2.A.2: Use mathematical representations to support and verify the concepts that the total momentum of a system of objects is conserved when there is no net force on the system.

9-12.PS2.A.3: Apply scientific principles of motion and momentum to design, evaluate, and refine a device that minimizes the force on a macroscopic object during a collision.

Engineering:

9-12.ETS1.A.1: Analyze a major global challenge to specify qualitative and quantitative criteria and constraints for solutions that account for societal needs and wants.

9-12.ETS1.A.2: Design a solution to a complex real-world problem by breaking it down into smaller, more manageable problems that can be solved through engineering.

9-12.ETS1.B.1: Evaluate a solution to a complex real-world problem based on prioritized criteria and trade-offs that account for a range of constraints, including cost, safety, reliability, and aesthetics as well as possible social, cultural, and environmental impacts.

9-12.ETS1.B.2 Use a computer simulation to model the impact of proposed solutions to a complex real-world problem with numerous criteria and constraints on interactions within and between systems relevant to the problem

ITEEA Standards

1N. Explain how the world around them guides technological development and engineering design.

2V. Analyze the stability of a technological system and how it is influenced by all the components in the system.

4P. Evaluate ways that technology can impact individuals, society, and the environment.

5H. Evaluate a technological innovation that arose from a specific society's unique need or want.

6F. Relate how technological development has been evolutionary, often the result of a series of refinements to basic inventions or technological knowledge.

7W. Determine the best approach by evaluating the purpose of the design.

7Y. Optimize a design by addressing desired qualities within criteria and constraints.

7Z. Apply principles of human-centered design.

7AA. Illustrate principles, elements, and factors of design.

7BB. Implement the best possible solution to a design.

7CC. Apply a broad range of design skills to their design process.

8P. Apply appropriate methods to diagnose, adjust and repair systems to ensure precise, safe, and proper functionality.

Measurable Student Learning Objectives:

- Students will be able to communicate the findings of the experiment in oral and written form.
- Students will be able to construct physical objects using hand tools and shop tools.
- Students will be able to differentiate between scalar and vector quantities.
- Students will be able to identify the magnitude, direction, and sense of a vector.
- Students will be able to understand how Newton's Laws are applied to determine the forces acting on an object.
- Students will be able to create free body diagrams of objects, identifying all forces acting on the object.
- Students will be able to calculate the x and y components of a given vector.

Nature of STEM:

This lesson embraces the nature of STEM by integrating all four areas of STEM. Using a problem-based learning approach, students must create their own bridge truss design that will support a load. Students collect data from the load tests, then compare their designs to designs from other students. Students develop critical thinking skills as they work through the engineering design process and analyze data from their structural tests. Students also learn about careers related to architecture as well as civil and structural engineering.

Engaging Context/Phenomena:

At the beginning of the lesson, the teacher will show several videos of bridge failures so students can see what happens when designs don't measure up to expectations. The videos are quite shocking to watch. The teacher will also show images and discuss a recent NASA bridge project where they updated the bridge over the Indian River Lagoon that connects the Kennedy Space Center and the Cape Canaveral Space Force Station to mainland Florida.

Data Integration:

In this lesson, students attach a series of loads to their bridge truss design until the truss fails to hold the load successfully. Students must collect data from all the experiments, then compare the results to see which truss supported the most weight.

Differentiation of Instruction:

For students that struggle to build a truss out of balsa wood, there is an option to use popsicle sticks which are easier to use since the student isn't required to cut the wood.

For students that get frustrated with the mathematical calculations, they can use computer software to help analyze the forces acting on the bridge truss.

Students are allowed to work in groups for this lesson which can help students that struggle with the lesson rigor.

Real-life Connection:

The lesson is related to real life applications since societies use bridges all over the world. If a student is unfamiliar with bridge designs, the videos at the beginning of the lesson can help them visualize the structure. Class discussion about bridge design in other parts of the world can help to address cultural differences. Students are encouraged to research and discuss bridge designs in other countries.

Possible Misconceptions:

Students may not be familiar with some of the famous bridge failures. There are failures from the past that were catastrophic and resulted in human casualties. It's important for students to understand the safety concerns associated with bridge design.

Lesson Procedure:

5E Model	5E Objectives
<u>Engage</u>	Procedure: Do Now: Why is a triangle a good shape for a bridge truss? Students write their own response to the prompt on the board as they enter the room. After the Do Now, students watch a series of videos about famous bridge failures, then the teacher leads brief discussion about why it's important to build bridges safely and show a NASA bridge project completed recently. (5 to 10 minutes) Modifications: The videos will include closed captions so that students can read the dialogue while they listen to the audio. Standards Addressed: NGSS: Analyze complex real-world problems by specifying criteria and constraints for successful solutions. (HS-ETS1-1) Missouri Learning Standard: 9-12.ETS1.A.1: Analyze a major global challenge to specify qualitative and quantitative criteria and constraints for solutions that account for societal needs and wants. ITEEA Standards: 1N. Explain how the world around them guides technological development and engineering design. 4P. Evaluate ways that technology can impact individuals, society, and the environment. 5H. Evaluate a technological innovation that arose from a specific society's unique need or want.

	6F. Relate how technological development has been evolutionary, often the result of a series of refinements to basic inventions or technological knowledge. Formative/Summative Assessments: Teacher will do an informal assessment of prior knowledge based on Do Now prompt. Resources: https://www.nasa.gov/image-article/kennedy-space-centers-nasa-causeway-bridge-construction/ https://www.wsp.com/en-us/projects/the-nasa-causeway-bridge-gateway-to-the-universe https://www.cflroads.com/project/440424-1 https://www.smithsonianmag.com/history/seven-of-the-worst-bridge-disasters-in-world-history-180984032/
<u>Explore</u>	Procedure: Students will build their own bridge truss out of balsa wood, then test various loads to determine the maximum amount supported by the truss. The teacher will monitor students as they build their trusses and provide support when necessary. After students test the structures, there will be a discussion where students reflect on the process. Modifications: Teacher will provide popsicle sticks for students that struggle to build with balsa wood. Group work is also allowed for students that struggle to build their own truss. Standards Addressed: NGSS: HS-ETS1-2. Design a solution to a complex real-world problem by breaking it down into smaller, more manageable problems that can be solved through engineering. 7W. Determine the best approach by evaluating the purpose of the design. 7Y. Optimize a design by addressing desired qualities within criteria and constraints. 7Z. Apply principles of human-centered design.

	7AA. Illustrate principles, elements, and factors of design. 7BB. Implement the best possible solution to a design. 7CC. Apply a broad range of design skills to their design process. 8P. Apply appropriate methods to diagnose, adjust and repair systems to ensure precise, safe, and proper functionality. Formative/Summative Assessments: The teacher will evaluate the truss designs after students finish building them. Resources: Project Lead the Way Activity 4.1.7 (Method of Joints): Students can view images of truss designs in the on-line curriculum.
<u>Explain</u>	Procedure: The teacher will model how to calculate the forces acting on a 3-membered bridge truss using the method of joints approach. Students will take their own notes which they will turn in for class participation points. Modifications: The teacher will allow extended time for students that struggle to complete notes during class time. Standards Addressed: Common Core Math: CCSS.MATH.CONTENT.HSG.SRT.D.11(+) Understand and apply the Law of Sines and the Law of Cosines to find unknown measurements in right and non-right triangles (e.g., surveying problems, resultant forces). ITEAA: 2V. Analyze the stability of a technological system and how it is influenced by all the components in the system. 9-12.PS2.A.1: Analyze data to support and verify the concepts expressed by Newton's 2nd law of motion, as it describes the mathematical relationship among the net force on a macroscopic object, its mass, and its acceleration.

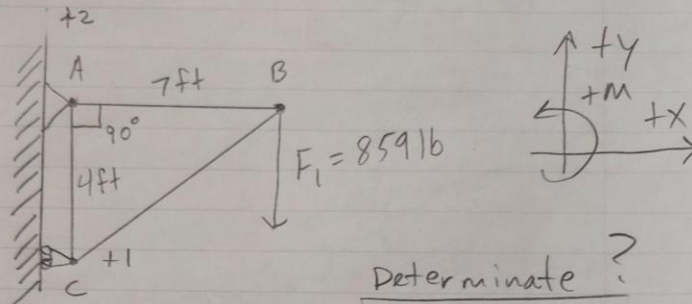
	9-12.PS2.A.2: Use mathematical representations to support and verify the concepts that the total momentum of a system of objects is conserved when there is no net force on the system. 9-12.PS2.A.3: Apply scientific principles of motion and momentum to design, evaluate, and refine a device that minimizes the force on a macroscopic object during a collision. Formative/Summative Assessments: The teacher will check the class notes submitted by students after class ends. Students will get participation points for completing the notes. Resources: Project Lead the Way Activity 4.1.7 (Method of Joints): Students can view examples of calculations in the on-line curriculum.
<u>Elaborate</u>	Procedure: The teacher will model how to design a bridge using the Bridge Designer software. Students will be tasked to design their own bridge with a challenge to keep the budget below a specified amount (The software calculates the cost automatically). Modifications: Students are allowed to work with a partner if they are struggling to design a bridge on their own. Extended time will be given for students that need extra time to complete the assignment. Standards Addressed HS-ETS1-2. Design a solution to a complex real-world problem by breaking it down into smaller, more manageable problems that can be solved through engineering. HS-ETS1-3. Evaluate a solution to a complex real-world problem based on prioritized criteria and trade-offs that account for a range of constraints, including cost, safety, reliability, and aesthetics as well as possible social, cultural, and environmental impacts. HS-ETS1-4. Use a computer simulation to model the impact of proposed solutions to a complex real-world problem with numerous criteria and constraints on interactions within and between systems relevant to the problem.

	9-12.ETS1.B.2 Use a computer simulation to model the impact of proposed solutions to a complex real-world problem with numerous criteria and constraints on interactions within and between systems relevant to the problem Formative/Summative Assessments: Students will submit a screen shot of their bridge design that includes the amount of money spent to build the bridge. The teacher will check the designs to ensure the bridges meet the criteria in the instructions. Resources: https://bridgedesigner.org/
<u>Evaluate</u>	Procedure: Students are given a 3-membered truss with different characteristics from the one in the example calculation and tasked to determine forces acting on the truss using the method of joints approach. Students will also write a reflection on the lesson including claims, evidence, reasoning, and conclusion. Modifications: Students that struggle to solve the calculations will be given extended time to complete the assignment. The teacher can also provide one-on-one assistance to help students that get stuck on solutions to the calculations. Standards Addressed: Common Core Math: CCSS.MATH.CONTENT.HSG.SRT.D.11(+) Understand and apply the Law of Sines and the Law of Cosines to find unknown measurements in right and non-right triangles (e.g., surveying problems, resultant forces). 9-12.ETS1.A.2: Design a solution to a complex real-world problem by breaking it down into smaller, more manageable problems that can be solved through engineering. 9-12.ETS1.B.1: Evaluate a solution to a complex real-world problem based on prioritized criteria and trade-offs that account for a range of constraints, including cost, safety, reliability, and aesthetics as well as possible social, cultural, and environmental impacts. ITEAA: 2V. Analyze the stability of a technological system and how it is influenced by all the components in the system. CCSS.ELA-LITERACY.W.9-10.1.A

	<i>Introduce precise claim(s), distinguish the claim(s) from alternate or opposing claims, and create an organization that establishes clear relationships among claim(s), counterclaims, reasons, and evidence.</i> CCSS.ELA-LITERACY.W.9-10.1.B <i>Develop claim(s) and counterclaims fairly, supplying evidence for each while pointing out the strengths and limitations of both in a manner that anticipates the audience's knowledge level and concerns.</i> CCSS.ELA-LITERACY.W.9-10.1.C <i>Use words, phrases, and clauses to link the major sections of the text, create cohesion, and clarify the relationships between claim(s) and reasons, between reasons and evidence, and between claim(s) and counterclaims.</i> CCSS.ELA-LITERACY.W.9-10.1.D <i>Establish and maintain a formal style and objective tone while attending to the norms and conventions of the discipline in which they are writing.</i> CCSS.ELA-LITERACY.W.9-10.1.E <i>Provide a concluding statement or section that follows from and supports the argument presented.</i> Formative/Summative Assessments: The teacher will grade the calculations and the written reflection based on the rubric at the end of this document. Resources: Project Lead the Way Activity 4.1.7 (Method of Joints).
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Teacher Background: This lesson is based on a lesson in the Project Lead the Way curriculum. The lesson requires knowledge of trigonometry, vector forces, and the method of joints approach to truss analysis. Use the notes in the images shown below as an example of the method of joints approach to truss analysis.

Example Truss 1 (3 member, 1 load)



Determinate ?

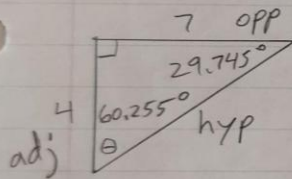
$$J = 3$$

$$M = 3$$

$$R = 3$$

$$2(3) = 3 + 3 \quad \checkmark$$

$$6 = 6$$



$$\tan \theta = \frac{\text{opp}}{\text{adj}}$$

$$\tan \theta = \frac{7}{4}$$

$$\theta = \tan^{-1}\left(\frac{7}{4}\right)$$

$$\theta = 60.255^\circ$$

$$\theta + \phi = 90$$

$$60.255^\circ + \phi = 90^\circ$$

$$\phi = 29.745^\circ$$

2

FBD

Calculating the external reaction forces

$\sum \vec{M} = 0$ (around A) $M = F \cdot d_1$

$\vec{M}_{R_{AX}} + \vec{M}_{R_{AY}} + \vec{M}_{F_1} + \vec{M}_{R_{cx}} = 0$

$R_{AX}(0) + R_{AY}(0) - 859 \text{ lb}(7 \text{ ft}) + R_{cx}(4 \text{ ft}) = 0$

$0 + 0 - 6013 \text{ lb} \cdot \text{ft} + (4 \text{ ft})R_{cx} = 0$

$4 \text{ ft}(R_{cx}) = 6013 \text{ lb} \cdot \text{ft}$

$R_{cx} = 1503.25 \text{ lb} \quad \checkmark$

3

FBD $\rightarrow$
 $\sum F_x = 0$

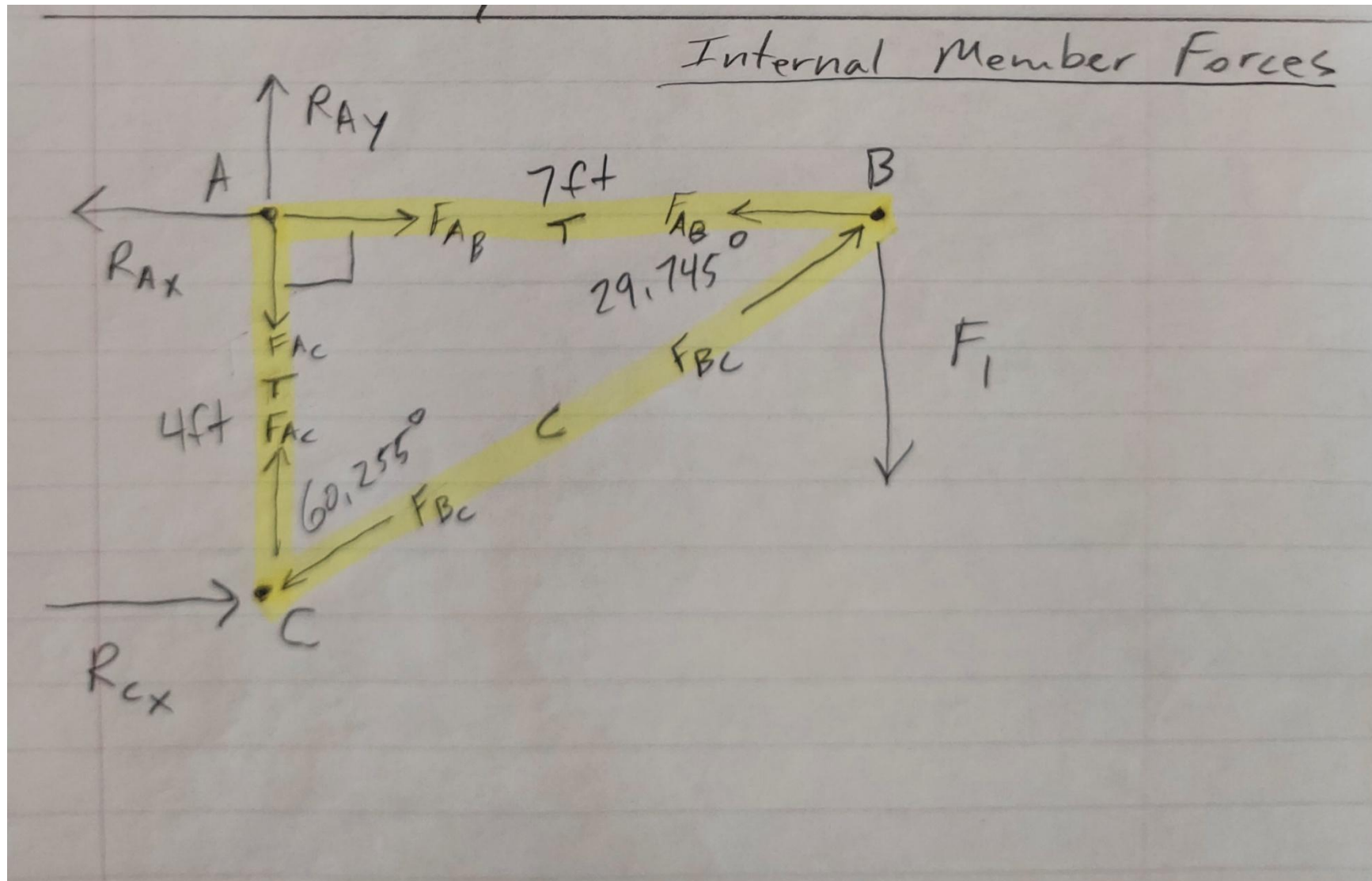
$$-R_{Ax} + 1503.25 \text{ lb} = 0$$

$$R_{Ax} = 1503.25 \text{ lb}$$

$\rightarrow$
 $\sum F_y = 0$

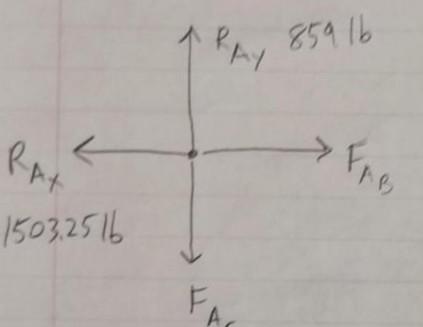
$$+R_{Ay} - 859 \text{ lb} = 0$$

$$R_{Ay} = 859 \text{ lb}$$



4

FBD
Joint + A



A free body diagram of Joint + A. A central point has four force vectors acting on it: an upward vector labeled R_{Ay} 859 lb, a downward vector labeled F_{Ac} , a leftward vector labeled R_{Ax} 1503.25 lb, and a rightward vector labeled F_{AB} .

$\sum \vec{F}_x = 0$

$$-1503.25 \text{ lb} + F_{AB} = 0$$
$$F_{AB} = 1503.25 \text{ lb}$$

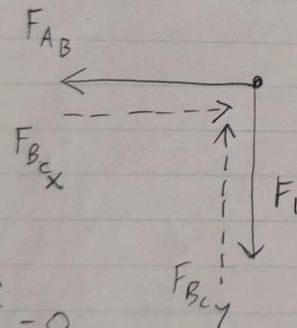
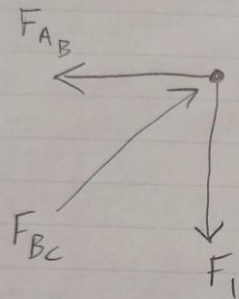
$\sum \vec{F}_y = 0$

$$859 \text{ lb} - F_{Ac} = 0$$
$$F_{Ac} = 859 \text{ lb}$$

5

FBD

Joint B



$$\sum \vec{F}_x = 0$$

$$F_{BCx} - F_{Ax} = 0$$

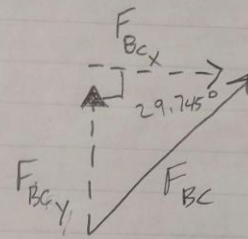
$$F_{BCx} - 1503.2516 = 0$$

$$F_{BCx} = 1503.2516$$

$$\sum \vec{F}_y = 0$$

$$F_{BCy} - 85916 = 0$$

$$F_{BCy} = 85916$$



$$F_{BC}^2 = F_{BCx}^2 + F_{BCy}^2$$

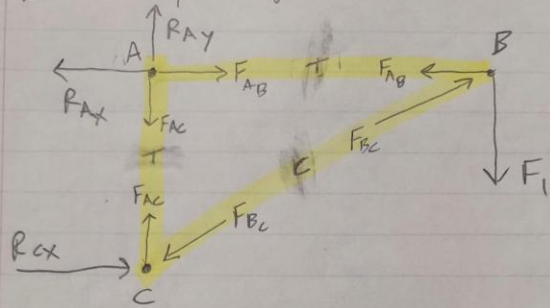
$$F_{BC} = \sqrt{F_{BCx}^2 + F_{BCy}^2}$$

$$F_{BC} = \sqrt{(1503.25)^2 + (859)^2}$$

$$F_{BC} = 1731.3716$$

6

Free Body Diagram Summary



Forces

<u>Symbol</u>	<u>Magnitude</u>	<u>Direction</u>
$\vec{F}_1$	859 lb	-y
$\vec{R}_{Ax}$	1503.25 lb	-x
$\vec{R}_{Ay}$	859 lb	+y
$\vec{R}_{Cx}$	1503.25 lb	+x
$\vec{F}_{AB}$	1503.25	T
$\vec{F}_{AC}$	859 lb	T
$\vec{F}_{BC}$	1731.37 lb	C

The following link can provide some additional guidance for teachers:

<https://skyciv.com/docs/tutorials/truss-tutorials/tutorial-for-truss-method-of-joints/>

Lesson Rubric:

	Advanced	Proficient	Basic
Build Seven-membered Truss with Craft Sticks	Built seven-membered truss capable of supporting multiple loads.	Built seven-membered truss that failed to support any load.	Attempted to build a truss but did not follow instructions properly.
Use Method of Joints Calculations to Analyze Truss Structure	Successfully used the method of joints calculation to analyze more than 90% of the forces acting on bridge truss.	Used method of joints calculation to determine more than 70% of the forces acting on bridge truss.	Calculated less than 50% of the forces acting on the bridge truss.
Design Bridge using Bridge Designer Software	Designed bridge that can support a load under the assigned budget.	Designed bridge that can support a load without meeting the budget.	Attempted to design bridge without supporting a load.
Written Reflection of Lesson	Implemented claim, evidence, and reasoning approach with clear language to reflect on lesson including a conclusion.	Attempted to implement claim, evidence, and reasoning approach to reflect on lesson without a clear argument or conclusion.	Did not provide claim, evidence, and reasoning approach or conclusion in the lesson reflection.