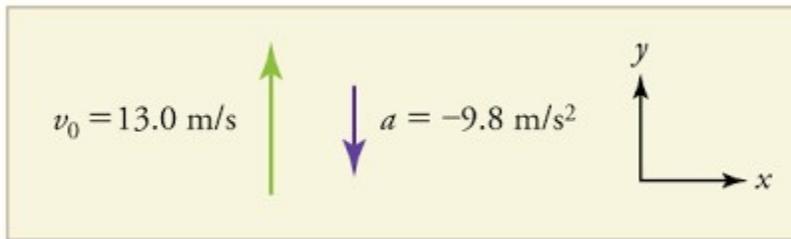


A cannon shoots a projectile off of a 5m high platform. The projectile is travelling at 20m/s at an angle of 60 degrees. How far does the projectile go before making contact with the ground?

How far will it fly worksheet

The distance an object will travel is influenced by several different factors. We will look at each of those factors individually before making connections between them to determine the distance an object will travel.

Firstly an object is always affected by the pull of gravity. From the time it leaves its station to its resting place gravity is always accelerating the object downward. This as a result is factored into our calculations for acceleration in the Y direction. Since the direction is down toward the Earth we give gravity the negative sign. The Earth pulls on all objects the same which is -9.8m/s^2 . Please view the picture below:



Notice that velocity in the upward direction is positive.

Next the velocity will also affect the range of your projectile. We will demonstrate this by using the software virtual cannon program via the link below:

https://phet.colorado.edu/sims/html/projectile-motion/latest/projectile-motion_en.html

To begin we will set the angle at 45 degrees by clicking and dragging the cannon until the end is pointed in the 45 degree angle. The height of the cannon will stay at ground level as well and we'll only adjust the velocity via the scroll bar below the cannon.

Trial	Angle	Height	Velocity of Projectile	Distance
1	45	0	3	
2	45	0	6	
3	45	0	9	
4	45	0	12	

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5	45	0	15	
6	45	0	18	
7	45	0	21	
8	45	0	24	
9	45	0	27	
10	45	0	30	

Using the cross-arrows/ blue box next to the measuring tape, align the cross up with the impact dot of the projectile to get the distance. Continue for all velocities.

Learning check

1. How did changing the velocity affect the amount of distance the object traveled?

The next thing that can affect distance is the angle of the projectile or the angle at which the velocity is travelling. Similarly to above we can run trials to observe how the angle will change the distance of an object.

Trial	Angle	Height	Velocity of Projectile	Distance
1	90	0	15	
2	80	0	15	
3	70	0	15	
4	60	0	15	
5	50	0	15	
6	45	0	15	
7	40	0	15	

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8	30	0	15	
9	20	0	15	
10	10	0	15	

This time the only thing affecting the distance of the projectile is the angle.

Learning Check

- How did changing the angle affect the amount of distance of the projectile? What angle would get you the farthest amount of distance?

The next variable we will look at is the height of the cannon and how that might affect the distance of the projectile. Did placing the cannons and catapults on the castle walls really give the defenders an advantage? We can determine this by using the same method above. This time we are going to keep the velocity at 15m/s and the angle at 60 degrees. The height will be adjusted by dragging the cannon upwards.

Trial	Angle	Height	Velocity of Projectile	Distance
1	60	0	15	
2	60	5	15	
3	60	6	15	
4	60	7	15	
5	60	8	15	
6	60	9	15	
7	60	10	15	
8	60	11	15	
9	60	12	15	
10	60	13	15	

Learning Check

A cannon shoots a projectile off of a 5m high platform. The projectile is travelling at 20m/s at an angle of 60 degrees. How far does the projectile go before making contact with the ground?

3. How did changing the height of the cannon impact the distance of the projectile?

Finally there are other factors that determine distance as well such as propulsion and design of the projectile. By looking at the graphical data below from NASA's website please answer the following questions:

<https://www.grc.nasa.gov/WWW/K-12/rocket/rktenglab.html>

4. How did the amount of thrust (velocity) affect the amount of time the rocket travelled?

Putting everything together

In order to calculate the distance of the projectile we need to account for all the different factors above and we can do that with a few equations:

$$X_f = X_i + V_x(T) + \frac{1}{2}A_x(T)^2$$

$$X_f = V_x(T)$$

$$Y_f = Y_i + V_y(T) + \frac{1}{2}A_y(T)^2$$

$$V_x = \cos(\text{angle}) * V$$

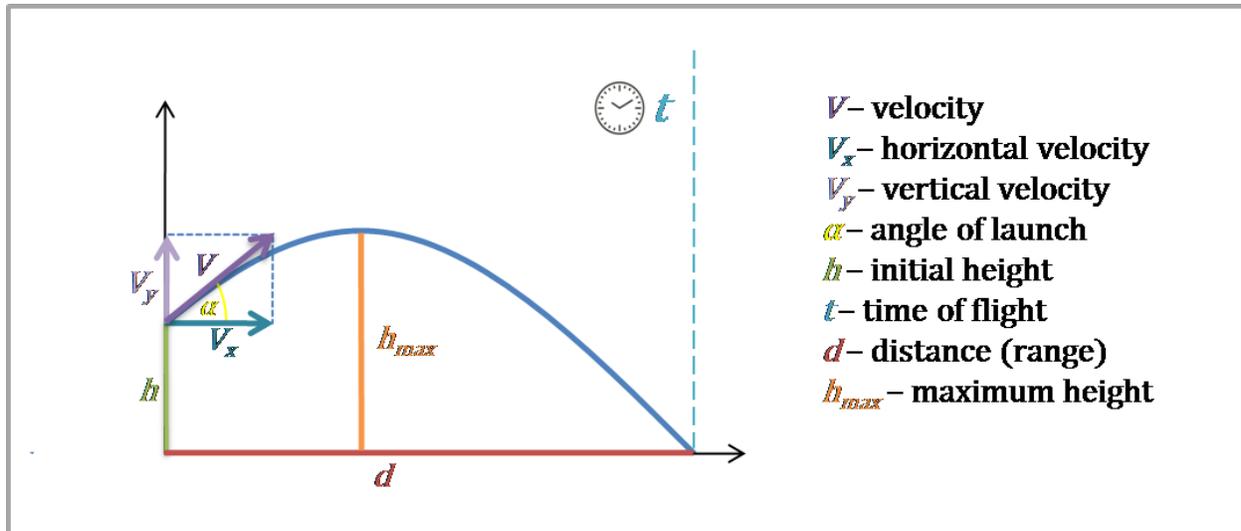
$$V_y = \sin(\text{angle}) * V$$

Example problem:

A cannon shoots a projectile off of a 5m high platform. The projectile is travelling at 20m/s at an angle of 60 degrees. How far does the projectile go before making contact with the ground?

A cannon launches a projectile from a platform 10m in height. The velocity of the projectile is 15m/s at an angle of 50 degrees. How far did the projectile travel before making contact with the ground.

The first thing I would do is draw a picture of what is going on and list out the variables that we know. Each variable in the equations are listed as letters and you can view the picture below to see which variables correspond to each letter.



To begin with this problem we will list out some knowns.

- $V = 15\text{m/s}$
- $H = 10\text{m}$
- $H_f = 0\text{m}$ (ground)
- $A_y = -9.8\text{m/s}$ (gravity always 9.8 with respect to Earth)
- Angle = 50 degrees
- $D_i = 0\text{m}$
- $D_f = ?$
- $T = ?$
- $V_x = ?$
- $V_y = ?$

Once all the variables known are listed you can then use the equations to determine what you don't know. For example we will first determine the V in the x and y directions with these two equations:

$$V_x = \cos(50) 15\text{m/s} \dots \dots \dots V_x = 9.64\text{m/s}$$

$$V_y = \sin(50) 15\text{m/s} \dots \dots \dots V_y = 11.49\text{m/s}$$

A cannon shoots a projectile off of a 5m high platform. The projectile is travelling at 20m/s at an angle of 60 degrees. How far does the projectile go before making contact with the ground?

Now we can fill in with our variables that we know:

$$V = 15\text{m/s}$$

$$Y_0 = 10\text{m}$$

$$Y_f = 0\text{m (ground)}$$

$$A_y = -9.8\text{m/s}^2 \text{ (gravity always } 9.8 \text{ with respect to Earth)}$$

$$\text{Angle} = 50 \text{ degrees}$$

$$D_i = 0\text{m}$$

$$D_f = ?$$

$$T = ?$$

$$V_x = 9.64\text{m/s}$$

$$V_y = 11.49\text{m/s}$$

To finally determine distance we need to determine the amount of time the projectile was in the air. In order to determine that we use this equations since we now know V_y .

$$Y_f = Y_0 + V_y(T) + \frac{1}{2}A_y(T)^2$$

Filling in with our variables we get

$0 = 10\text{m} + 11.49(T) + -4.9(T)^2$ *looking close this is a quadratic equation problem to determine T. This would make sense as all projectiles form a parabola when travelling their trajectory in the air.

After using the quadratic equation we determine T to be equal to. T = 3.02s

Now that we know T we can solve for the distance.

$$D_f = V_d(T)$$

$$D = 9.64 \times 3.02 \dots \dots \dots D \text{ or } X \text{ is equal to } 29.11\text{m}.$$

To double check, set the parameters of the cannon animation to match the variables known. Then move the target to 17m and see if our calculations are correct.

Now you try

A cannon shoots a projectile off of a 5m high platform. The projectile is travelling at 20m/s at an angle of 60 degrees. How far does the projectile go before making contact with the ground?

After you make your calculations plug the scenario into the animation and see if you hit the target.