A Mathematical Exploration of Covid 19 and Logistics Curves

COVID 19: The Novel Coronavirus pandemic has shocked the world over the past three months, causing financial meltdown alongside revealing severe weaknesses in the global healthcare system. Let's explore this phenomenon by the numbers!

Engage (5 Minutes): Explore the application developed by John Hopkins University that monitors the spread of COVID 19. Write down 3 key observations that resonate with you, and prepare to your thoughts with your table group.

Website: https://coronavirus.jhu.edu/map.html

Observation Notes:

Explore (15 Minutes): COVID 19 would best be explored through the mathematical function known as a logistic curve.

- Using sliders in Desmos to determine the best model for the infection rate curve for both Mainland China (orange) and Other Locations (yellow). There is no "correct" answer here, and answers may vary!
- What effect does each parameter in the logistics curve have?
- Make a prediction on how many cases there will be globally outside of China. Use data and the logistics curve equation to support your guess.

Desmos Link: https://www.desmos.com/calculator/fbiokj6yp6

Mainland China Curve	**Other Locations Curve**	**What effect does each parameter have?**
$L =$ ________ $a =$ ________ $b =$ ________	$L =$ ________ $a =$ ________ $b =$ ________ $c =$ ________	L: a: b: c:
Sketch the Curve of the Model	**Sketch the Curve of the Model**	

Follow-Up Questions: Think about the following, and be able to defend your case using data during group share-out.

1. Based on your model, how many total infections do you expect for 'Other Locations' outside of Mainland China?
2. What real life explanation justifies the inclusion of parameter c for 'Other locations'?
3. What policies can global governments enact to help curb the curve as seen in the Mainland China curve?

Think about these questions independently. You will have chance share your thoughts at the end of 15 minutes!

Small Group / Whole Group Discussion (5/5 minutes):
At this time, share your thoughts within your group regarding the **follow-up** questions. Add the following question to your conversation. We will come together as a whole class in 5 minutes.

- Was everyone's total infection rate for 'Other Locations' the same? If not, how do we decide which model to consider when making public policy?

Explanation (Time Depends): Let's dive deeper into the mathematics of Logistics Curves. This Section will depend on what course this lesson is taught! We will proceed from a Calculus BC Perspective, however the Calculus may be truncated and still operate as a functional lesson.

Calculus BC: Begin Here
A logistic curve is a differential equation in which rate of population growth is the product of the current population and the difference between the carrying capacity of the current population. This is commonly modeled as follows:

Logistics Curve: $\frac{dp}{dt} = kp(L-p) = Mp(1-\frac{p}{L})$ where $M = kL$

Use **partial fractions** to solve the **separable differential equation** for $p(t)$

$$\frac{dp}{dt} = kp(1-\frac{p}{L})$$

$$\frac{1}{p(1-\frac{p}{L})}dp = Mdt$$

$$\int \frac{1}{p(1-\frac{p}{L})}dp = \int Mdt$$

Partial Fractions

$$\frac{1}{p(1-\frac{p}{L})} = \frac{A}{p} + \frac{B}{(1-\frac{p}{L})}$$

$$= \frac{A\left(1-\frac{p}{L}\right) + Bp}{p\left(1-\frac{p}{L}\right)}$$

$$\frac{1+0p}{p(1-\frac{p}{L})} = \frac{p\left(B-\frac{A}{L}\right) + A}{p\left(1-\frac{p}{L}\right)}$$

$$\therefore A = 1, \qquad B - \frac{A}{L} = 0, \qquad B = \frac{1}{L}$$

Substitute A and B

$$\int \frac{1}{p} + \frac{\frac{1}{L}}{(1-\frac{p}{L})}dp = \int Mdt$$

$$u = \frac{p}{L} \Rightarrow du = \frac{1}{L}dp \Longrightarrow Ldu = dp$$

$$\ln|p| + \int \frac{\frac{1}{L}}{(1-u)}Ldu = Mt + C$$

$$\ln|p| + \int \frac{1}{(1-u)}du = Mt + C$$

$$\ln|p| - \ln\left|1-\frac{p}{L}\right| = Mt + C$$

$$\ln\left|\frac{p}{\frac{L-p}{L}}\right| = Mt + C$$

$$\ln\left|\frac{pL}{L-p}\right| = Mt + C$$

For $\frac{pL}{L-p} > 0$

$$\frac{pL}{L-p} = e^{Mt+C}$$

$$pL = e^{Mt} \cdot e^{C} \cdot (L-p)$$

$$p = L \cdot e^{Mt} \cdot C - p \cdot e^{Mt} \cdot C$$

$$p + p \cdot e^{Mt} \cdot C = L \cdot e^{Mt} \cdot C$$

$$p(1 + e^{Mt} \cdot C) = L \cdot e^{Mt} \cdot C$$

$$p = \frac{C \cdot L \cdot e^{Mt}}{1 + e^{Mt} \cdot C}$$

$$p = \frac{C \cdot L \cdot e^{Mt}}{1 + e^{Mt} \cdot C} \cdot \frac{C^{-1} \cdot e^{-Mt}}{C^{-1} \cdot e^{-Mt}}$$

$$p = \frac{L}{C^{-1} \cdot e^{-Mt} + 1}$$

$$p = \frac{L}{1 + C \cdot e^{-Mt}}$$

$$p = \frac{L}{1 + C \cdot e^{-K \cdot L \cdot t}}$$

From here we have shown that.

$$\frac{dp}{dt} = kp(L-p) = Mp(1-\frac{p}{L}) \Longrightarrow p = \frac{L}{1 + C \cdot e^{-K \cdot L \cdot t}} = \frac{L}{1 + C \cdot e^{-Mt}}$$

Let's dive further still!

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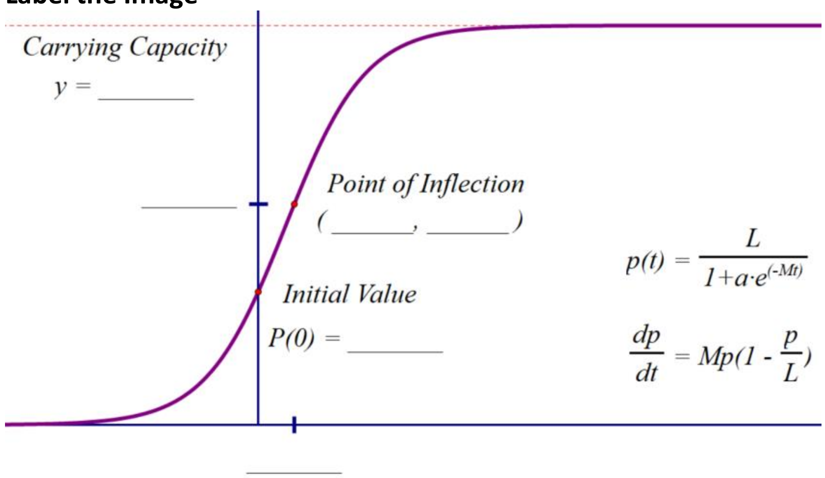
AP Calculus AB Start Here.

Note: Logistics Curve are a BC Topic Only, but serve as a great review of differential equations, which is an AB Topic.

Label the Image



I. Determine $P(0)$

II. Show $\lim_{t\to\infty} p(t) = L$

III. Show $\lim_{t\to-\infty} p(t) = 0$

IV. Show that $\frac{dp}{dt} > 0$ for all t given $M > 0$

V. Show that $\frac{d^2p}{dt^2} = 0$ at $p = \frac{L}{2}$

VI. Show that $\frac{d^2p}{dt^2} > 0$ for $p < \frac{L}{2}$ and $M > 0$

VII. VI. Show that $\frac{d^2p}{dt^2} < 0$ for $p > \frac{L}{2}$ and $M > 0$

VIII. Determine at which *x* the P.O.I is attained

Show all Steps Below!

AP Calculus Released Multiple Choice Practice!

AP Calculus BC 2017

27. The number of students in a cafeteria is modeled by the function P that satisfies the logistic differential equation $\frac{dP}{dt} = \frac{1}{2000}P(200 - P)$, where t is the time in seconds and $P(0) = 25$. What is the greatest rate of change, in students per second, of the number of students in the cafeteria?

(A) 5 (B) 25 (C) 100 (D) 200

AP Calculus BC 2016 Practice Exam

28. The function N satisfies the logistic differential equation $\frac{dN}{dt} = \frac{N}{10}\left(1 - \frac{N}{850}\right)$, where $N(0) = 105$. Which of the following statements is false?

(A) $\lim_{t \to \infty} N(t) = 850$

(B) $\frac{dN}{dt}$ has a maximum value when $N = 105$.

(C) $\frac{d^2N}{dt^2} = 0$ when $N = 425$.

(D) When $N > 425$, $\frac{dN}{dt} > 0$ and $\frac{d^2N}{dt^2} < 0$.

AP Calculus BC 2015

91. The number of students in a school who have heard a rumor at time t hours is modeled by the function P, the solution to a logistic differential equation. At noon, 50 of the school's 500 students have heard the rumor. Also at noon, P is increasing at a rate of 20 students per hour. Which of the following could be the logistic differential equation?

(A) $\frac{dP}{dt} = \frac{1}{1125}P(500 - P)$

(B) $\frac{dP}{dt} = \frac{1}{480}P(500 - P)$

(C) $\frac{dP}{dt} = \frac{1}{192}P(500 - P)$

(D) $\frac{dP}{dt} = \frac{2}{45}P(500 - P)$

(E) $\frac{dP}{dt} = \frac{5}{48}P(500 - P)$

AP Calculus BC 2015

13. A population of wolves is modeled by the function P and grows according to the logistic differential equation $\frac{dP}{dt} = 5P\left(1 - \frac{P}{5000}\right)$, where t is the time in years and $P(0) = 1000$. Which of the following statements are true?

I. $\lim_{t \to \infty} P(t) = 5000$

II. $\frac{dP}{dt}$ is positive for $t > 0$.

III. $\frac{d^2P}{dt^2}$ is positive for $t > 0$.

(A) I only

(B) II only

(C) I and II only

(D) I and III only

(E) I, II, and III

AP Calculus 2014 BC

77. The number of antibodies y in a patient's bloodstream at time t is increasing according to a logistic differential equation. Which of the following could be the differential equation?

(A) $\frac{dy}{dt} = 0.025t$

(B) $\frac{dy}{dt} = 0.025t(5000 - t)$

(C) $\frac{dy}{dt} = 0.025y$

(D) $\frac{dy}{dt} = 0.025(5000 - y)$

(E) $\frac{dy}{dt} = 0.025y(5000 - y)$

AP Calculus BC 2012

14. Let k be a positive constant. Which of the following is a logistic differential equation?

(A) $\frac{dy}{dt} = kt$

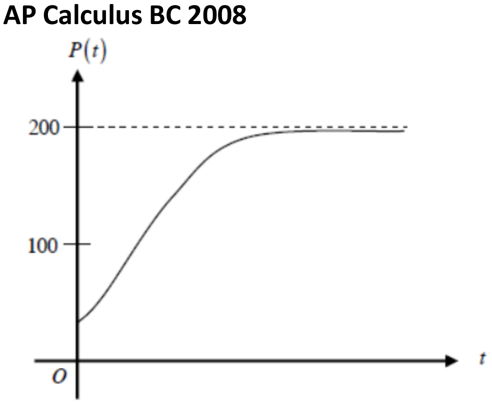
(B) $\frac{dy}{dt} = ky$

(C) $\frac{dy}{dt} = kt(1 - t)$

(D) $\frac{dy}{dt} = ky(1 - t)$

(E) $\frac{dy}{dt} = ky(1 - y)$

AP Calculus BC 2008



24. Which of the following differential equations for a population P could model the logistic growth shown in the figure above?

(A) $\frac{dP}{dt} = 0.2P - 0.001P^2$

(B) $\frac{dP}{dt} = 0.1P - 0.001P^2$

(C) $\frac{dP}{dt} = 0.2P^2 - 0.001P$

(D) $\frac{dP}{dt} = 0.1P^2 - 0.001P$

(E) $\frac{dP}{dt} = 0.1P^2 + 0.001P$

A Mathematical Exploration of Covid 19 and Logistics Curves

Elaboration/Extension (1 Week): Your next task is a short project! The expectations of the project are below.

- Find a connection to logistics curve in one of your other classes, such as economics, biology, chemistry, social studies, …
- Use collected data from that connection to build a logistic curve
- Feel free to use technology to calculate a logistics curve regression
- Prepare a presentation to discuss the connection you found
- Include any socially relevant topics related to your logistics curve.
- What would you like to know more about, given the limitations of the data? How could you improve the data collection process?
- Produce a product that could be presented to a small group of about 5-7. Examples include a poster, ppt, word doc, …

The project presentation format will be as follows.

1. 4 student presenters will position themselves around the room
2. Presenters will present their project to a small audience of their peers for 5 minutes.
3. The audience will rotate 1 time, then the presenters will present again.
4. After presenting to two groups, the presenters will now join the audience.
5. 4 new presenters will position themselves around the room.

May The Force be with you!