

# Jesiah Jeffers - Mandatory 5 Unit Development

Subject: High School Geometry

Unit: Measurement and Modeling in 2 and 3 Dimensions

## A. Statement of Purpose.

This unit is designed to have students become skilled at...

1. Finding the volume and surface area of any prism, cylinder, pyramid, cone, or sphere.
2. Understanding how scale factor works within a model.
3. Breaking a 3-D model into its 2-D elements.
4. Modeling with surface area and volume in mind to represent real world phenomena.

Lesson 6 brings all of these concepts together with a NASA science lesson. By introducing students to wingspans and flight times of planes, students engage with 2-D elements of 3-D objects to hypothesize what effects wingspan has on flight time. Students will research the science behind wingspan and connect it to the learning of the unit.

## B. Instruction:

Title: Measurement and Modeling in 2 and 3 Dimensions

Time needed: 9 class periods (45-50 min each)

Standards addressed in this unit:

### **Common Core Geometry Standards**

**G-SRT.B.5** - Use congruence and similarity criteria for triangles to solve problems and to prove relationships in geometric figures.

**G-GMD.A.3** - Use volume formulas for cylinders, pyramids, cones, and spheres to solve problems.

**G-CO.B.6** - Use geometric descriptions of rigid motions to transform figures and to predict the effect of a given rigid motion on a given figure; given two figures, use the definition of congruence in terms of rigid motions to decide if they are congruent.

**G-GMD.A.1** - Give an informal argument for the formulas for the circumference of a circle, area of a circle, volume of a cylinder, pyramid, and cone.

**G-CO.D.12** - Make formal geometric constructions with a variety of tools and methods (compass and straightedge, string, reflective devices, paper folding, dynamic geometric software, etc.).

**G-SRT.B.5** - Use congruence and similarity criteria for triangles to solve problems and to prove relationships in geometric figures.

### **Next Generation Science Standards**

**HS-ESS1-1** - Develop a model based on evidence to illustrate the relationships between systems or between components of a system.

**HS-ESS1-4** - Use mathematical or computational representations of phenomena to describe explanations.

**HS-ESS1-2** - Construct an explanation based on valid and reliable evidence obtained from a variety of sources (including students' own investigations, theories, simulations, peer review) and the assumption that theories and laws that describe the natural world operate today as they did in the past and will continue to do so in the future.

Essential Questions Addressed in this Unit:

1. How are volume formulas related to real world phenomena?

2. How can visualizing solids help you solve real world problems?
3. How can you represent a 3-D model using 2-D elements?
4. How can you use modeling to represent and understand real world phenomena?

## **Lesson Activities:**

### ● **Lesson 1 & 2 - Introduction to Volume**

*Essential Question Addressed: How are volume formulas related to real world phenomena?*

*Lesson Objectives:*

1. Students will know the formulas for the volume of a prism, cylinder, pyramid, cone, and sphere.
2. Students will begin to analyze models in 3 dimensions.

Students will be introduced to 2-D and 3-D objects through the PACKING PROBLEM. Students will individually create and analyze diagrams to determine the best possible way to pack toys into a suitcase. The teacher will monitor and assess the students ability to think in 2 and 3 Dimension together and student ability to diagram models. Students will calculate basic volume of a rectangular prism (suitcase). After the activity, students pair up and share their diagrams and answers with a classmate and make any adjustments they need to. The teacher leads a class discussion on the activity and introduces the theme of the unit: Measuring and Modeling in 2 and 3 Dimensions. The teacher prompts and lists the names of all 3-D shapes that the class can think of (prisms, cylinders, pyramids, cones, spheres). Students draw examples of each type and write the volume formula down for each shape.

### ● **Lesson 3 & 4 - Cross Sections & Solids of Rotation**

*Essential Question Addressed: How can you represent a 3-D model using 2-D elements?*

*Lesson Objectives:*

1. Students will know cross sections and solids of rotation.
2. Students will understand the nets of 3 dimensional figures to analyze them in 2 dimensions.

Students are introduced to Nets for 3-D objects through a matching activity. Students will work in groups of 3 or 4. They are given an envelope of 20 different nets/3-D shape pairs. Students will be given 10 minutes to identify as many pairs as they can as a group. One student will record the groups answers on the handout. The teacher will monitor student activity and offer as little help at this point as possible. After 10 minutes, the teacher will show the correct pairings and lead a discussion on nets of 3-D figures. The teacher will then introduce 2 other ways to understand 3-D objects in 2-D (cross sections and rotating solids). Students will do a Geogebra program on chromebooks that will lead students through creating cross sections of different shapes and exploring what they look like in 2-D as well as taking any 2-D figure and rotating it about an axes to explore what 3-D shapes they can make. They will be responsible to find what shapes rotating around an axes make the basic 3-D shapes that we will be analyzing this unit.

### ● **Lesson 5 & 6 - Surface Area and Relating 2-D to 3-D**

*Essential Question Addressed: How can you use modeling to represent and understand real world phenomena?*

*Lesson Objectives:*

- 1. Students will model real world phenomena.*
- 2. Students will understand that surface area and volume are related between different aspects of 3 dimensional objects.*
- 3. Students will know the formulas for the surface area of a prism, cylinder, pyramid, cone, and sphere.*

The teacher will begin the lesson with the nets from the previous lesson. The teacher will lecture on how the area of each piece of the net added together is calculating the surface area of the 3 Dimensional figure. The teacher will demonstrate how to calculate the surface area for all of the basic figures (prism, cylinder, pyramid, cone, and sphere). Students will then calculate the surface area for the rest of the figures from the net lesson themselves. The teacher will provide help and guide students that are struggling at this point. Students will pick a figure they made from their previous Geogebra activity in 3-D and draw the net and calculate the surface area for their figure. They will check their work in the Geogebra program for and make any corrections to their nets and calculations as needed.

The teacher will do a brief presentation on wing span and surface area and explain the activity. Students will research via the NASA education site how different areas of different wing span shapes affect the flight time of different planes. They will develop and test hypotheses using paper airplanes and trial flights. Students will graph their results and come up with their conclusions based on their models flight times. This lesson will end with a survey of the project. \*Depending on the results, there may be a need for a review lesson.

## ● Lesson 7 & 8 - Scale Factor

*Essential Question Addressed: How can visualizing solids help you solve real world problems?*

*Lesson Objectives:*

- 1. Students will be able to define scale factor.*
- 2. Students will understand how scale factor works within a model.*

Students will explore the effects of changing different dimensions of the 2-D pieces of objects on perimeter, area, and volume (3-D). The teacher will lead guided practice for changing dimensions and graphing the change in 2-D and 3-D. Students will fill in their guided notes. The teacher will then have the students grab individual white boards and give students practice problems and prompting the students and checking for understanding. Students will hold up their whiteboards and receive correction where needed. The teacher will then introduce the group art project. Students will be individually given a part of a photo (small enough section to not tell what the picture is). Students will scale their own photo up by a factor of 5 and draw it. They will then group with class members who were given other parts of the same photo. If done right, the students should be able to puzzle together a large scale photo.

## ● Lesson 9 - Modeling to Meet Constraints

*Essential Questions Addressed:*

*How are volume formulas related to real world phenomena?*

*How can visualizing solids help you solve real world problems?*

*How can you represent a 3-D model using 2-D elements?*

*How can you use modeling to represent and understand real world phenomena?*

*Lesson Objectives:*

*1. Students will determine which 2 dimensional and 3 dimensional aspects to analyze to solve a real world problem.*

Students will be assessed through a more traditional math test over volume and surface area formulas and their uses, analyzing and diagramming nets, scale factor, and diagrams of cross sections. The second part of the end of the unit assessment will be a performance task. The teacher will introduce the Humboldt Manufacturing Problem: HM produces small connectors (shown in a diagram). These are manufactured by fusing two 3-centimeter-tall cylinders to a rectangular prism. Each piece is made up of solid material. Students will draw and express different cross sections, find the volume and surface area of various parts of the figure, and research the densest material it could be made of in order for the connector not to exceed 0.2kg.

**Materials:**

See below for created worksheets for each lesson. \*(I still need to create a couple more of them for when I teach this unit).

Packing Problem - Lesson 1&2

Name \_\_\_\_\_

You are trying to pack in preparation for a trip and need to fit a collection of children's toys in a box. Each individual toy is a composite figure of four cubes, and all of the toys are shown in the figure. Arrange the toys in an orderly fashion so that they will fit in the

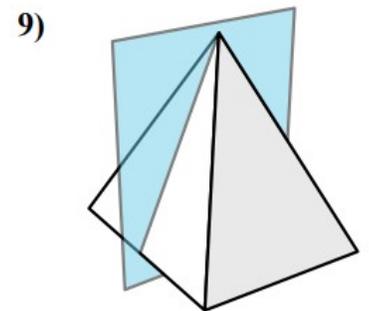
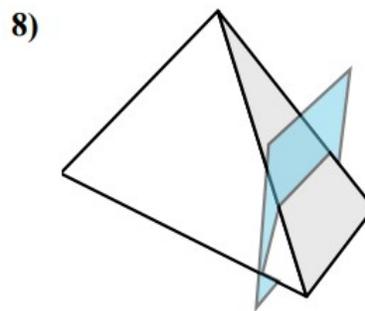
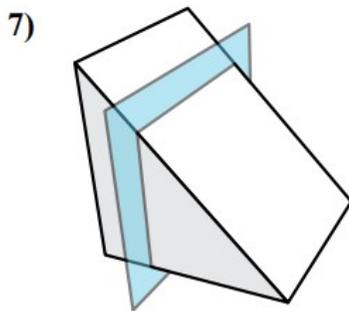
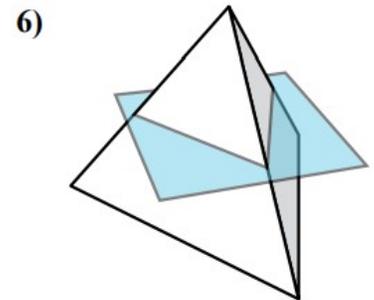
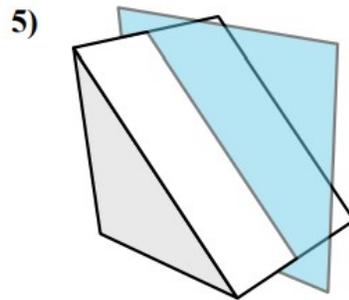
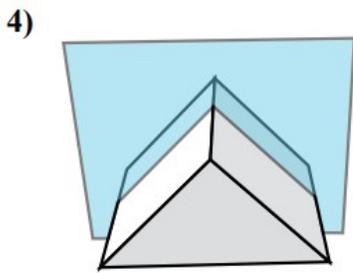
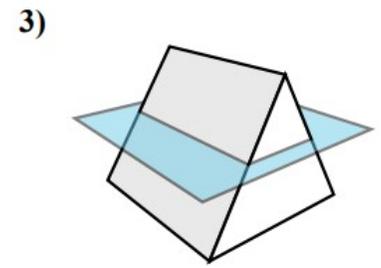
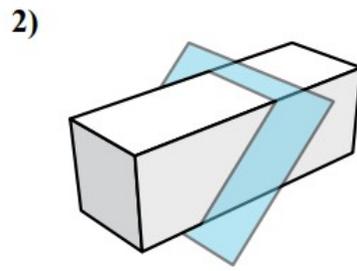
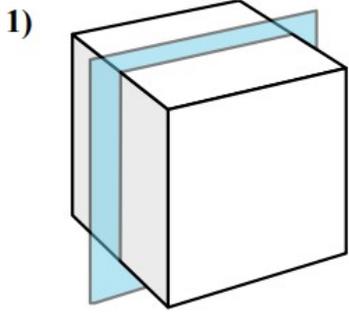
smallest box possible.

Draw the arrangement.

What is the volume of the box if each of the cubes have side lengths of 10 cm?

Cross Sections: From 3-D to 2-D - Lesson 3&4

Draw the 2-D shape created by the cross section of each solid.



Area and Flying Survey - Lesson 5&6

Name \_\_\_\_\_

Please answer the following questions to the best of your ability.

1. What did you learn in this project (explain)?

2. What concept was new and how would you explain it?

3. What are some suggestions for improving this project for the future?

4. If you were to design your own paper airplane, what would the shape of your wings be and why?

Area and Flying Data and Graph - Lesson 6

Name \_\_\_\_\_

Data Collection:

**Your Data:**

Wings - Take a look at the wings of your paper airplane. Break down the wings into basic geometric shapes. Draw the shapes below and find the area of the shapes in square centimeters. Then determine the total surface area of your wings.

Basic Shape Break Down	Areas	
	Piece	Area
Total Area _____ cm <sup>2</sup>		

**Flight Trials Part 1:**

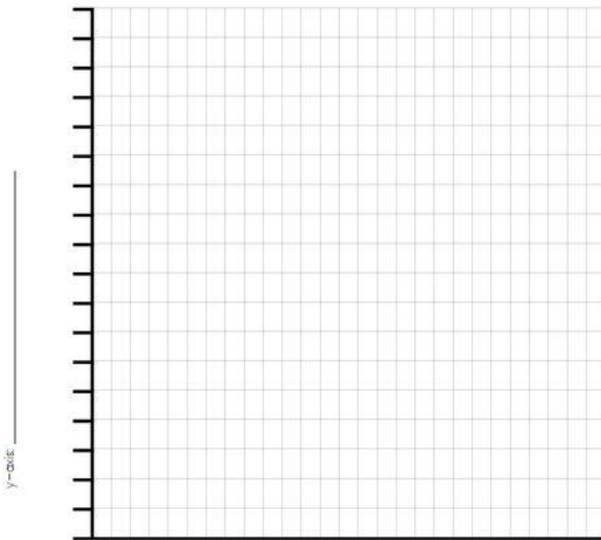
Trial	Distance Trial	Time Aloft Trial
1	cm	seconds
2	cm	seconds
3	cm	seconds
4	cm	seconds
5	cm	seconds

Top 3 Total	cm	seconds
Average	cm	seconds

**Class Data:**

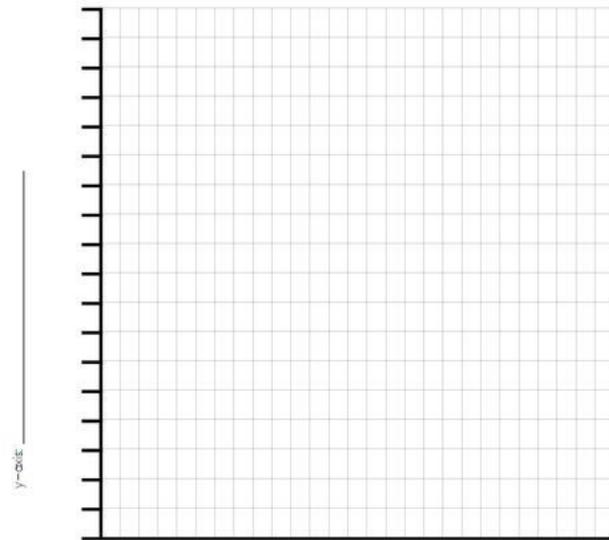
Plot your data on the \_\_\_\_\_ class graphs. Copy the data from your classmates onto the two graphs below. Be sure to label your axes and scale the graphs.

Title: \_\_\_\_\_



x-axis: \_\_\_\_\_

Title: \_\_\_\_\_



x-axis: \_\_\_\_\_

1. What type of equation would best fit our data (explain)?

Distance Data -

Time Aloft Data -

2. What would happen if the wings were made smaller?

**Flight Trials Part 2:**

Trial	Distance Trial	Time Aloft Trial
1	cm	seconds
2	cm	seconds
3	cm	seconds
4	cm	seconds
5	cm	seconds

Top 3 Total	cm	seconds
Average	cm	seconds

**Resources:**

Kanold, T. D., Burger, E. B., Dixon, J. K., Larson, M. R., & Leinwand, S. (2015). *Geometry*. Orlando, FL: Houghton Mifflin Harcourt Publishing Company.

- T. B. (n.d.). Factors that Affect Lift. Retrieved June 4, 2019, from <https://www.grc.nasa.gov/www/k-12/VirtualAero/BottleRocket/airplane/factors.html>
- T. B. (2014, June 12). How to Make a Dart Airplane. Retrieved June 4, 2019, from [https://www.grc.nasa.gov/WWW/K-12/Summer\\_Training/Elementary97/dart.html](https://www.grc.nasa.gov/WWW/K-12/Summer_Training/Elementary97/dart.html)
- Mathematics Academic Standards. (n.d.). Retrieved June 3, 2019, from <https://www.cde.state.co.us/comath/statestandards>
- T. B. (2014, June 12). Paper Airplane Activity. Retrieved June 4, 2019, from <https://www.grc.nasa.gov/WWW/K-12/aerosim/LessonHS97/paperairplaneac.html>
- T. B. (n.d.). Wing Area. Retrieved June 4, 2019, from <https://www.grc.nasa.gov/www/k-12/VirtualAero/BottleRocket/airplane/area.html>
- T. B. (n.d.). Wing Geometry Definitions. Retrieved June 4, 2019, from <https://www.grc.nasa.gov/www/k-12/VirtualAero/BottleRocket/airplane/geom.html>