

# ENGINEERING DESIGN NOTEBOOK

## Determining Instantaneous Velocity - Calculus



Caitlyn Gironda

## Table of Contents

<b>Preface to Phase I</b>	<b>3</b>
<b>Phase I- Research and Planning</b>	<b>3</b>
<b>Phase II- Implementation</b>	<b>6</b>
<b>Class Engineering Design Notebook</b>	<b>8</b>
<b>Final Teacher Reflections</b>	<b>18</b>
<b>Appendix A- Previous Phase I Document</b>	<b>21</b>
<b>Appendix B- Blank Student Engineering Notebook</b>	<b>23</b>

## Preface to Phase I

Originally, I had planned this activity to be used during our particle motion unit. Unfortunately, I missed days at school due to personal illness and had to change my implementation timeline. I instead implemented another engineering design challenge on a topic we cover earlier in the year. I have included a new Phase I for this challenge. See Appendix A for the original Phase 1 documentation.

### Phase I – Research and Planning

1. Identify the “Big” concept to be covered by the engineering design challenge.

In this challenge, students are asked to determine the instantaneous velocity of a marble at a given point. The “big” concept that will be addressed, from a Calculus perspective, is the relationship between instantaneous rates of change and average rates of change. Many students are familiar with finding an average rate of change, also known as slope  $\left(\frac{f(b)-f(a)}{b-a}\right)$ , from previous math courses and from their previous experience with kinematics in both physics and middle school science classes. In this challenge, students will brainstorm, test, and select the most precise method for approximating the instantaneous rate of change. This will serve as the foundation for our understanding of the derivative, which students must understand is the limit of the average rate of change as the size of the interval approaches 0.

2. Research appropriate learning standards associated with the topic.

**EK.1.1B1:** Numerical and Graphical Information Can be Used to Estimate Limits

**EU 2.1:** The derivative of a function is defined as the limit of a difference quotient and can be determined using a variety of strategies

**LO 2.1A:** Identify the derivative of a function as the limit of a difference quotient

**EK 2.1A1:** The difference quotient  $\frac{f(a+h)-f(a)}{h}$  and  $\frac{f(x)-f(a)}{x-a}$  express the average rate of change of a function over an interval

**EK 2.1A2:** The instantaneous rate of change of a function at a point can be expressed by  $\lim_{h \rightarrow 0} \frac{f(a+h)-f(a)}{h}$  or  $\lim_{x \rightarrow a} \frac{f(x)-f(a)}{x-a}$ , provided that the limit exists. These are common forms of the definition of the derivative and are denoted  $f'(a)$ .

**MPAC 1: Reasoning with definitions and theorems**

Students can:

- Use definitions and theorems to build arguments, to justify conclusions or answers, and to prove results
- Develop conjectures based on exploration with technology

**MPAC 2: Connecting Concepts**

- Relate the concept of limit to all aspects of calculus
- Use the connection between concepts or processes

**MPAC 3: Implementing algebraic/computational processes**

Students can:

- select appropriate mathematical strategies;

- apply technology strategically to solve problems;
- attend to precision graphically, numerically, analytically, and verbally and specify units of measure; and
- connect the results of algebraic/computational processes to the question asked.

**MPAC 4: Connecting multiple representations**

Students can:

- associate tables, graphs, and symbolic representations of functions;
- develop concepts using graphical, symbolical, or numerical representations with and without technology;
- identify how mathematical characteristics of functions are related in different representations;
- extract and interpret mathematical content from any presentation of a function (e.g., utilize information from a table of values);
- construct one representational form from another (e.g., a table from a graph or a graph from given information); and
- consider multiple representations of a function to select or construct a useful representation for solving a problem.

**MPAC 6: Communicating**

Students can:

- clearly present methods, reasoning, justifications, and conclusions;
- use accurate and precise language and notation;
- explain the meaning of expressions, notation, and results in terms of a context (including units);
- explain the connections among concepts;
- critically interpret and accurately report information provided by technology; and analyze, evaluate, and compare the reasoning of others.

3. Identify and discuss the different types of problem solving and declarative/procedure knowledge needed.

There are various types of knowledge that will be needed during this challenge. Some of the declarative knowledge that students will need includes an understanding of average rate of change as a ratio of change in the dependent variable to the change in the independent variable. They will also need knowledge of basic factors that impact motion, especially gravity and friction. Students will also need knowledge of tools since they will be selecting all of their own materials.

This challenge will require less procedural knowledge since there is not one specific path to success. Students can take a variety of approaches to make an accurate prediction, including taking measurements at various instances, using video analysis, and more. I expect that students will use a variety of techniques, build auxiliary ramps, stoppers, etc, and surprise me with their creativity!

4. Explore objectives and ancillary concepts/content covered by the project.

This topic may also address students' knowledge of particle motion. This will be a major topic as we continue in the year, so it will be interesting as a pre-assessment for those topics. Students will reason using the relationship between position and velocity in order to determine average rates of change, as well as addressing displacement, forces, and many other physics ideas.

## 5. Identify possible activities.

There are many possible activities that could be completed using this challenge. First, students can determine their most accurate estimate for the instantaneous velocity. Students may approach this in a variety of ways, so it may be useful for groups to compare their solutions to the solutions of others, then try to synthesize this information into a “best” method. Students could also determine average rates of change over various intervals and look at the limit of these as the intervals become smaller and smaller, though this would need to be much more scaffolded for students.

## 6. Select the best activity for your classroom

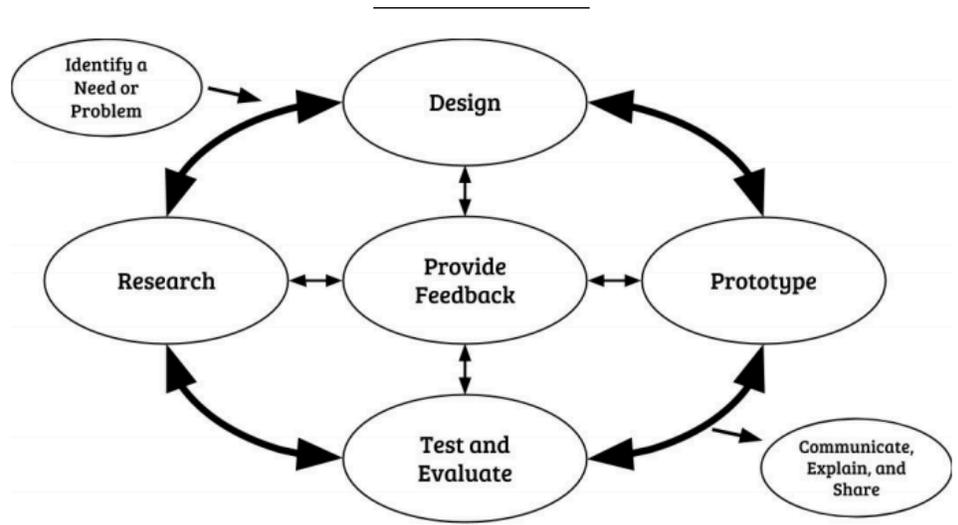
In order to give students maximum freedom to design and test, I am going to ask students to design a method to most accurately determine instantaneous velocity. Students will seek peer feedback, modify their method, and then present their approach to the class. After their presentation, we will test each method and reflect on our successes and areas for improvement. I would like to extend this into a discussion on the method that proved most efficient of all the groups and then conduct one final experiment as a class where students use the idea of a limit to approximate the instantaneous rate of change most accurately. This will tie directly into our discussion of the derivative.

## Phase II – Implementation

### Engineering Design Process

For this activity, I chose to use the Massachusetts Department of Education's Design Process. I particularly liked this one because it showed an entry and exit point to the cycle, but also emphasized that feedback was needed at all times and that the cycle did not just flow in 1 direction.

Before beginning the task, students worked both individually and collaboratively to reflect on the engineering design process and clarify it's use.



Examples of Student Work:

Write at least 3 questions you have from looking at the Engineering Design Process.

Questions	Answers
DO YOU GO FROM DESIGN → PROTOTYPE OR DESIGN → RESEARCH?	YOU CAN GO TO EITHER SIDE
WHO SHOULD YOU BE GETTING FEEDBACK FROM?	DEPENDS ON THE EXPERIMENT
SHOULD YOU MAKE MORE THAN ONE PROTOTYPE?	YES, KEEP MAKING THEM + TESTING THEM
DOES THE ORDER YOU DO THINGS IN MATTER?	KIND OF - YOU CAN KEEP REPEATING A STEP THOUGH.

Write at least 3 questions you have from looking at the Engineering Design Process.

Questions	Answers
IS COMMUNICATING THE LAST STEP?	SHARING IS THE LAST STEP, BUT COMM IS KEPT THROUGHOUT THE WHOLE PROCESS.

Write at least 3 questions you have from looking at the Engineering Design Process.

Questions	Answers
Can which way the process goes change?	yes, when you take feedback you may need to reiterate one of the steps.
Does research come before identifying a problem?	You must identify the problem before researching.

Write at least 3 questions you have from looking at the Engineering Design Process.

Questions	Answers
Are all products designed w/ this process?	This is a good process to follow in order to find a solution.
How do you make a prototype?	It is different for each thing
How often do you need to get feedback?	After each step it is a good idea to get feedback.

## Implementation Timeline

Due to limited class time, this design challenge will be completed over a total of 180 minutes (A 20 minute intro and two full 80 minute blocks). Students will learn about the design process during the first 20 minute period so they are prepared to enter into the process immediately in each block. During the first block, we will identify the problem, brainstorm, design, build, and begin to test. During the second 80 minute block, we will conclude our testing, share our solutions, reflect, and extend.

# Class Engineering Design Notebook

## Identify the problem

To begin, students were asked to individually restate the problem and constraints in their own words. Then, they were asked to share their explanation with 2 other partners and note anything they'd like to improve about their own restatement. They also could ask questions and clarify during this time. Lastly, groups were asked to write a group restatement of the problem and constraints.

### Examples of Student Work:

Explain the Problem & Constraints, In Your Own Words: Find the velocity of a marble being rolled down a track at an instant. No using phones.	
Partner: <u>Jack</u>	Partner: <u>NICK</u>
Notes: <ul style="list-style-type: none"><li>• Exact</li><li>• Only materials in the room</li></ul>	Notes: <ul style="list-style-type: none"><li>• Certain point / time</li></ul>
Group Restatement of the Problem & Constraints: With only materials in the room and no phones, find the velocity of a rolling marble at a specific point in time.	

Explain the Problem & Constraints, In Your Own Words: How can you find the velocity of a moving marble using anything in the classroom except an actual velocity measuring thing?	
Partner: <u>Eva</u>	Partner: <u>Conor</u>
Notes: <ul style="list-style-type: none"><li>- EXACT</li><li>- Bspi or radar gun</li><li>- Resources in room</li></ul>	Notes: <ul style="list-style-type: none"><li>- given instant</li></ul>
Group Restatement of the Problem & Constraints: Find the exact velocity at a given instant of a marble with resources in the classroom <u>except</u> a bspi or radar detector.	

**Explain the Problem & Constraints, In Your Own Words:**  
 A marble will be rolled and we must be able to find the velocity of the ball without using a radar gun.

Partner: <u>Quinn</u>	Partner: <u>Paul</u>
Notes: <u>Velocity at a certain instant</u>	Notes:

**Group Restatement of the Problem & Constraints:**  
 A marble will be rolled and we must find the velocity of the ball at a certain instant without the use of a radar gun.

**Explain the Problem & Constraints, In Your Own Words:**  
 - must identify velocity of marble at a given instant, using only classroom objects - not including radar-gun-esque devices.

Partner: <u>Sarah</u>	Partner: <u>Eva</u>
Notes: <u>-moving velocity</u>	Notes: <u>-exact</u>

**Group Restatement of the Problem & Constraints:**  
 we must identify the exact velocity of a moving marble @ a given instant, using any classroom resources barring radar-gun devices.

**Identify the Problem**

**Explain the Problem & Constraints, In Your Own Words:**  
 = WE NEED TO FIND OUT THE EXACT VELOCITY OF A MARBLE  
 - WE CANNOT USE A ~~SEE SPY~~ RADAR GUN  
 - CAN USE ALL OTHER RESOURCES IN THE ~~CLASS~~ ROOM

Partner: <u>SARAH</u>	Partner: <u>CONOR</u>
Notes: <u>How you can find the velocity of MARBLE w/o USING RADAR GUN</u>	Notes: <u>IDENTIFY VELOCITY OF MARBLE</u>

**Group Restatement of the Problem & Constraints:**  
 FIND THE VELOCITY OF A MARBLE AT A GIVEN INSTANT WITH RESOURCES IN THE CLASSROOM EXCEPT RADAR GUNS/ ~~SEE SPY~~

## Brainstorming

During this phase, students were required to come up with at least 3 different suggestions of methods of approximating. We emphasized that there were no “bad” ideas and they were allowed to use their personal devices to research, as needed. Many fell back on knowledge from their previous physics classes. Students struggled with getting started with brainstorming, since many believed their first idea was the “only” idea. As you’ll see below, many students had a similar first idea. It was interesting to see that students asked for more brainstorming time after they’d begun testing and when asked what phase they wish they’d approached differently, the class almost unanimously said this phase. Students were asked to select their favorite idea with which to proceed to the next phase.



### Examples of Student Work:

Document all ideas your group suggests!

Idea	Advantages	Failure Points
Roll the marble along a specified length of track, timing how long it takes to roll the distance	- slower ~ easier to time	- human error (reflex based) - Friction - screws w/ velocity - non-constant push-force
Drop marble from specified height, & time how long it takes to hit the ground. Take the $\sqrt{\quad}$ of the # of seconds measured, b/c known acceleration & $a = m/s^2$	- more mathematical, constant results	- more complicated math manipulation - reflex-based human error

Roll marble off set length of ramp to maintain starting velocity, & time how long it takes to fall from bottom of ramp to floor (see previous idea) (~measure drop distance & time)	- constant starting $v$	- reflex-based human timing error
Roll marble off ramp onto flat course, & measure how long it takes to roll a short distance on the flat course	- minimizes effects of friction - constant starting $v$	- reflex-based human timing errors - friction of track on marble screwing w/ $v$ measurements

Document all ideas your group suggests.

Idea	Advantages	Failure Points
measure the distance and time it, then do $v = \frac{\text{distance}}{\text{time}}$	Gives a good approximate velocity.	-could have errors in accuracy for timing -would be an average
Roll entire track length, but measure and time only a specific piece in middle of track.	More accurate velocity at one point.	-Hard to precisely time

Idea	Advantages	Failure Points
Push the marble and divide its distance over time	Gives average velocity	
Shorter distance of rolling to figure out speed	Gives average velocity over shorter distance meaning more acc velocity	

The above brainstorm was particularly interesting, as it starts to address the idea of taking a limit as the interval approaches 0. This is an idea we will later build on in our class discussion.

measure out 10 m, measure time it takes to travel 1m, then 2m then 3m, etc.	we could see a trend in how it slows down (measure velocity @ different distances)	
---	--	--

# Design

During this phase, students were challenged to do as much pre-planning as possible. They were asked to sketch a visual representation of their plan, including labels, notes, measurements, or any other important information. They also needed a list of materials. This was particularly difficult for students as they wanted to just jump in!

## Examples of Student Work:

In the space below, sketch a visual representation of your plan, including labels, notes, measurements, or any other important information. Also, create a list of materials you will use.

Materials:

- marble
- track
- measuring tape
- phone w/ camera
- tape

In the space below, sketch a visual representation of your plan, including labels, notes, measurements, or any other important information. Also, create a list of materials you will use.

Materials:

- Stopwatch
- Tape measure
- Run 3x
- Marble

Velocity	From	To
1-2	1-2	
2-3	2-3	
3-4	3-4	
4-5	4-5	
5-6	5-6	
6-7	6-7	
7-8	7-8	

In the space below, sketch a visual representation of your plan, including labels, notes, measurements, or any other important information. Also, create a list of materials you will use.

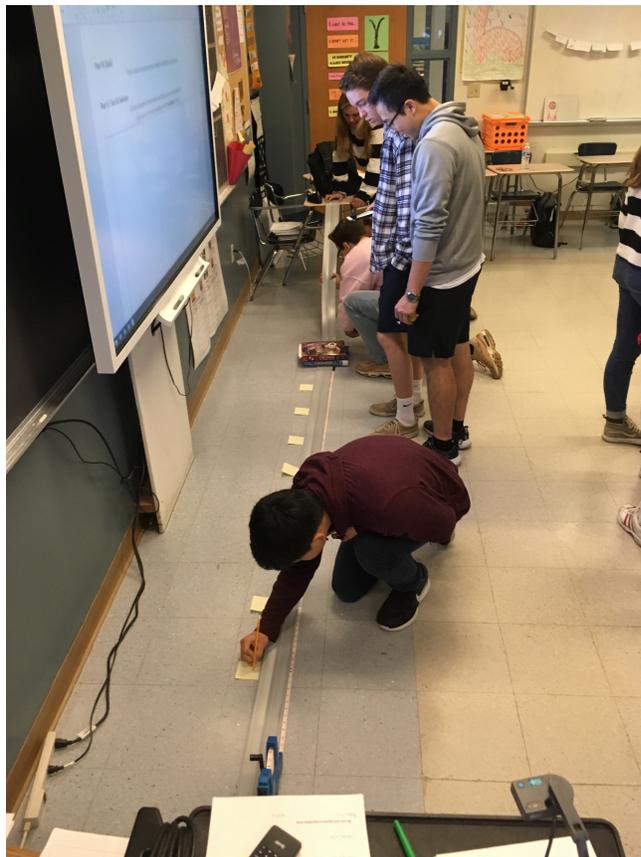
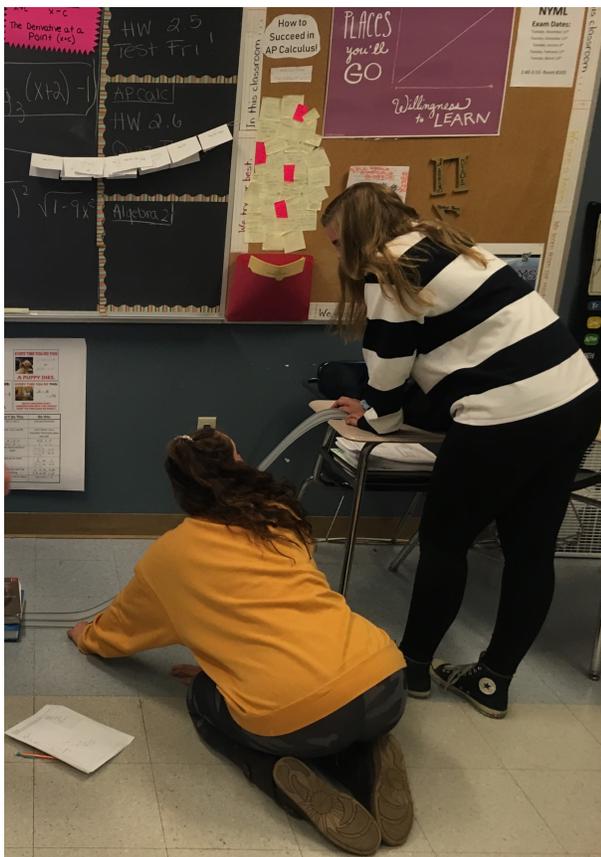
Materials:

- Roller/Mezge Stick
- BLACK BOARD TRACK
- TRIPLE PHONE
- Play Doh
- MARBLE
- TRIANGLE BLOCK
- if dont have USE A BINDER

## Build

During this phase, students were given time to build. Almost all groups used some sort of additional materials to aid their trials. Many built ramps, relying on potential energy from gravity to give a predictable velocity at a point. They needed to both build their ramps and calculate using them. Some built “rails” for their ramps using PlayDoh to keep the marble aligned. One group mapped out even intervals using post it notes to be able to perform video analysis more easily. Groups could source any materials they wanted, other than photogates (which we would use to confirm their solution) and radar gun apps on their phone.

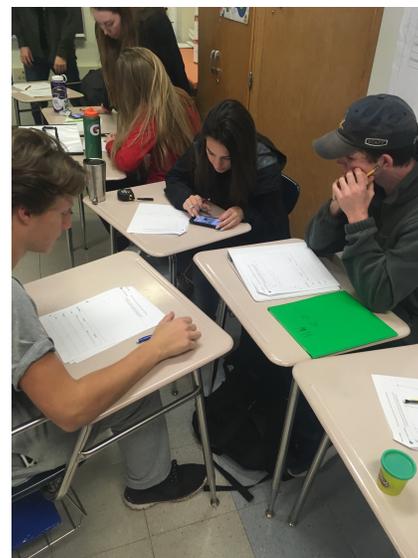
Photos of Student Work:



## Test & Evaluate

During this phase, each group got a test run on the track. They still weren't able to use the photogate, but were able to troubleshoot, practice their calculation techniques, get feedback, and watch other groups work (if they were observant!). Groups that did video analysis, like the one at right, made sure they had all the information they needed and added additional markers to the track at spots that would make their analysis better.

During this phase, groups also received feedback using the “Praise Question Polish” protocol, working in small groups. Each person had to offer one thing they liked about another group's idea, one question they had about it, and one thing they think the group could polish. This feedback could be used in the “redesign” phase.



## Examples of Student Work:

Test #1		Test #2		Test #1		Test #2	
Distance	Time	D	T				
1 ft	.21	1ft	.20	1: v = 4.76 f/s	1: v = 5 f/s		
2 ft	.36	2ft	.18	2: v = 3.51 f/s	2: v = 1.45 f/s		
3 ft	.40	3ft	.63	3: v = 3.09 f/s	3: v = 1.49 f/s		
4 ft	.43	4ft	1.23	4: v = 2.66 f/s	4: v = 1.23 f/s		
5 ft	.73			5: v = 2.35 f/s			
6 ft	.59			6: v = 2.21 f/s			
7 ft	.72			7: v = 2.03 f/s			
8 ft	.77			8: v = 1.9 f/s			

Chose a tile to record the marble passing through (1 foot)

Decided to take a video to get a more accurate reading

Put the timer in the video and put it in slow motion

Trial

1) .14 s, 12 in = .305m  $v = \frac{d}{t} = \frac{.305m}{.14s} = 2.18 m/s$

2) .18 s, 12 in = .305m  $v = \frac{d}{t} = \frac{.305m}{.18s} = 1.69 m/s$

3) .15 s, 12 in = .305m  $v = \frac{d}{t} = \frac{.305m}{.15s} = 2.03 m/s$

$\begin{matrix} 2.18 m/s \\ + \\ 1.69 m/s \\ + \\ 2.03 m/s \\ \hline = 1.97 m/s \end{matrix}$

Test 1		Test 2		Test 3	
d(ft)	t(sec)	d(ft)	t(sec)	d(ft)	t(sec)
1	.43	1	.40	1	.49
2	.65	2	.49	2	.57
3	.98	3	.61	3	.72
4	1.01	4	.68	4	.98
5	1.11	5	.70	5	1.04

- did not apply the same force each time

$v @ 3ft = 1.02 m/s$        $v @ 3ft = 1.04 m/s$        $v @ 3ft = 1.39 m/s$

Did first one but messed up so we didn't record that.

We only recorded a foot and used a slow motion timer to get the velocity for that foot

1) time: .14s distance: 1 foot } Did fail first the

2) time: .18s distance: 1 foot } same

3) time: .15s distance: 1 foot }

1)  $2.17 \frac{m}{s}$       3)  $2.03 \frac{m}{s}$

2)  $1.69 \frac{m}{s}$

**Average velocity  $1.97 \frac{m}{s}$**

$\begin{matrix} .305 \\ .14sec \\ \hline 2.17 \frac{m}{s} \end{matrix}$      
 $\begin{matrix} .1ft + .1ft \\ .14s + .18m \\ \hline 2.14 \frac{m}{s} \end{matrix}$      
 $\begin{matrix} .1ft \\ .15m \\ \hline 5.55 \frac{m}{s} \end{matrix}$

<b>Praise</b>	Slow-mo is a good idea
<b>Question</b>	How will you find the velocity using the slow motion?
<b>Polish</b>	Make sure there was no human error on dropping it from a certain height.

<b>Praise</b>	Good data from 3 different trials
<b>Question</b>	Wouldn't using a different force everytime would produce diff answers?
<b>Polish</b>	Use a ramp to get the same initial velocity instead of human force

<b>Praise</b>	<del>data</del> we have a speed @ multiple instants
<b>Question</b>	How do you know your data is accurate if no force is constant applied?
<b>Polish</b>	Apply a <u>constant</u> force each time

<b>Praise</b>	TRIGGER THAT THEY CONSIDERED GOOD FACTORS <del>THE</del> $v_{in} = 0$
<b>Question</b>	ARE THEY INCLUDING THE DIST OF THE TOP OF RAMP?
<b>Polish</b>	THE VELOCITY IS GOING TO DECREASE AT THE RAMP

## Redesign

Each group had an opportunity to make changes to their design, based on their trials and the feedback they received. This was the point where many groups emphasized that they wish they'd used their brainstorming time more efficiently, as seen in their reflection later in the process.

What step of the process (I-VII) would you approach differently? Why would this result in a better final design?

Part II brainstorm. It may give me better ideas if I thought harder.

What step of the process (I-VII) would you approach differently? Why would this result in a better final design?

II brainstorming because we could compare different ideas.

## Share the Solution

During this phase, students were asked to share their design in a presentation to the class before their final trial. Since students had done mini-presentations to each other during the PQP phase, it was interesting to see the redesigns that had taken place. We then tested each technique and the groups had the following results

Group	Technique	Estimate (m/s)	Actual (m/s)
1	Roll marble off ramp and then measure over a 1 ft interval to determine time	0.71	0.64
2	Roll marble off ramp and measure how long it takes to travel a short distance	0.572	0.41
3	Roll marble off ramp and a small increment of the track to measure the distance per time	2.7	2.75
4	Measured out multiple intervals and timed each, realized they should have measured the same 8 distances instead of 8 different intervals	0.76	0.37
5	Use a small increment of the track to measure the distance per time	2.03	1.92
6	Use a small increment of the track to measure the distance per time	1.97	1.89
7	Use a small increment of the track to measure the distance per time	0.34	0.6

## Student Reflection

In the final stage, students were asked to reflect on their solution, the process, and their work as a group.

### Examples of Student Work:

Looking back, what is one decision you would make again, if given the opportunity?

I think getting multiple  
velocities @ multiple instants  
makes it easier to interpolate  
data.

#### Part VIII. Reflect On Your Experience

Would you consider your design a success? Why or why not? Be specific!

I would have made sure we took a video  
of the whole track because we could not  
choose the placement of the velocity calculator.  
Also our sticky note of where to drop the  
marble was taken.

#### Part VIII. Reflect On Your Experience

Would you consider your design a success? Why or why not? Be specific!

No, because it only measured the <sup>average</sup> last quarter of a meter  
instead of the velocity of that exact time.

#### Part VIII. Reflect On Your Experience

Would you consider your design a success? Why or why not? Be specific!

No because it was very difficult to measure  
the time it took to roll a distance

What method would give you the **most** accurate answer? Will this ever give you the actual instantaneous velocity? Why or why not?

This method (the one we tested) would give  
the most accurate answer. It would not  
give the actual instantaneous velocity because  
we have to measure a distance rather  
than an instant

What step of the process (I-VII) would you approach differently? Why would this result in a better final design?

Part II brainstorm. It may give me better ideas if I thought harder.

What was one conflict of ideas or personalities that your group faced? Did you overcome this conflict? If so, how? If not, what would you do differently next time?

I didn't care enough to put in effort, Aleida helped me and made me work.

Looking back, what is one decision you would make again, if given the opportunity?

- I WOULD STILL USE THE RAMP / INCLINE IDEA
- STILL MEASURE THE TIME IT TOOK
- STILL PAY ATTENTION TO THE DISTANCE

What is one decision that you would have made differently? Based on what evidence?

- MADE A STEEPER INCLINE B/C WE HAD TO CHANGE OUR MARBLE B/C THE DIST. WAS TOO FAR FOR THE BEE SPY TO MEASURE ANYTHING

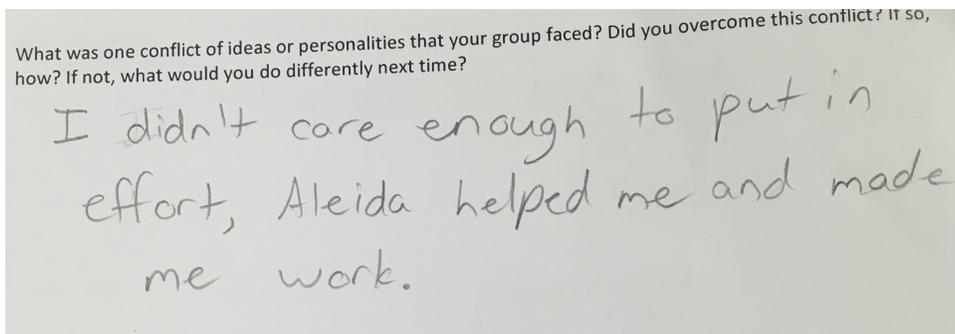
# Final Teacher Reflections

- a. What went well with the engineering design challenge?

It was amazing to see the ingenuity my students displayed! Though they were resilient to looking at multiple ideas, I saw them continue to refine and redirect as they moved through the process and could get a glimpse into how the engineering design process works. I was also very happy with the techniques my students chose. My goal was to have students discover the idea that taking an average rate of change will never be exact, but as we decrease the size of our interval the approximation will become more and more accurate. Their work on refining their own ideas led them to this naturally and my typically desire to “over-scaffold” this to funnel them where I’d like them to go wasn’t necessary- they got their on their own!

- b. What did not go well with the engineering design challenge?

I had a few students who struggled to engage with the activity. They relied on their group to carry the load, which was frustrating for the group. Part of this is because of a particular student who I have this year who is challenging and who impacts the rest of the class in a profound way. I would like to learn better strategies to help disengaged students.



My students were also resilient to getting started. The brainstorm process wasn’t as impactful as it could have been because they didn’t want to talk to their assignment group first block and because they assumed their first idea was the best one. I think this needs to be more of a culture in the classroom if it’s something that I want them to be able to jump in and do easily.

Timing was also a difficult task. I wished I’d had more time to extend the lesson, but in a college level class at the high school there is never enough time.

- c. What concepts were covered (list standards and topics where appropriate)

The major topics I’d hoped to cover were all addressed in the activity, but most closely the following were addressed:

**EU 2.1:** The derivative of a function is defined as the limit of a difference quotient and can be determined using a variety of strategies

**LO 2.1A:** Identify the derivative of a function as the limit of a difference quotient

**EK 2.1A1:** The difference quotient  $\frac{f(a+h)-f(a)}{h}$  and  $\frac{f(x)-f(a)}{x-a}$  express the average rate of change of a function over an interval

**EK 2.1A2:** The instantaneous rate of change of a function at a point can be expressed by  $\lim_{h \rightarrow 0} \frac{f(a+h)-f(a)}{h}$  or  $\lim_{x \rightarrow a} \frac{f(x)-f(a)}{x-a}$ , provided that the limit exists. These are common forms of the definition of the derivative and are denoted  $f'(a)$ .

**MPAC 1: Reasoning with definitions and theorems**

Students can:

- Develop conjectures based on exploration with technology

**MPAC 2: Connecting Concepts**

- Relate the concept of limit to all aspects of calculus
- Use the connection between concepts or processes

**MPAC 3: Implementing algebraic/computational processes**

Students can:

- select appropriate mathematical strategies;
- apply technology strategically to solve problems;
- attend to precision graphically, numerically, analytically, and verbally and specify units of measure; and
- connect the results of algebraic/computational processes to the question asked.

**MPAC 6: Communicating**

Students can:

- clearly present methods, reasoning, justifications, and conclusions;
- use accurate and precise language and notation;
- explain the meaning of expressions, notation, and results in terms of a context (including units);
- explain the connections among concepts;
- critically interpret and accurately report information provided by technology; and analyze, evaluate, and compare the reasoning of others.

d. How did the ED process help teach the science and mathematics concepts?

The iterative nature of the ED process perfectly prompted students to engage in the activity, even though they were a bit reluctant at first. Through trial and experimentation, they went “back to the drawing board” and revised their thinking. This all contributed greatly to learning the mathematical concept as students were able to naturally determine that the smaller the interval, the more accurate the resulting approximation. The process of finding the derivative involves taking the limit of the average rate of change as the size of the interval approaches 0. Students were able to anchor the rigorous notation they would need to use in the conceptual understanding they developed during this lab.

e. Did I choose an appropriate engineering design process? Should I simplify or make more complex?

I think using the Massachusetts Engineering Design Process was an appropriate choice for this level of students. However, I wish I’d had more time to work with the process. Because students had such limited time, we were not able to have feedback at every step or to spend as much time redesigning as I would like. This was more an issue of my own instructional time, not the design process itself.

f. How can I improve this activity to use with future students?

In the future, I would like to leave more time for this activity-. I was truly impressed with how students developed the idea of taking the limit independently of any instruction, but wish we'd had more time for feedback, redesign, and reflection. With more reflection and the proper questions, students might be able to better develop their understanding. This would be a better anchor as we moved into the teaching of the actual concept.

# APPENDIX A- Previous Phase I Document

“Launch It” Design Challenge

Modified for a School Level Calculus Class

[https://www.nasa.gov/pdf/418003main\\_OTM\\_Launch\\_It.pdf](https://www.nasa.gov/pdf/418003main_OTM_Launch_It.pdf)

1. Identify the “Big” concept to be covered by the engineering design challenge.

In the Launch It challenge, students are challenged to design and build a balloon powered rocket. The “big” concept this will address in my Calculus class is the application of derivatives to physical phenomena. Projectile motion is a very common application of differential calculus. We will be able to analyze the motion of the rockets using video analysis to use as feedback for our trials. This will include function analysis,

2. Research appropriate learning standards associated with the topic.

## AP Calculus Standards

**LO 2.2A:** Use derivatives to analyze properties of a function

**EK 2.2A1:** First and second derivatives of a function can provide information about the function and its graph including intervals of increase or decrease, local (relative) and global (absolute) extrema, intervals of upward or downward concavity, and points of inflection.

**EK 2.2A2:** Key features of functions and their derivatives can be identified and related to their graphical, numerical, and analytical representations.

**EK 2.2A3:** Key features of the graphs of  $f$ ,  $f'$ , and  $f''$  are related to one another.

**EK 2.3C1:** The derivative can be used to solve motion problems involving position, speed, velocity, and acceleration.

**EK 2.3C3:** The derivative can be used to solve optimization problems, that is, finding a maximum or minimum value of a function over a given interval.

**EK 2.3D1:** The derivative can be used to express information about rates of change in applied contexts.

## **MPAC 3: Implementing algebraic/computational processes**

Students can:

- select appropriate mathematical strategies;
- sequence algebraic/computational procedures logically;
- complete algebraic/computational processes correctly;
- apply technology strategically to solve problems;
- attend to precision graphically, numerically, analytically, and verbally and specify units of measure; and
- connect the results of algebraic/computational processes to the question asked.

## **MPAC 4: Connecting multiple representations**

Students can:

- associate tables, graphs, and symbolic representations of functions;
- develop concepts using graphical, symbolical, or numerical representations with and without technology;
- identify how mathematical characteristics of functions are related in different representations;
- extract and interpret mathematical content from any presentation of a function (e.g., utilize information from a table of values);
- construct one representational form from another (e.g., a table from a graph or a graph from given information); and

- consider multiple representations of a function to select or construct a useful representation for solving a problem.

### **MPAC 6: Communicating**

Students can:

- clearly present methods, reasoning, justifications, and conclusions;
- use accurate and precise language and notation;
- explain the meaning of expressions, notation, and results in terms of a context (including units);
- explain the connections among concepts;
- critically interpret and accurately report information provided by technology; and
- analyze, evaluate, and compare the reasoning of others.

3. Identify and discuss the different types of problem solving and declarative/procedure knowledge needed.

There are various types of knowledge that will be needed during this challenge. Some of the declarative knowledge that students will need will include a basic understanding of Newton's 3<sup>rd</sup> law, that for every action there is an equal and opposite reaction. Students will need to make sure the force of the balloon is enough to accelerate their rocket and will need to be mindful of the direction of that force as well. A mental model of typical projectile motion may aid students in this. Students should be aware of the impact of gravity, the impact angle of launch may have on motion, and more from their previous experience. These will all inform their mental models and thus their approach.

This challenge will require less procedural knowledge since there is not one specific path to success. Students can take a variety of approaches to building their rocket to have it reach the target. They can be shaped differently, so long as they can be launched by the balloon mechanism we will be using.

4. Explore objectives and ancillary concepts/content covered by the project.

One of the ancillary topics that can be addressed from this engineering process is the actual equations for projectile motion. These can be generated using integration and the value of acceleration due to gravity. While a more advanced topic, this could be addressed later in the year when we student the Fundamental Theorem of Calculus.

Another one of the ancillary concepts that can be covered by this project is the idea of optimization. Not only can we have students pick a target, but we can also have them try to reach a maximum height, a maximum distance, or carry a maximum weight.

5. Identify possible activities.

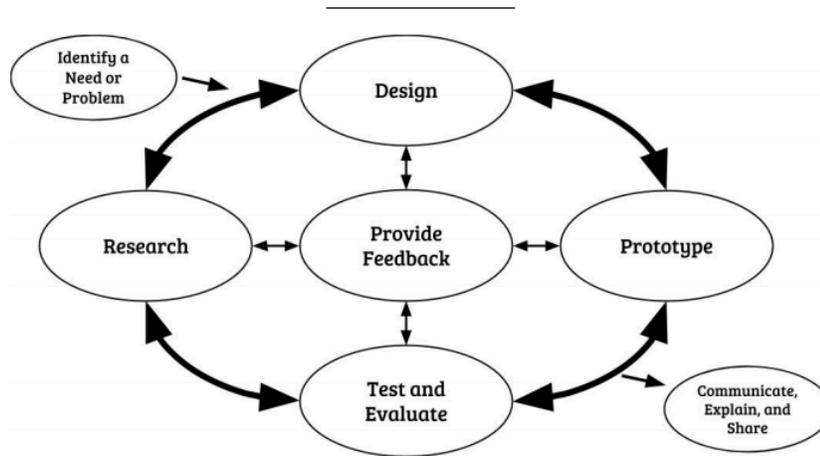
Students have a variety of activities they could complete during this challenge. One is simply attempting to hit the target. Further extension could come from video analysis using Logger Pro software. Using this software, we can find important features of the motion of the rocket, such as the maximum or the time travelled. We can also find a function to describe the motion, so we can then use Calculus to find velocities, acceleration, and other important information. We could also predict the motion after changing a variable, given the data we collect.

6. Select the best activity for your classroom.

For this activity, I am going to begin with building the rockets to hit a target. We will use video analysis from that to then revise our rockets to try to go the maximum distance. If students are able to do this easily, further extension may be added later.

# APPENDIX B – Blank Student Engineering Notebook

## Part I. The Engineering Design Process



Write **at least 3** questions you have from looking at the Engineering Design Process.

Questions	Answers

Can you think of a scenario in your life where you implement this process? Explain how!  
It can be anything, not **just** school related!

# Engineering Design Notebook

## Part I. Identify the Problem

Explain the Problem & Constraints, In Your Own Words:

Partner: \_\_\_\_\_

Notes:

Partner: \_\_\_\_\_

Notes:

Group Restatement of the Problem & Constraints:

## Part II. Brainstorm

Document **all** ideas your group suggests!

Idea	Advantages	Failure Points


If you need more space, attach additional paper.

Which idea did your group select? Why?

--

**Part III. Design**

In the space below, sketch a visual representation of your plan. Include labels, notes, measurements, or any other important information. Also, create a list of materials you will use.

Materials: _____ _____ _____ _____ _____ _____ _____ _____ _____ _____
--

**Part IV. Build**

Please make sure your group number is visible in all pictures!

**Part V. Test & Evaluate**

Closely document notes from each test you do on your system!

Label each test with a number and run **at least 3 tests!**

**PQP Feedback**

	<b>Group #1</b>	<b>Group #2</b>
	Name _____	Name _____
<b>Praise</b>		
<b>Question</b>		
<b>Polish</b>		

**Part VI. Redesign**

Complete this chart about your group's design, in light of what you observed and the feedback you received from other groups.

+	Δ

**Part VII. Share the Solution**

Be prepared to present your method before your team's turn in the final showcase!

**Part VIII. Reflect On Your Experience**

Would you consider your design a success? Why or why not? Be specific!

Looking back, what is one decision you would make again, if given the opportunity?

What is one decision that you would have made differently? Based on what evidence?

What step of the process (I-VII) would you approach differently? Why would this result in a better final design?

What was one conflict of ideas or personalities that your group faced? Did you overcome this conflict? If so, how? If not, what would you do differently next time?