Charles Canestaro

Physical Science in Motion

Lesson Plan: Modified *Project Enterprise*

## **Background Information**

This activity is an adaptation of the activity *Project Enterprise*, from the NASA publication *Adventures in Rocket Science* [https://www.nasa.gov/pdf/265386main_Adventures_In_Rocket_Science.pdf](https://www.nasa.gov/pdf/265386main_Adventures_In_Rocket_Science.pdf). Due to materials available at Cortland High School as well as the clientele benefiting from the instruction, alterations were made to this activity to make it a more suitable fit.

**Subject:** AP Physics C: Mechanics

**Grade Level/Student Population:** Seniors taking AP Physics as a second year physics course (prior completion of algebra-based college level physics and/or NYS Regents Physics). All students are currently enrolled in Calculus I/II **and/or** have completed a two week introduction to calculus and are proficient in taking derivatives (power rule, product rule, chain rule, trig derivatives), indefinite integrals (power rule, basic trig functions), and definite integrals (power rule and using a graphing calculator).

**Level of Inquiry (LOL)**: This activity is purposed designed to be a guided inquiry activity. The instructor will state the problem (given as a statement from NASA) and the instructor will state multiple prompts and constraints on the activity so that students achieve the desired learning outcomes. Due to the engineering nature of this activity, the activity was chosen to be guided in place of bounded so that students have specific constraints that they are made aware of which they must engineer around.

## **Learning Outcomes**

[1] Students will develop a model for air resistance, including a qualitative understanding of the variables which affect the force of air resistance, including: exposed surface area, viscosity of the fluid, shape of the object, and velocity.

[2] Students will gain an understanding of the NASA engineering design process and solve a complex problem given constraints.

## **Standards:**

## **AP Physics C: Mechanics Learning Objectives**

Students should understand the effect of drag forces on the motion of an object, so they can:

(1) Find the terminal velocity of an object moving vertically under the influence of a retarding force dependent on velocity.

(2) Describe qualitatively, with the aid of graphs, the acceleration, velocity, and displacement of such a particle when it is released from rest or is projected vertically with specified initial velocity.

(3) Use Newton's Second Law to write a differential equation for the velocity of the object as a function of time.

(4) Use the method of separation of variables to derive the equation for the velocity as a function of time from the differential equation that follows from Newton's Second Law.

(5) Derive an expression for the acceleration as a function of time for an object falling under the influence of drag forces.


**Lesson Plan**

**Engage:**

The instructor will play a short clip of rocket launches (for example [https://www.youtube.com/watch?v=kNOhNLk0mRQ](https://www.youtube.com/watch?v=kNOhNLk0mRQ))

The instructor will ask the following question to the class: "When any projectile is fired into the air, is the time of accent [1] greater then, [2] less then, or [3] equal to the time of descent?" (note to educators: while this problem may seem trivial, when considering air resistance, the problem becomes much more complex than simply saying time up equals time down).

The lesson should begin with the prompt provided on page 140 of *Adventures in Rocket Science*. Students will be tasked with designing, testing, modifying, and launching a rocket. In place of a water rocket, air pressure will be used because a device currently exists in Cortland High School for such a purpose. The proposal deadline will be modified to 1 week to accommodate discussion of air resistance following the activity. The budget sheet and member organization will be utilized with only minor changes to accommodate the air rockets (specifically instead of 1L and 2L bottles, paper and cardstock will be used with the same prices).

During the introduction portion of the activity, the instructor will state that in addition to studying the engineering design process and constraints on a problem, students will be attempting to engineer the most aerodynamic rocket. A discussion should occur where the instructor discusses with students what factors they think affect drag on a rocket as well as how they would go about **demonstrating** that they actually created the most aerodynamic rocket (aka lots of math to follow!). The instructor should lay out specific objectives for students to consider, such as shape, mass, size, area, weather and temperature outside, and initial launch speed. For each variable discussed, the instructor should ask how students plan to incorporate testing of that variable. Finally, the last objective of the lab will be to study air resistance through measurements of time in air and maximum height achieved.

**Explore:**

Students will complete days 1-5 as described in the lesson plan. The instructor should budget one forty minute class for day 1, one forty minute class for day 2, and two forty minute classes for days 3-5.

During the next three class sessions, students should first explore the center of mass and pressure as described in the outline. During these sessions, students will also build altimeters using the template on page 125 of *Adventures in Rocket* Science. Students only need the template, as they will be expected to do the trig themselves to find the height. After that, students will be allowed to "purchase" test runs at varying air pressure (ranging from 0psi to 50psi in the firing chamber. Note, the launcher is a build similar to [https://makezine.com/2010/07/11/how-to-building-the-compressed-air/](https://makezine.com/2010/07/11/how-to-building-the-compressed-air/)). Students will keep a log of air pressure, time of flight, and max height for aid in their analysis as well as further discussion later on.

After allowing 1-2 days for testing and re-design, students will perform their final launches. Points will be awarded as according to the rubric. In addition, the following pieces will be added:

[1] Cost to height ratio for each rocket

[2] A challenge on the spot to try and get a rocket to reach a certain desired height or to be in the air for a certain time (for example, if all rockets make it to at least 50 ft high, the challenge may be to fire a rocket which goes exactly 25 feet in the air). This will allow students to utilize their collected data to test the correctness of the fit they were trying to impose on their data of pressure, height, and time (this is especially important for later on when the class compares objects in flight with and without air resistance).

[3] Measurements of time up, time down, and maximum height achieved. Use of a video camera will be especially helpful here.

**Explain:**

Following the activity, students will be asked to construct graphs of displacement, velocity, and acceleration vs. time for their rocket, using collected data where they can to aid their graphs. In addition, students will estimate the terminal velocity of their rocket, using data collected from launches. As a class, we will compare rockets (nosecone shape, mass, fins, etc.) to see if any data sticks out which appears to have an influence on height. Students will estimate the drag coefficient for each rocket using the estimated terminal velocity.

For the time problem, students should measure that the time down is larger then the time up. Students should be asked to do two things: [1] consider their graphs of velocity vs. time. If they correctly represented air resistance, then they would show the object reaching a relatively slow terminal velocity on the way down, while on the way up. An alternative way to think about it is through an energy approach, noting that work done by drag removes energy from the rocket system. [2] Students can estimate (very roughly) the launch velocity of their rocket using the max height and

time up and also calculate the average velocity going up and compare it to the terminal velocity.

**Elaborate:**

This activity should be extended to considering the differential equation describing the motion of a particle under a drag force proportional to an object's velocity. Using Newton's 2nd law, we can describe the acceleration of an object under velocity dependent air resistance as:

$$\frac{dv}{dt} = g - \frac{b}{m} v$$

Where dv/dt=acceleration, g=9.81m/s$^2$, m=mass, v=velocity, and b is the drag coefficient of an object which is again dependent on surface area, viscosity of the liquid it's moving through, and shape of the object. In class, the instructor should rearrange variables to get the above equation in the form:

$$\frac{dv}{\left(v - \frac{mg}{b}\right)} = \frac{-b}{m} dt$$

Which, after integrating both sides, gives

$$\ln\left|v - \frac{mg}{b}\right| = \frac{-b}{m} t + C$$

Next, every term can be raised by e to give

$$v - \frac{mg}{b} = e^{\frac{-b}{m}t} \cdot e^{C}$$

Now, setting the initial constraint that at t=0, v(0)=$v_0$, you can solve for the constant of integration and after some rearranging, arrive at the equation for the velocity of an object under air resistance given by

$$v(t) = \left(v_0 - \frac{mg}{b}\right) e^{\frac{-b}{m}t} + \frac{mg}{b}$$

If we now consider an object starting from rest and consider the motion of the particle after a large amount of time, we see that the first term approaches zero, meaning that the velocity approaches the value mg/b, which is a constant, which we will call the terminal velocity of an object. Thus, by knowing the mass of the rocket and measuring it's terminal velocity, we can calculate the drag coefficient of each rocket by using

$$b = \frac{mg}{v_t}$$

Where $v_t$ is the terminal velocity of the rocket. Thus, each group can calculate the drag coefficient of each rocket and determine who had the most aerodynamic rocket. The lesson can then be concluded to discuss that at high speeds, air resistance is often better approximated by velocity-squared dependent drag, at which point a discussion of the Wikipedia page on drag (or the NASA website https://www.grc.nasa.gov/www/k-12/airplane/drageq.html) where students can examine the various drag coefficients of different shapes. The instructor should connect the low drag coefficient numbers to objects like the space shuttles, modern airplanes, bike helmets, and even the new Tesla Semi. Students should be allowed time to summarize their conclusions and research various aerodynamic objects to connect to their findings.