4.3 Exercises

4. $f(x) = \ln(x^3+1)$

$f'(x) = \frac{d}{dx}(\ln(x^3+1))$

$f'(x) = \frac{d}{dg}(\ln(g))\frac{d}{dx}(x^3+1)$

$f'(x) = \frac{1}{g} \times 3x^2$

$f'(x) = \frac{1}{x^3+1} \cdot 3x^2$

$f'(x) = \frac{3x^2}{x^3+1}$

8. $f(x) = \ln(x^4+1)^2$

$f'(x) = \frac{d}{dx}(\ln((x^4+1)^2))$

$f'(x) = \frac{d}{dg}(\ln(g))\frac{d}{dx}((x^4+1)^2)$

$f'(x) = \frac{1}{g} \cdot 2(x^4+1)(4x^3)$

$f'(x) = \frac{1}{(x^4+1)^2} \cdot 2(x^4+1)(4x^3)$

$f'(x) = \frac{8x^3}{x^4+1}$

12. $f(x) = x^3 e^x$

$f'(x) = \frac{d}{dx}(x^3 e^x)$

$f'(x) = \frac{d}{dx}(x^3)e^x + x^3 \cdot \frac{d}{dx}(e^x)$

$f'(x) = 3x^2 e^x + x^3 e^x$

16. $f(x) = \ln(e^x - 2x)$

$f'(x) = \frac{d}{dx}(\ln(e^x - 2x))$

$f'(x) = \frac{d}{dg}(\ln(g))\frac{d}{dx}(e^x - 2x)$

$f'(x) = \frac{1}{g}(e^x - 2)$

$f'(x) = \frac{1}{e^x - 2x}(e^x - 2)$

$f'(x) = \frac{e^x - 2}{e^x - 2x}$

20. $f(x) = \ln e^x$

$f'(x) = \frac{d}{dx}(\ln(e^x))$

$f'(x) = \frac{d}{dx}(x)$

$f'(x) = 1$

24. $f(x) = ex$

$f'(x) = \frac{d}{dx}(ex)$

$f'(x) = e$

28. $f(x) = x^3 e^x - 2\ln(x) + (x^2+1)^3$

$f'(x) = \frac{2x^3 e^x + x^3 e^x - 2 + 6x^5 + 12x^4 + 6x^2}{x}$

32. $f(x) = e^x \ln(x+1)$

$f'(x) = \frac{d}{dx}(e^x \ln(x+1))$

$f'(x) = \frac{d}{dx}(e^x)\ln(x+1) + e^x \frac{d}{dx}(\ln(x+1))$

$f'(x) = e^x \ln(x+1) \cdot e^x \cdot \frac{1}{x+1}$

$f'(x) = e^x \ln(x+1) + \frac{e^x}{x+1}$

36. $f(x) = \sqrt{e^{2t}+4}$

$f'(t) = \frac{d}{dt}(\sqrt{e^{2t}+4})$

$f'(t) = \frac{d}{dg}(\sqrt{g})\frac{d}{dt}(e^{2t}+4)$

$f'(t) = \frac{1}{2\sqrt{g}} e^{2t} \cdot 2$

$f'(t) = \frac{1}{2\sqrt{e^{2t}+4}} e^{2t} \cdot 2$

$f'(t) = \frac{e^{2t}}{\sqrt{e^{2t}+4}}$

40. $f(t) = \frac{t^2}{1+t^2}$

$f'(t) = \frac{e^{2t}\cdot 2}{(1+e^t)^2}$

44. $f(x) = \frac{e^x - e^{-x}}{e^x + e^{-x}}$

$f'(x) = \frac{4e^{2x}}{(e^{2x}+1)^2}$

52. $f'(x) = [(1+\ln(x))(2x) - x^2(1/x)]/(1+\ln(x))^2$

$= [2x + 2x\ln(x) - x]/(1+\ln(x))^2$

$= [x + 2x\ln(x)]/(1+\ln(x))^2$

$= [x(1+2\ln(x))]/(1+\ln(x))^2$

$y - f(1) = f'(1)(x-1)$

$y - [1^2/(1+\ln(1))] = 1(x-1)$

$y = x$

56.

a. $f'(x) = x \cdot 2(x^2/2)$

b. $f'(2) = 2(2^2/2) = 2e^2$

$f(2) \approx 14.778$

60.

$= (-5x^4)e^{(-x^5/4)} + x^{-5}e^{(-x^5/4)}(-5x^4)$

$= e^{(-x^5/4)}(-5x^4 + 5x^{10})$

64.

$dy = f'(x)dx = (-4)e^{(-1)}(-1)^2(0.25) = -0.5e^{-1}$

$x = 2$ & $dx = 0.25$

68. $dy = f'(x)dx = (2 \cdot 3)(3^2+1)(0.25) = 0.12$

$x = 3$ & $dx = 0.25$

88. $N(t) = 50{,}000(1 - e^{(-0.4t)})$

a. $N'(0) = dN/dt\,(t=0)$

$= 50{,}000(-1)(-0.4)e^{(-0.4(0))}$

b. $N'(8) = dN/dt\,(t=8)$

$= 50{,}000(-1)(-0.4)e^{(-0.4(8))}$

$= 6{,}273.08$