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MAT 246-CALCULUS

Final Exam

Directions: Provide complete responses to each question and show all work. Good luck!

Functions

(1) If $f(x) = x^2 - 11x + 30,\ then\ find\ the\ following$: **(10 points)**

a. $f(-2)$

$f(-2) = -11(-2) + 30$

$= 22 + 30$

$f(-2) = 52$

b. $f(b-2)$

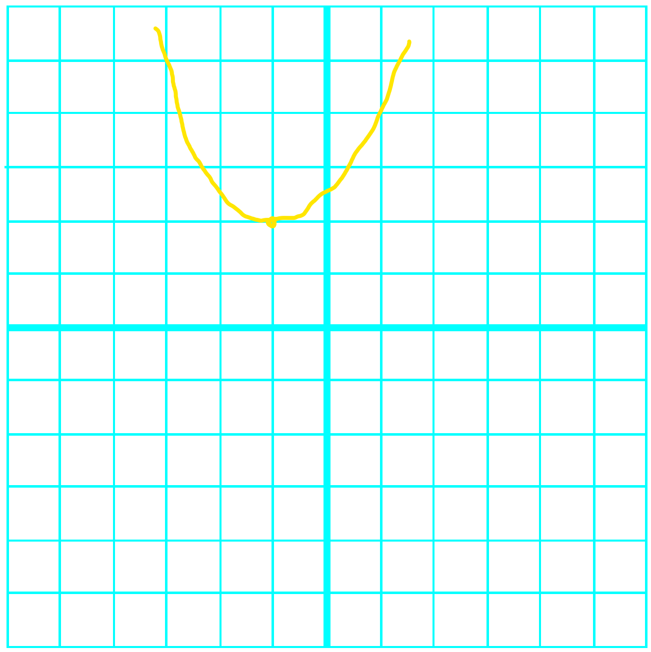
$f(b-2) = -11(b-2) + 30$

$= -11b + 22 + 30$

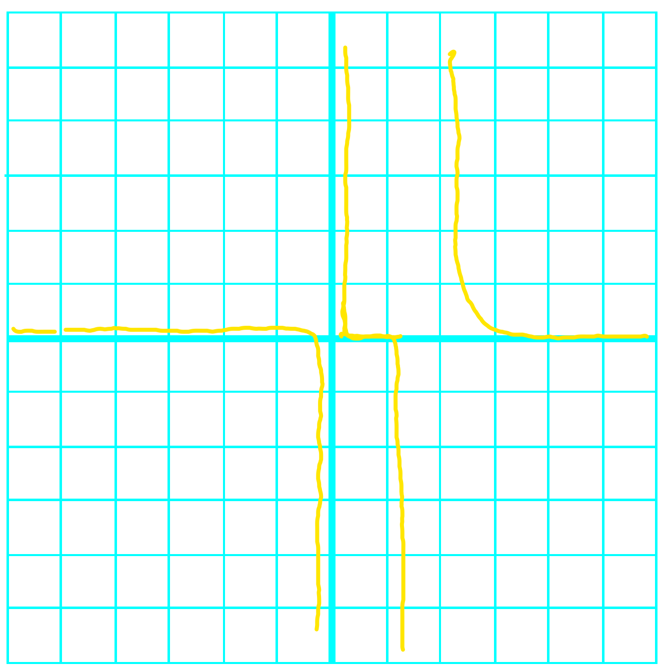
$f(b-2) = -11b + 52$

(2) Graph the following functions and determine their respective limits as $x \to 0$: **(10 points)**

a. $f(x) = x^4 + 2x^2 + 2x + 6$ Plot and find the limit as $x \to 0$



b. $f(x) = \frac{x^2 - x - 90}{x^2 - 21x + 110}$ Plot and find the limit as $x \to 10$



(3) Given the following functions:

$$h(x) = \frac{x^2 - 4}{x + 2}$$

$$j(x) = \frac{2x - 14}{x - 7}$$

Express the following as rational functions (i.e. functions with a common denominator in simplest form): **(10 points)**

a. $h(x) \times j(x)$

$$\frac{x^2-4}{x+2} \cdot \frac{2x-14}{x-7}$$

$$= \frac{(x-2)(x+2)}{x+2} \cdot \frac{2(x-7)}{x-7}$$

$$= (x-2)2 = 2x-4$$

b. $j(h(x))$

$$\frac{2\left(\frac{x^2-4}{x+2}\right)-14}{\frac{x^2-4}{x+2}+2}$$

$$= \frac{2\frac{(x-2)(x+2)}{x+2}-14}{\frac{(x-2)(x+2)}{x+2}+2}$$

$$= \frac{2(x-2)-14}{x-2+2}$$

$$= \frac{2x-4-14}{x}$$

$$= \frac{2x-18}{x}$$

(4) Determine what the value(s) of x when $f(x) = 0$ for the following functions:

a. $f(x) = x^2 + 16x - 720$

$$x = \frac{-16 \pm \sqrt{16^2-4(1)(-720)}}{2}$$

$x = -8 + 2\sqrt{61}$ & $x = -8 - 2\sqrt{61}$

b. $f(x) = 2x^3 - 8x^2$

$2x^2(x-4)=0$

$x=0$ & $x=4$

Derivatives

(5) Differentiate the following functions: **(25 points)**

a. $f(x) = \frac{1}{x}$

$\frac{d}{dx}(1/x) = -1/x^2$

b. $f(x) = (5x+1)^5$

$\frac{d}{dx}(u^5) = \frac{d}{dx}(5x+1)^5$

$= 5(5x+1)^4 d/dx(5x+1)$

$= 5(5x+1)^4(d/dx(5x) + d/dx(1))$

$= 5(5x+1)^4(5+0)$ $= 25(5x+1)^4$

c. $f(x) = (5x-7)^2(x-2)^3$

$u' = 2(5x-7)(5)$

$v' = 3(x-2)^2(1)$

$f'(x) = 2(5x-7)(5)(x-2)^3 + (5x-7)^2(3(x-2)^2)$

$f'(x) = 10(5x-7)(x-2)^3 + 3(5x-7)^2(x-2)^2$

d. $f(x) = e^{\frac{1}{x}}$

$u' = d/dx(1/x) = -1/x^2$

$f'(x) = e^{(1/x)}(-1/x^2)$

$f'(x) = -e^{(1/x)}/x^2$

e. $f(x) = \frac{x^4 - x^2}{x^3}$

$f(x) = x^4 - x^{-1}$

$f'(x) = 4x^3 - d/dx(x^{-1})$

$f'(x) = 4x^3 + 1/x^2$

Integration

(6) Calculate the following integrals: **(25 points)**

a. $\int 5x\, dx$

$5 \cdot \int x\, dx$

$5 \cdot \frac{x^2}{2}$

$\frac{5x^2}{2}$

$\frac{5x^2}{2} + C$

b. $\int (x^{\frac{5}{2}} + x^{\frac{3}{2}} + x + 1)\, dx$

$\int x^{\frac{5}{2}}\, dx + \int x^{\frac{3}{2}}\, dx + \int x\, dx + \int 1\, dx$

$\frac{2x^3\sqrt{x}}{7} + \frac{2x^2\sqrt{x}}{5} + \frac{x^2}{2} + x$

$\frac{2x^3\sqrt{x}}{7} + \frac{2x^2\sqrt{x}}{5} + \frac{x^2}{2} + x + C$

c. $\int \frac{10}{x^{-9}}\, dx$

$10 \cdot \int \frac{1}{x^{-9}}\, dx$

$10 \cdot \frac{x^{10}}{10}$

x^{10}

$x^{10} + C$

d. $\int_{-1}^{1}(4x^3 + 3x^2 - 6x - 1)dx$

$$\int_{-1}^{1} 4x^3 + 3x^3 - 6x - 1\,dx$$

$$= \int_{-1}^{1} 7x^3 - 6x - 1\,dx$$

$$= \int_{-1}^{1} 7x^3\,dx - \int_{-1}^{1} 6x\,dx - \int_{-1}^{1} 1\,dx$$

$$= -2$$

e. $\int_{0}^{ln10}(2e^x - 2e^{-x})dx$

$$\int_{0}^{In10} 2e^x - 2e^{-x}\,dx$$

$$= \int_{0}^{In10} 2e^x\,dx - \int_{0}^{In10} 2e^{-x}\,dx$$

$$= 2e^{-In10} + 2e^{In10} - 4$$

Applications

(7) Sketch the graph of $y = x^3 - 4x^2$, using properties of first and second derivatives. **(30 points)**

a. Find the critical points and describe interval(s) where the function is increasing and decreasing?

$y' = 3x^2 - 8x = x(3x - 8)$

$x = 0$ & $x = 8/3$

decreasing $(-\infty, 0)$

increasing $(0, 8/3)$

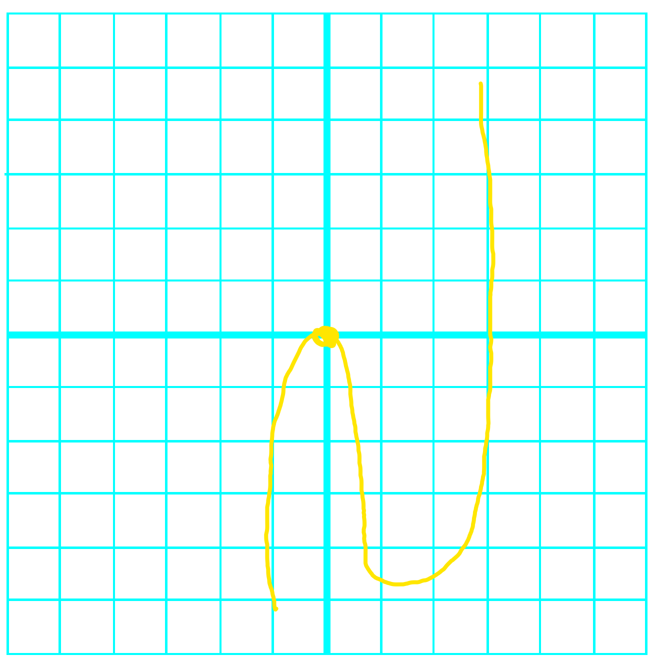
increasing $(8/3, \infty)$

b. Find the point of inflection and describe the concavity at the around the critical points and point of inflection?

$y'' = 6x - 8$

$x = 4/3$

c. Sketch the function on the following Cartesian plane:



(8) (*Cost and Profit*) A one-product firm estimates its daily total revenue function (in suitable units) is $R(x) = x^3 - 11.5x^2 + 26x + 40$ and its total cost function is $C(x) = 52x$. Find the value of *x* that maximizes the daily profit. **(20 points)**

$P(x) = x^3 - 11.5x^2 + 26x + 40 - 52x$
$= x^3 - 11.5x^2 - 26x + 40$

$P'(x) = 3x^2 - 23x - 26 = 0$

$x = [-(-23) \pm \sqrt{(-23)^2 - 4(3)(-26)}] / 2(3)$

$x = [23 \pm \sqrt{1093}] / 6$

$P''(x) = 6x - 23$

The value of x that maximizes the profit is approx. 1.78

(9) For the following function: **(20 points)**

$$f(x) = \frac{64}{x^2}$$

a. What is the average rate of change of f (x) over the interval 1 to 1.2?

$(64/1.2^2 - 64/1^2)/(1.2-1)$
$=(64/1.44 - 64)/0.2$
$= -45.71$

b. What is the instantaneous rate of change of f (x) when x = 4?

$f'(x) = -128/x^3$
$f'(4) = -128/4^3$
$= -2$

x = 4 is -2

(10) Find the equation of the tangent line to the curve for the following function at (2, 1): **(20 points)**

$$f(x) = x^5 + x^3 + x + 1$$

$f'(x) = 5x^4 + 3x^2 + 1$

$f'(2) = 5(2)^4 + 3(2)^2 + 1 = 85$

$x = 2$ is 85

$y - 1 = 85(x - 2)$

$y = 85x - 169$

$(2,1)$ is $y = 85x - 169$

(11) A financial advisor invests \$10,000 at 10% interest compounded continuously. **(40 points)**

a. Give the formula for, $A(t)$, the compound amount after t years?

$A(t) = 10{,}000e^{(0.1t)}$

b. How much will be in the account after 10 years?

$A(t) = 10{,}000e^{(0.1 \cdot 10)} \approx 27{,}182.82$

c. After 10 years, at what rate will $A(t)$ be growing?

$dA/dt = 0.1 \cdot 10{,}000e^{(0.1t)}$

$dA/dt|_{t=10} = 0.1 \cdot 10{,}000e^{(0.1 \cdot 10)} = 2{,}718.28$

d. How long will it take for the initial investment to increase by four hundred percent (i.e. 400%)?

$t = \ln(4)/0.1 \approx 27.73$ years