Behavioral Science Assignments #8 and #9

Student's Name

Institutional Affiliation

Course Name and Number

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Date

Behavioral Science Assignment #8

Qn. 1

The mean of the sampling distribution of the sample mean is equal to the population mean, which is 82. Standard error of the mean = population standard deviation / square root of sample size. The population standard deviation is 18, and the sample size is 25. Therefore: standard error of the mean = 18 / sqrt (25) = 3.6.

Qn. 2

Formula:

standard deviation = (standard deviation of population) / sqrt (sample size)

- substituting;

standard deviation of sampling distribution = 172 / sqrt (16) = 43

Qn. 3

The formula: $z = (\bar{x} - \mu) / (\sigma / \sqrt{n})$

where:

$\bar{x}$ = sample mean

μ = population mean

σ = population standard deviation

n = sample size

z = (97 - 98.2) / (25.2 / sqrt (89)) = -1.6103

z = (91 - 98.2) / (25.2 / sqrt (89)) = -3.4253

$P(-3.4253 < z < -1.6103) = 0.0495$

By rounding off;

$P(91 < \bar{x} < 97) = P(-3.4253 < z < -1.6103) = 0.0495$ (rounded to four decimal places)

- Therefore, the probability that the sample mean of a sample of size 89 elements selected from this population will be 91 and 97 is 0.0495.

Qn. 4

Using the formula: $z = (\bar{x} - \mu) / (\sigma / \text{sqrt}(n))$

Substituting the values:

z = (49.35 - 43.4) / (12.1 / sqrt (21)) = 1.81

z = (43.26 - 43.4) / (12.1 / sqrt (21)) = -0.23

- By use of a standard normal distribution table or a calculator to find the area under the standard normal curve between z = -0.23 and z = 1.81. The area is approximately 0.7557.

Qn. 5

Number of seniors = 15

Total number of students = 61

Proportion of students who are seniors = 15/61 ≈ 0.246

Proportion of students who are seniors ≈ 0.246

Qn. 6

(a) The mean of the sampling distribution of this sample proportion is equal to the population proportion. Therefore, mean of sample proportion = 0.40

(b) The formula:

Standard deviation = sqrt [p*(1-p) / n]

Standard deviation = sqrt [0.40*(1-0.40) / 115]

= sqrt [0.24 / 115]

= 0.0456

Qn. 7

A standard deviation equal to the population standard deviation divided by the square root of the sample size: Standard deviation of x = σ / sqrt(n) where σ is the population standard deviation, and n is the sample size.

Standard deviation of x = 106 / sqrt (13)

≈ 29.141

Qn. 8

To determine the minimum sample size required to use the Central Limit Theorem, we can use the following formula:

n = [(z*σ) / E] ^2

n = [(1.96 * sqrt (0.72*(1-0.72))) / 0.02] ^2

n ≈ 752

Qn. 9

Using the formula:

$\mu_{\bar{x}} = \mu = 51$

where $\mu_{\bar{x}}$ - sampling distribution of the sample mean

μ- population mean.

$\sigma_{\bar{x}} = \sigma/\sqrt{n}$

$\sigma_{\bar{x}} = 26/\sqrt{16} = 6.5$

Qn. 10

To find the sampling distribution of X1 + X2 / 2,

x

Probability

0 0.20

1 0.60

2 0.20

Using this distribution for X1 + X2:

X1 + X2 = 0:

P (X1 = 0) * P (X2 = 0) = 0.20 * 0.20 = 0.04

X1 + X2 = 1:

P (X1 = 0) * P (X2 = 1) + P (X1 = 1) * P (X2 = 0) = 0.20 * 0.60 + 0.60 * 0.20 = 0.48

X1 + X2 = 2:

P (X1 = 1) * P (X2 = 1) = 0.60 * 0.60 = 0.36

Now the probabilities for X1 + X2 / 2:

X1 + X2 / 2 = 0:

P (X1 + X2 = 0) = 0.04

X1 + X2 / 2 = 0.5:

P (X1 + X2 = 1) = 0.48

P (X1 + X2 = 0) + P (X1 + X2 = 1) = 0.04 + 0.48 = 0.52

P (X1 + X2 / 2 = 0.5) = 0.48 / 0.52 = 0.9231 (rounded to four decimal places)

X1 + X2 / 2 = 1:

P (X1 + X2 = 2) = 0.36

P (X1 + X2 = 1) + P (X1 + X2 = 2) = 0.48 + 0.36 = 0.84

P (X1 + X2 / 2 = 1) = 0.36 / 0.84 = 0.4286 (rounded to four decimal places)

X1 + X2 / 2 = 1.5:

There is no way to get a sum of 1.5 with two whole numbers, so this probability is 0.

X1 + X2 / 2 = 2:

P (X1 + X2 = 2) = 0.36

P (X1 + X2 / 2 = 2) = 0.36 / 0.84 = 0.4286 (rounded to four decimal places)

Therefore, the completed table for the sampling distribution of X1 + X2 / 2 is:

x

Probability

0.0 0.0400

0.5 0.9231

1.0 0.4286

1.5 0

2.0 0.4286

Behavioral Science Assignment #9

Qn. 1

Formula:

Confidence interval = sample mean ± (critical value) x (standard error). standard error = standard deviation / sqrt (sample size)

- ✓ sample size n = 97
- ✓ sample mean $\bar{x}$ = 47
- ✓ population standard deviation σ = 7.

For the z-value from a standard normal distribution table with a tail area of 0.005 (0.01/2) is 2.58.

standard error = σ / sqrt(n) = 7 / sqrt (97) = 0.715

Therefore, Confidence interval = 47 ± 2.58 x 0.715

= (45.05, 48.95)

- ❖ lower limit =45.05
- ❖ upper limit = 48.95

Qn. 2

Formula: CI = $\bar{X}$ ± z(α/2) * (σ/√n)

$\bar{X}$ = sample mean = $71

z(α/2) = z-score with level of confidence = 2.33 (from z-tables)

σ = $19

n = 82

α = 0.02 (98% is 1 - 0.98 = 0.02)

Using the values,

$CI = 71 \pm 2.33 * (19/\sqrt{82})$

$CI = 71 \pm 4.01$

CI = (66.99, 75.01)

- Lower limit = 66.99
- Upper limit = 75.01

Qn. 3

To find the sample size within 4.6 of the population mean, formula for the margin of error:

Margin of error = z* (sigma / sqrt(n)).

z* is the z-score corresponding to the desired level of confidence, sigma is the population standard deviation, and n is the sample size. The z-score corresponding to this level of confidence is 2.576.

Therefore; 4.6 = 2.576 * (18.22 / sqrt(n))

n = (2.576 * 18.22 / 4.6) ^2

$n \approx 114.94$

- sample size of at least 115 is need to achieve a margin of error within 4.6 of the population mean with 99% confidence.

Qn. 4

Formula: $n = (z^2 * \hat{p} * \hat{q}) / E^2$

n = sample size

z = z-score for the desired confidence level (99% in this case)

$\hat{p}$ = sample proportion (0.680 in this case)

$\hat{q} = 1 - \hat{p}$

E = margin of error (0.024 in this case)

Substituting:

n = (2.576^2 * 0.680 * 0.320) / 0.024^2

n ≈ 891.64

sample size is 892; for 0.024 of the population proportion with 99% confidence.

Qn. 5

The formula:

n = (z^2 * p * (1-p)) / E^2

where:

n- sample size

z - z-score associated with the confidence level (95% in this case), which is 1.96

p - sample proportion, which is 0.30 (30% in decimal form)

E - maximum margin of error, which is 0.045

n = (1.96^2 * 0.30 * (1-0.30)) / 0.045^2

n = 267.39

Qn. 6

Margin of error = Critical value x Standard error.

Standard error = standard deviation / square root of sample size

Standard error = 21.8 / sqrt (50)

Standard error = 3.087

Next, the critical value for a 95% confidence interval with 49 degrees of freedom (50 employees sampled - 1): Critical value = 2.01

Therefore, the margin of error:

Margin of error = 2.01 x 3.087

Margin of error = 6.2057

- ❖ The true mean number of hang-ups per employee on that day is between 34.7943 (41.0 - 6.2057) and 47.2057 (41.0 + 6.2057).

Qn. 7

The point estimate is:

Point estimate = Number of defective items / Total number of items in the sample

Point estimate = 5 / 53

Point estimate = 0.0943 (to four decimal places)

- ❖ =0.0943 or approximately 9.43%.