Name Amaya Campbell
Date

MAT 246
Assessment# 2

## Assessment#2 –Chapter 3 & 4

**Directions:** Provide complete responses to each question.

1. Sketch the following function using the first and second derivatives:

$$y = x^3 - x^2 + 8x + 12$$

   a. Find the critical points using the first derivative and identify the intervals where the function is changing (i.e. increasing, decreasing, or constant)

$f(x) = x^3 - x^2 + 8x + \cancel{12}$

$f'(x) = 3x^2 - 2x + 8$

$0 = 3x^2 - 2x + 8$

$x = \frac{-1-i\sqrt{23}}{3}, \frac{1+i\sqrt{23}}{3}$

Critical point:

$\frac{-1-i\sqrt{23}}{3}, -\frac{1+i\sqrt{23}}{3}$

• decrease increase

b. Find the point of inflection(s) and determine the concavity using the second derivative.

$f''(x) = 3x^2 - 2x + 8$

$\frac{dy^2}{dx^2} = 3x^2 + 2x + 8$

$\frac{dy^2}{dx^2} = 6x + 2$

$6x + 2 = 0$

$-2 \quad +2$

$\frac{6x}{6} = \frac{2}{6}$

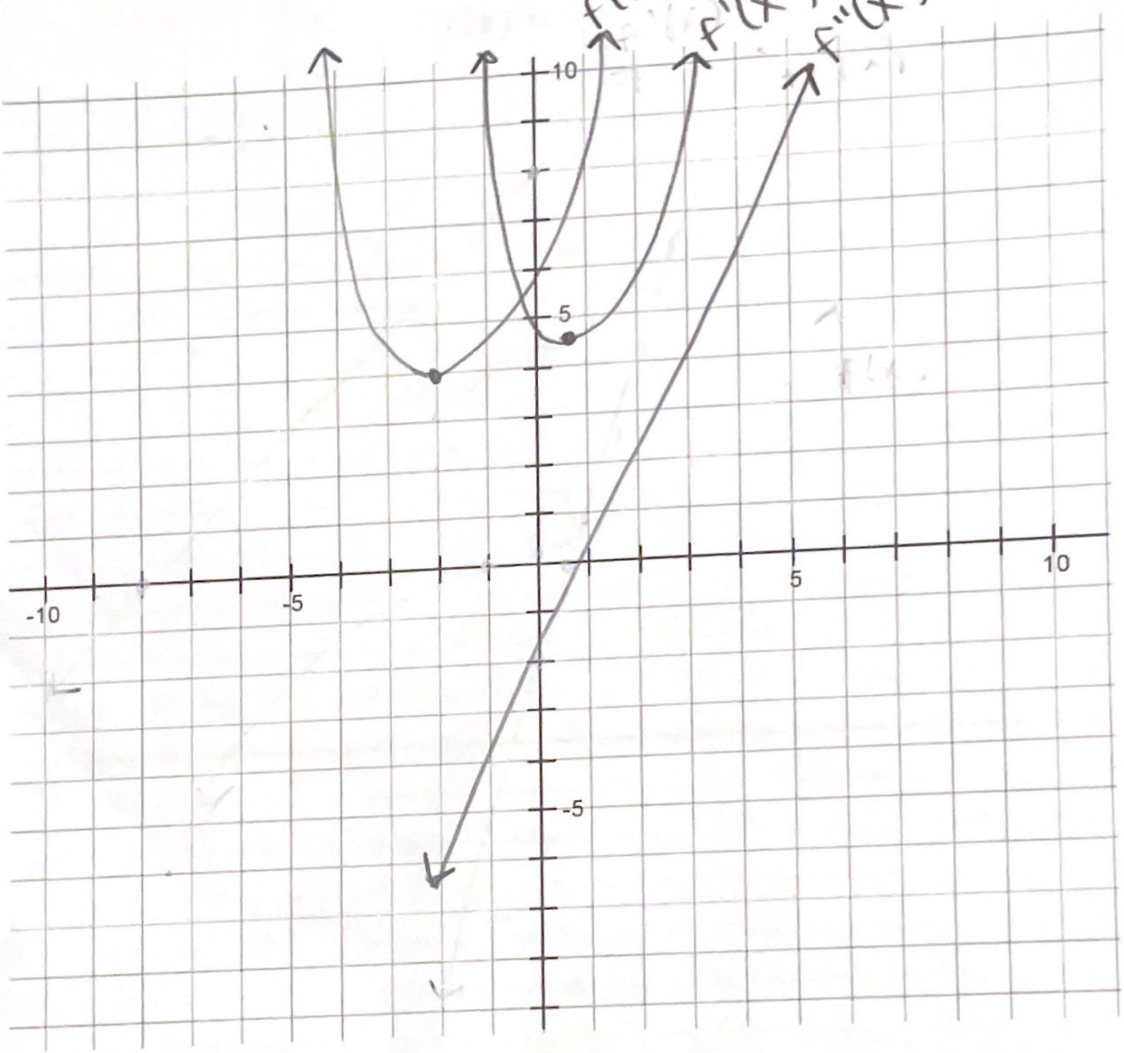
$x = \frac{1}{3}$

$-\frac{1}{3}$ point of inflection with upward concave

Name
Date

c. Sketch the graph

$y = x^3 - x^2 + 8x + 12$




$f' = \frac{df}{dx} = \boxed{3x^2 - 2x + 8}$

$3(0) - 2(0) + 8$

8 increasing function

$f'' = \frac{df}{dx} = 6x - 2$

$\boxed{6x - 2}$

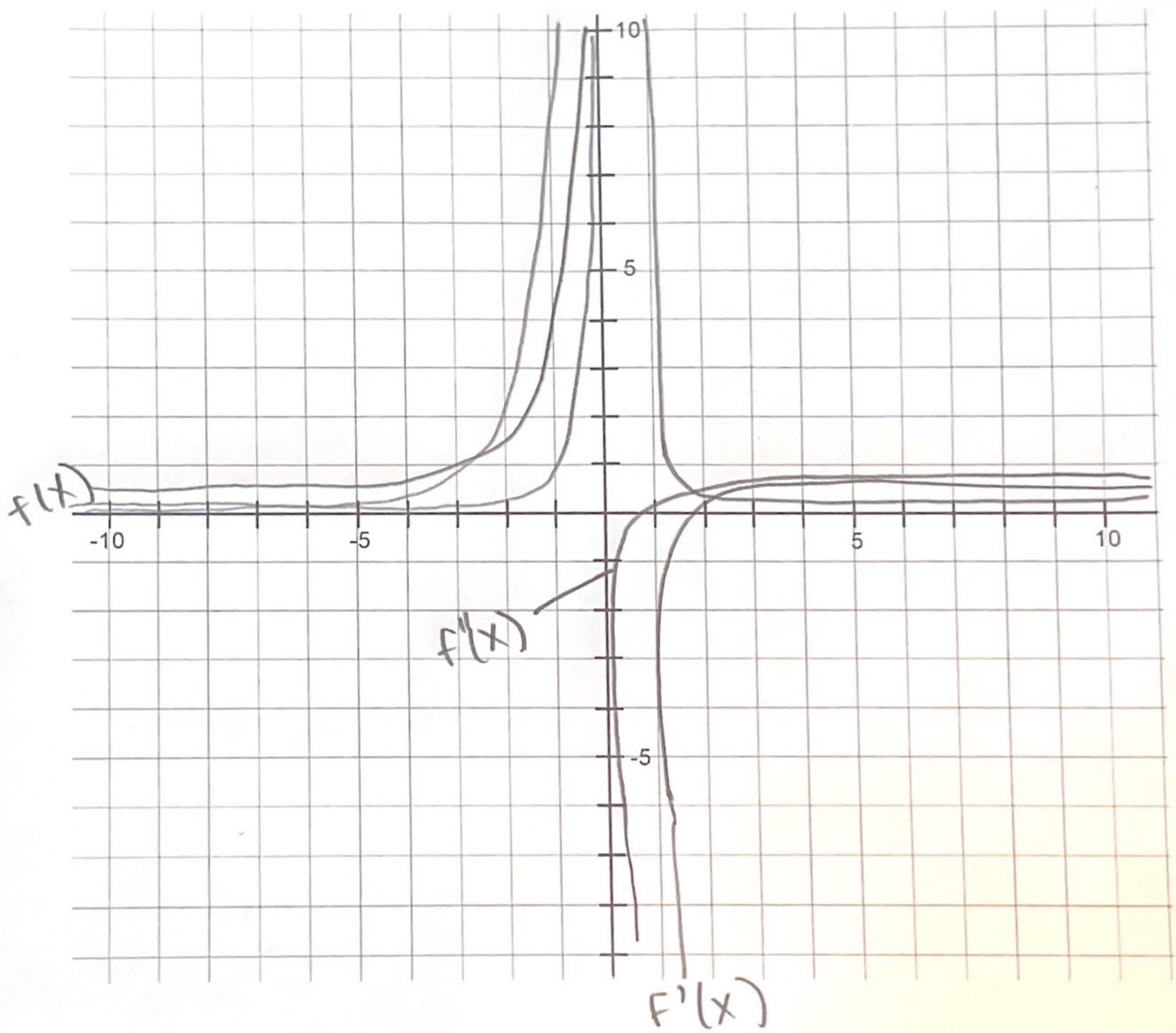
Name
Date

MAT 246
Assessment# 2

2. Sketch the following function (using first and second derivative method):

$$f(x) = \frac{-2(x-1)}{(x+4)^2}$$

$f'(x) = \dfrac{2(x-6)}{(x+4)^3}$ $\quad f''(x) = \dfrac{-4(x-11)}{(x+4)^4}$



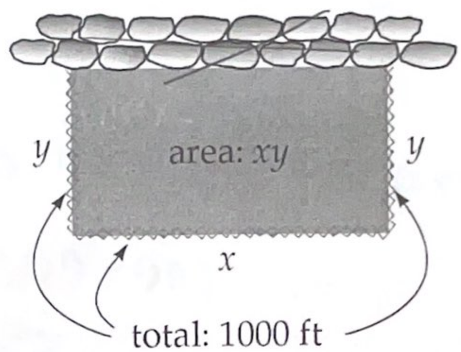
Name
Date

MAT 246
Assessment# 2

4. A farmer has 1000 feet of fence and wants to build a rectangular enclosure along a straight wall. If the side along the wall needs no fence, find the dimensions that make the enclosure as large as possible. Also find the maximum area.

$P = 2y + x = 1000$

$A = xy$




$2y + x = 1000$
$\quad\;\; -x$

$\frac{2y}{2} = \frac{1000-x}{2}$

$y = \frac{1000-x}{2}$

$A(x) = x\left(\frac{1000-x}{2}\right)$

$= -\frac{1}{2}x^2 + 500x + 0$ graphing calculator

maximum point: (500, 125000)

max area x $\quad$ A(x)

$2y + 500 = 1000$
$\quad -500 \;\; -500$

$\frac{2y}{2} = \frac{500}{2}$

$y = 250$

dimensions: x = 500 ft
y = 250 ft
maximum area = 125000

A = 250 (500)

5. An orange grower finds that if he plants 100 orange trees per acre, each tree will yield 50 bushels of oranges. He estimates that for each additional tree that he plants per acre, the yield of each tree will decrease by 4 bushels. How many trees should he plant per acre to maximize his harvest?

$100 + x$ trees/acre

$50 - 4x$ bushels/tree

$A = (100+x)(50-4x) = -4x^2 - 350x + 500$

$\frac{dx}{dy} = -4x^2 - 350x + 500$

$A' = -8x - 350$

$A'(x) = 0$

$-8x - 350 = 0$

$\frac{-8x}{-8} = \frac{350}{-8}$

$x = -43.75$ → maximum

$100 + (-43.75) = 56.25$

$\boxed{\text{plant } 56.25 \text{ orange trees per acre}}$

Name
Date

MAT 246
Assessment# 2

6. Find the value of $100,000 invested for 12 years at 10% compounded quarterly.

$$A = 100,000\left(1 + \frac{0.10}{4}\right)^{4 \cdot 12}$$

$$= 327,148.96$$

$$\boxed{\$327,148.96}$$

7. Find the value of $100,000 invested for 12 years at 10% compounded continuously.

$$P(t) = 100,000e^{0.10(12)}$$

$$= \boxed{\$\ 332011.69}$$

8. Find the present value of $50,000 to be paid 10 years from now at 10% interest compounded daily.

$$PVIF = \frac{50,000}{(1+.10)^{10}}$$

$$\boxed{= \$19277.16}$$

Name
Date

MAT 246
Assessment# 2

9. Evaluate the following expressions (without using a calculator):

a. $\log_4 64$ $= \boxed{3}$

$\log_4 64 = x$

$4^x = 4^3$

$x = 3$

$10^4 = 1000$

$2^6 = 64$

b. $\log_{10} 10000 - \log_2 64 + \ln e^5$

$\log_{10} 10000 - \log_2 64 + \ln e^5$

$10^4 = 10000 - 2^6 = 64 + 5$

$4 - 6 + 5$

$\boxed{3}$

c. $\frac{e^{\ln 88}}{\log_2 16}$

$\frac{e^{\ln 88} = 8}{\log_2 16 = 2^4 = 16} = \frac{88}{4} = \boxed{22}$