Name
Date

MAT 246
Assessment# 2

Assessment#2 –Chapter 3 & 4

Directions: Provide complete responses to each question.

1. Sketch the following function using the first and second derivatives:

$$y = x^3 - x^2 + 8x + 12$$

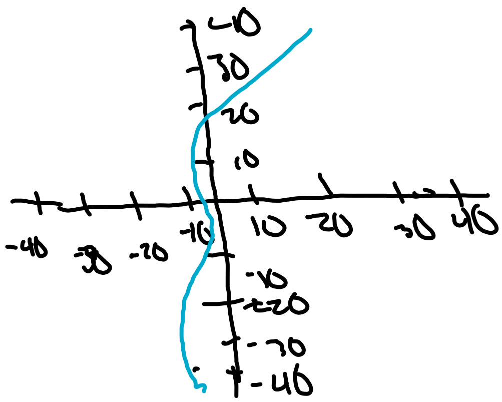
a. Find the critical points using the first derivative and identify the intervals where the function is changing (i.e. increasing, decreasing, or constant)

$y = x^3 - x^2 + 8x + 12$

$y' = (x^3 - x^2 + 8x + 12)$

$y' = 3x^2 - 2x + 8$

$y'' = 6x - 2$



1.a. no critical points

1.b. $\left(\frac{1}{3}, \frac{394}{27}\right)$

concave down on $(-\infty, \frac{1}{3})$ is negative

concave up on $(\frac{1}{3}, \infty)$ is positive

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b. Find the point of inflection(s) and determine the concavity using the second derivative.

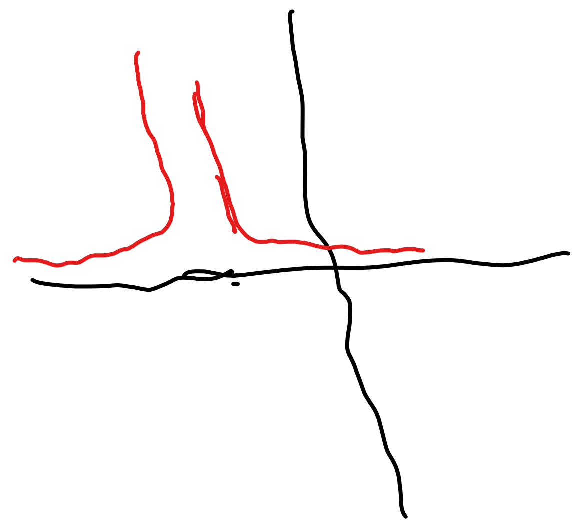
$$f(x) = \frac{-2(x-1)}{(x+4)^2}$$

$$f'(x) = \left(\frac{-2x+2}{(x+4)^2} \right.$$

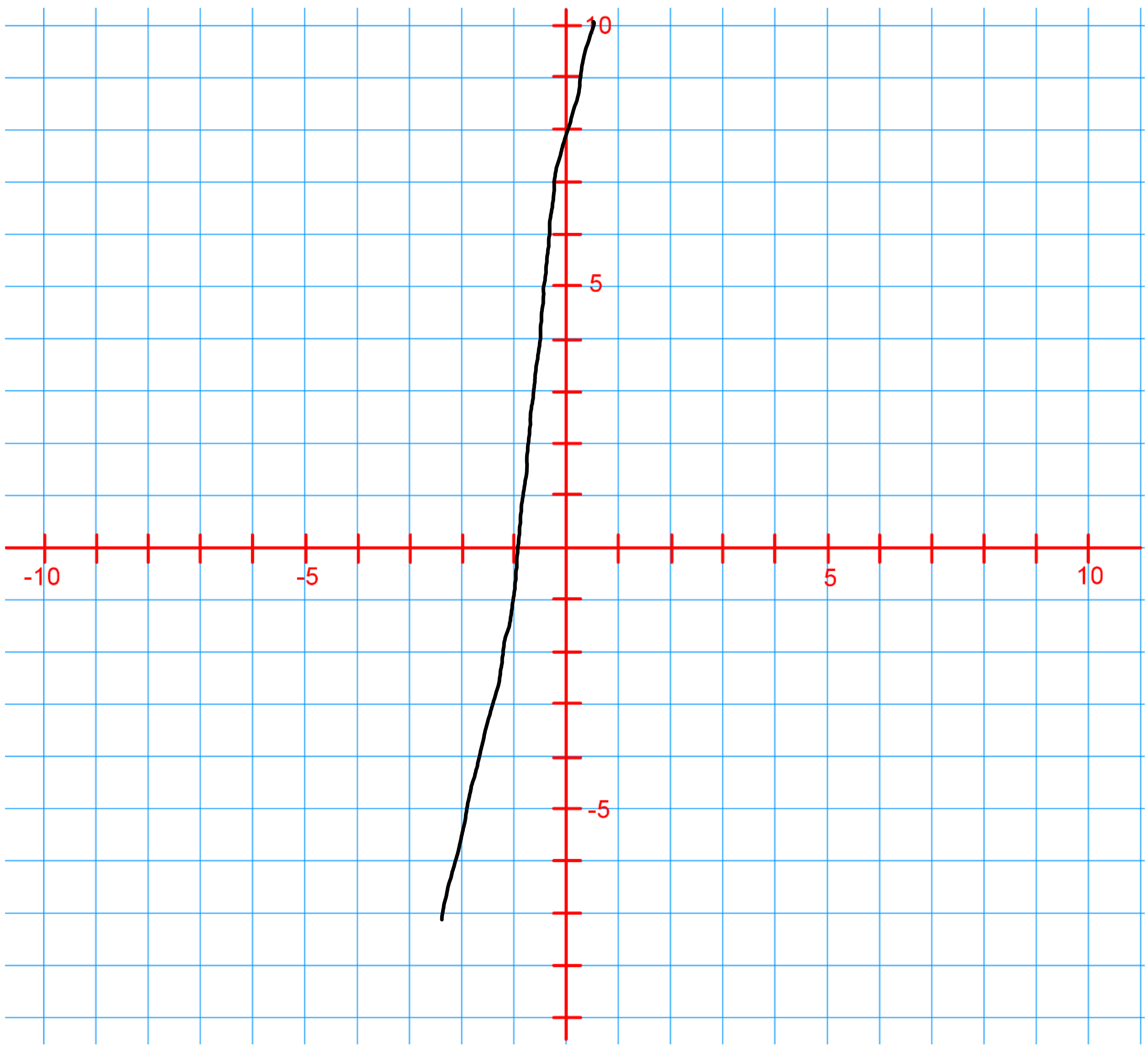
$$f'(x) = \frac{-2(x+4)^2 - (-2x+2) \cdot 2(x+4)}{((x+4)^2)^2}$$

$$f'(x) = \frac{2x-12}{(x+4)^3}$$

$$f''(x) = -\frac{4(3-x)}{(x+4)^4}$$



c. Sketch the graph



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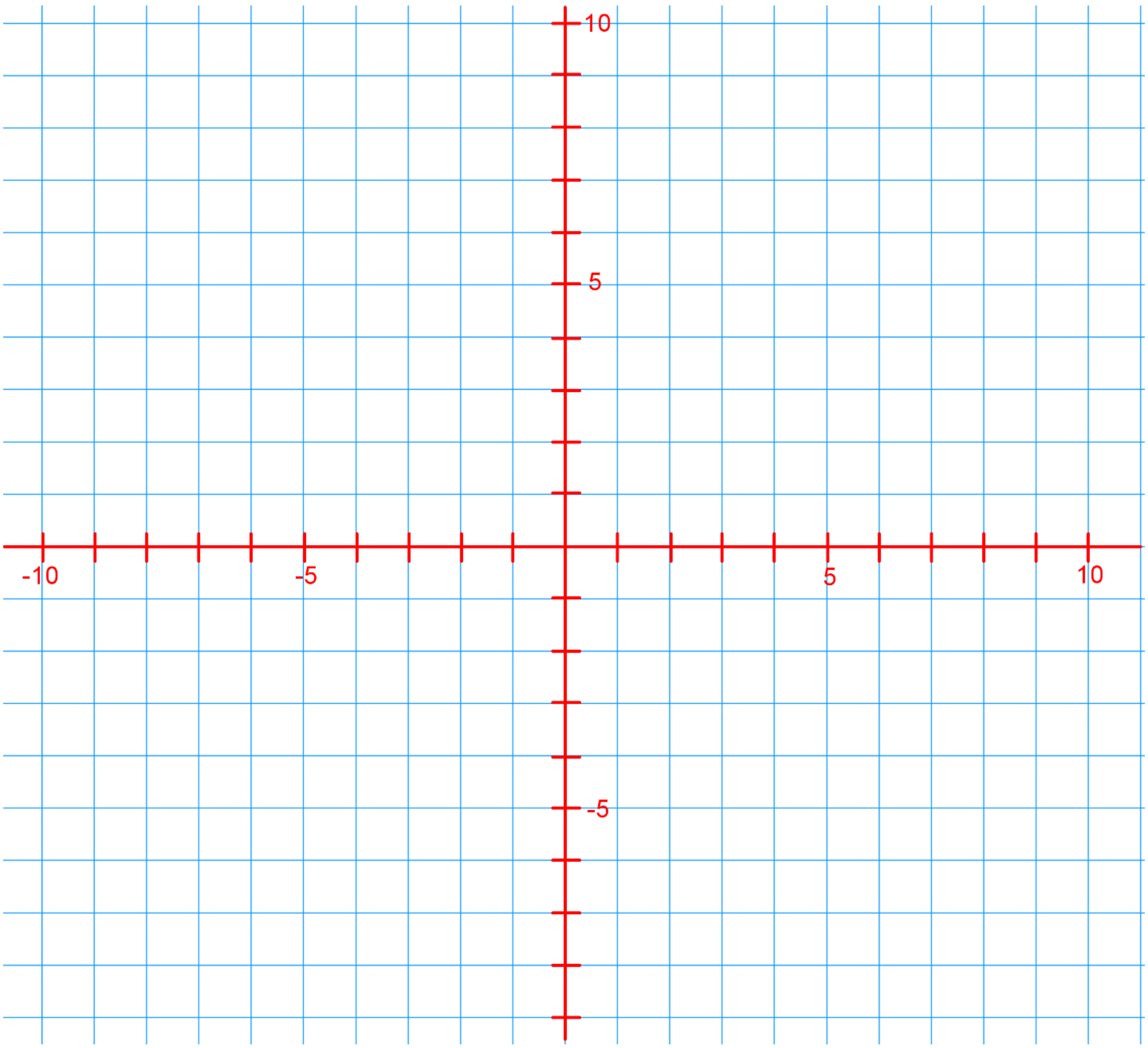
2. Sketch the following function (using first and second derivative method):

$$f(x) = \frac{-2(x-1)}{(x+4)^2}$$



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3. It costs the American Automobile Company \$10,000 to produce each automobile, and fixed costs (rent and other expenses that do not depend on the amount of production) are \$40,000 per week. The company's price function is:

$$p(x) = \$42{,}000 - 100x$$

where p is the price at which exactly x cars will be sold.

a. How many cars should be produced each week to maximize profit?

$(42{,}000 - 100x) \times x = 42{,}000x - 100x^2$

$42{,}000x - 100x^2 - 10{,}000x - 40{,}000 = -100x^2 + 32{,}000x - 40{,}000$

$= -200x + 32{,}000 = 2$

$x = 160$

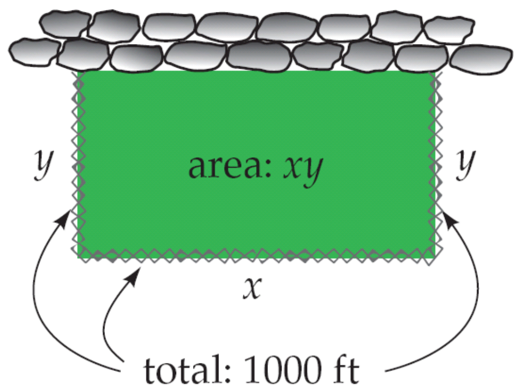
b. For what price should they be sold?

$P(160) = \$42{,}000 - 100(160) = \$26{,}000$

c. What is the company's maximum profit?

$\pi(160) = -100(160)^2 + \$32{,}000(160) - \$40{,}000 = \$1{,}600{,}000$

4. A farmer has 1000 feet of fence and wants to build a rectangular enclosure along a straight wall. If the side along the wall needs no fence, find the dimensions that make the enclosure as large as possible. Also find the maximum area.



$1000 = x + 2y$

$x = 1000 - 2y$

$A = xy = (1000 - 2y)y = 1000y - 2y^2$

$A' = 1000 - 4y = 0$

$y = 250$

$x = 1000 - 2(250) = 500$

$A = xy = (500)(250) = 125{,}000$ square feet

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5. An orange grower finds that if he plants 100 orange trees per acre, each tree will yield 50 bushels of oranges. He estimates that for each additional tree that he plants per acre, the yield of each tree will decrease by 4 bushels. How many trees should he plant per acre to maximize his harvest?

$50 - 4x$

$y = (100 + x)(50 - 4x) = 5000 - 300x - 4x^2$

$y' = -300 - 8x = 0$

$x = 37.5$

$y = (100 + 37)(50 - 4(37)) = 10,625$ bushels per acre

6. Find the value of $100,000 invested for 12 years at 10% compounded quarterly.

$A = 100{,}000(1+0.10/4)^{(4 \cdot 12)}$

$A = 100{,}000(1.025)^{48}$

$A = 100{,}000 \, 2.208426$

$A = 220{,}842.58$

7. Find the value of $100,000 invested for 12 years at 10% compounded continuously.

$A = 100{,}00e^{(0.10 \cdot 12)}$

$A = 100{,}000e^{1.2}$

$A = 100{,}000 \, 3.320116922736547 2$

$A = 332{,}011.69$

8. Find the present value of $50,000 to be paid 10 years from now at 10% interest compounded daily.

$PV = 50{,}000/(1+0.10/365)^{(365 \cdot 10)}$

$PV = 50{,}000/(1.000273972 6)^{3650}$

$PV = 50{,}000/1.3498588071$

$PV = 37{,}041.35$

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9. Evaluate the following expressions (without using a calculator):
   a. $\log_4 64$

$4^3 = 64$, $\log_4 64 = 3$

   b. $\log_{10} 10000 - \log_2 64 + \ln e^5$

$4 - \log_2 64 + \log_e e^5$

$4 - 6 + 5$

$-2 + 5$

3

   c. $\frac{e^{\ln 88}}{\log_2 16}$

$\frac{88}{\log_2(2^4)}$

$\frac{88}{4}$

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