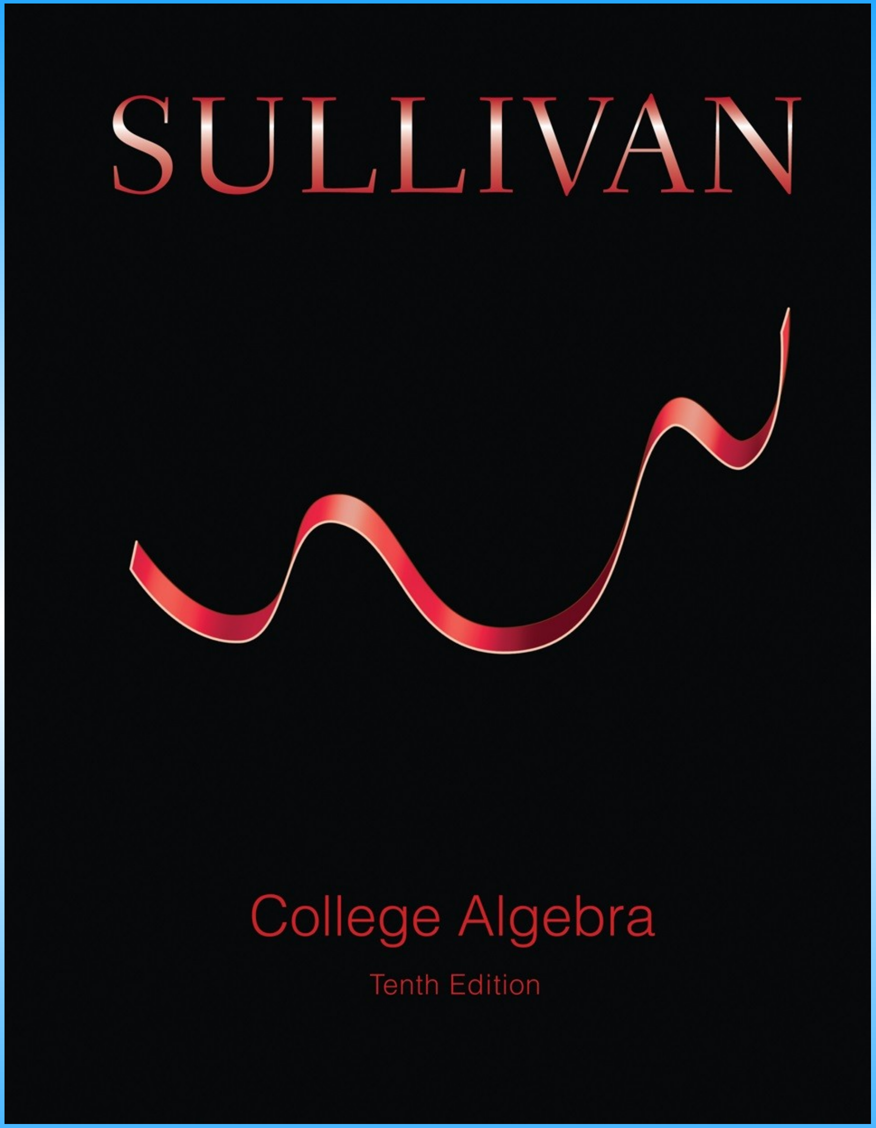
Chapter 1

Section 4



1.4 Radical Equations; Equations Quadratic in Form; Factorable Equations

PREPARING FOR THIS SECTION *Before getting started, review the following:*

- Square Roots (Section R.2, pp. 23–24)
- Factoring Polynomials (Section R.5, pp. 49–55)
- *n*th Roots; Rational Exponents (Section R.8, pp. 73–77)

Now Work the **'Are You Prepared?'** problems on page 117.

OBJECTIVES
1. Solve Radical Equations (p. 113)
2. Solve Equations Quadratic in Form (p. 114)
3. Solve Equations by Factoring (p. 116)

Solve Radical Equations

Example

Solving a Radical Equation

Find the real solutions of the equation: $\sqrt{x-1} = x - 7$

Solution

Square both sides since the index of a square root is 2.

$$\sqrt{x-1} = x - 7$$

$$\left(\sqrt{x-1}\right)^2 = (x-7)^2 \quad \text{Square both sides.}$$

$$x - 1 = x^2 - 14x + 49 \quad \text{Remove parentheses.}$$

$$x^2 - 15x + 50 = 0 \quad \text{Put in standard form.}$$

$$(x-10)(x-5) = 0 \quad \text{Factor.}$$

$$x = 10 \quad \text{or} \quad x = 5 \quad \text{Apply the Zero-Product Property and solve.}$$

✓**Check:**

$$x = 10: \quad \sqrt{x-1} = \sqrt{10-1} = \sqrt{9} = 3 \text{ and } x - 7 = 10 - 7 = 3$$

$$x = 5: \quad \sqrt{x-1} = \sqrt{5-1} = \sqrt{4} = 2 \text{ and } x - 7 = 5 - 7 = -2$$

The solution $x = 5$ does not check, so it is extraneous; the only solution of the equation is $x = 10$. The solution set is $\{10\}$.

Example

Solving a Radical Equation

Find the real solutions of the equation: $\sqrt{2x + 3} - \sqrt{x + 2} = 2$

Solution

First, isolate the more complicated radical expression (in this case, $\sqrt{2x+3}$) on the left side.

$$\sqrt{2x+3} = \sqrt{x+2} + 2$$

Now square both sides (the index of the radical on the left is 2).

$$\left(\sqrt{2x+3}\right)^2 = \left(\sqrt{x+2} + 2\right)^2 \qquad \text{Square both sides.}$$

$$2x + 3 = \left(\sqrt{x+2}\right)^2 + 4\sqrt{x+2} + 4 \qquad \text{Multiply out.}$$

$$2x + 3 = x + 2 + 4\sqrt{x+2} + 4 \qquad \text{Simplify.}$$

$$2x + 3 = x + 6 + 4\sqrt{x+2} \qquad \text{Combine like terms.}$$

Because the equation still contains a radical, isolate the remaining radical on the right side and again square both sides.

$$x - 3 = 4\sqrt{x+2} \qquad \text{Isolate the radical on the right side.}$$

$$(x-3)^2 = 16(x+2) \qquad \text{Square both sides.}$$

$$x^2 - 6x + 9 = 16x + 32 \qquad \text{Multiply out.}$$

$$x^2 - 22x - 23 = 0 \qquad \text{Put in standard form.}$$

$$(x-23)(x+1) = 0 \qquad \text{Factor.}$$

$$x = 23 \quad \text{or} \quad x = -1 \qquad \text{Apply the Zero-Product Property and solve.}$$

Solution continued

The original equation appears to have the solution set $\{-1, 23\}$. However, we have not yet checked.

✓Check:

$x = 23$: $\sqrt{2x + 3} - \sqrt{x + 2} = \sqrt{2(23) + 3} - \sqrt{23 + 2} = \sqrt{49} - \sqrt{25} = 7 - 5 = 2$

$x = -1$: $\sqrt{2x + 3} - \sqrt{x + 2} = \sqrt{2(-1) + 3} - \sqrt{-1 + 2} = \sqrt{1} - \sqrt{1} = 1 - 1 = 0$

The equation has only one solution, 23; the solution -1 is extraneous. The solution set is $\{23\}$.

Solve Equations Quadratic in Form

Example

Solving an Equation Quadratic in Form

Find the real solutions of the equation: $(x+2)^2 + 11(x+2) - 12 = 0$

Solution

For this equation, let $u = x + 2$. Then $u^2 = (x + 2)^2$, and the original equation,

$$(x + 2)^2 + 11(x + 2) - 12 = 0$$

becomes

$$u^2 + 11u - 12 = 0 \quad \text{Let } u = x + 2. \text{ Then } u^2 = (x + 2)^2.$$

$$(u + 12)(u - 1) = 0 \quad \text{Factor.}$$

$$u = -12 \quad \text{or} \quad u = 1 \quad \text{Solve.}$$

But we want to solve for x. Because $u = x + 2$, we have

$$x + 2 = -12 \quad \text{or} \quad x + 2 = 1$$

$$x = -14 \qquad\qquad x = -1$$

✓**Check:** $x = -14$: $(-14 + 2)^2 + 11(-14 + 2) - 12$

$$= (-12)^2 + 11(-12) - 12 = 144 - 132 - 12 = 0$$

$x = -1$: $(-1 + 2)^2 + 11(-1 + 2) - 12 = 1 + 11 - 12 = 0$

The original equation has the solution set $\{-14, -1\}$.

Example

Solving an Equation Quadratic in Form

Find the real solutions of the equation: $x + 2\sqrt{x} - 3 = 0$

Solution

For the equation $x + 2\sqrt{x} - 3 = 0$, let $u = \sqrt{x}$. Then $u^2 = x$, and the original equation,

$$x + 2\sqrt{x} - 3 = 0$$

becomes

$$u^2 + 2u - 3 = 0 \quad \text{Let } u = \sqrt{x}. \text{ Then } u^2 = x.$$

$$(u + 3)(u - 1) = 0 \quad \text{Factor.}$$

$$u = -3 \quad \text{or} \quad u = 1 \quad \text{Solve.}$$

Since $u = \sqrt{x}$, we have $\sqrt{x} = -3$ or $\sqrt{x} = 1$. The first of these, $\sqrt{x} = -3$, has no real solution, since the principal square root of a real number is never negative. The second, $\sqrt{x} = 1$, has the solution $x = 1$.

✓**Check:** $1 + 2\sqrt{1} - 3 = 1 + 2 - 3 = 0$

The original equation has the solution set $\{1\}$.

Solve Equations by Factoring

Example

Solving an Equation by Factoring

Solve the equation: $x^4 = 4x^2$

Solution

Begin by collecting all terms on one side. This results in 0 on one side and an expression to be factored on the other.

$$x^4 = 4x^2$$
$$x^4 - 4x^2 = 0$$
$$x^2(x^2 - 4) = 0 \quad \text{Factor.}$$
$$x^2 = 0 \quad \text{or} \quad x^2 - 4 = 0 \quad \text{Apply the Zero-Product Property.}$$
$$x = 0 \quad \text{or} \quad x^2 = 4$$
$$x = 0 \quad \text{or} \quad x = -2 \quad \text{or} \quad x = 2$$

✓Check: $x = -2$: $(-2)^4 = 16$ and $4(-2)^2 = 16$ — -2 is a solution.

$x = 0$: $0^4 = 0$ and $4 \cdot 0^2 = 0$ — 0 is a solution.

$x = 2$: $2^4 = 16$ and $4 \cdot 2^2 = 16$ — 2 is a solution.

The solution set is $\{-2, 0, 2\}$.