Name **MAT 246**
Date **Assessment# 1**

Assessment# 1 –Chapter 1 & 2

Directions: Provide complete responses to each question.

1. Write an equation of the line satisfying the following conditions. If possible, write your answer in the following form $y = mx + b$:
   a. Passing through the points (5, 3) and (7, 1)

$m = \frac{y_2 - y_1}{x_2 - x} = \frac{1-3}{7-5} = \frac{-2}{2} = -1$

$y - y_1 = -1(x - x_1)$

$y - 3 = -1(x - 5)$

$y - 3 = -x + 5$

$y = -x + 8$

   b. Horizontal and passing through the point (1.5, - 4)

$y = -4$

$y = 0x - 4$

   c. Passing through the point (12, 2) that is (i) parallel to the line $4y - 3x = 5$ and (ii) perpendicular to the line $4y - 3x = 5$

$4y = 3x + 5$

$y = \frac{3}{4}x + \frac{5}{4}$; slope: $\frac{3}{4}$

$y - 2 = \frac{5}{4}(x - 12)$

$y - 2 = \frac{5}{4}x - 15$

$y = \frac{5}{4}x - 13$

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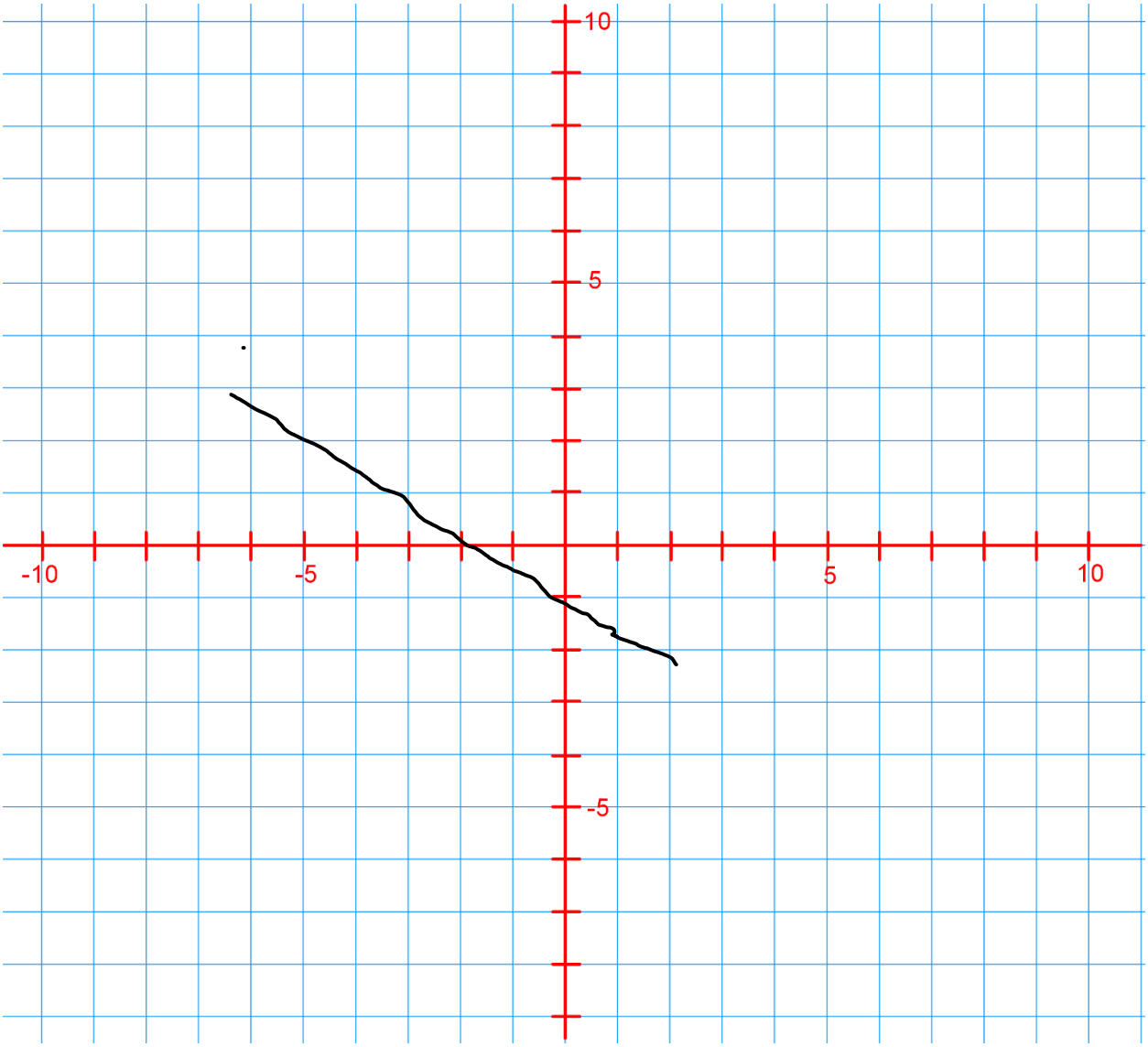
2. Sketch the following linear equation: $x + 2y = -2$



3. Evaluate each expression without using a calculator:

a. $4^{-2} \cdot 2^{-1}$

$2^{-4} \cdot 2^{-1}$

2^{-5}

$\frac{1}{2^5} = \frac{1}{32}$

b. $\left[\left(\frac{2}{3}\right)^{-2}\right]^{-4}$

$\left(\frac{2}{3}\right)^{8}$

$\frac{256}{6561}$

c. $\left(\frac{1}{32}\right)^{2/5}$

$\sqrt[5]{\left(\frac{1}{32}\right)^2}$

$\sqrt[5]{\frac{1}{32}^2} = \left(\frac{1}{2}\right)^2 = \frac{1}{4}$

d. $(-8)^{-1/3}$

undefined

4. Write each expression in power form ax^b for numbers, a and b.

a. $\frac{4}{x^5}$

$= 4 \cdot (x)^{-5}$

$= 4x^{-5}$

b. $\frac{4}{\sqrt[3]{8x^4}}$

$\frac{2^2}{\sqrt[3]{2^3 \cdot x^2}} = \frac{2^2}{\sqrt[3]{2^3}} = \frac{2^2}{2^3} = 2^{-1/3}$

c. $\frac{24}{(2\sqrt{x})^3}$

$\frac{24}{4x}$

$= \frac{6}{x}$

5. Solve each equation by factoring or the Quadratic Formula, as appropriate.

a. $x^2 - 6x - 7 = 0$

$x^2 + x - 7x - 7 = 0$

$x(x+1) - 7(x+1) = 0$

$(x+1)(x-7) = 0$

$x + 1 = 0$

$x - 7 = 0$

$x = -1 \quad , x = 7$

b. $x^2 + 2x = 15$

$x^2 + 2x - 15 = 0$

$x^2 + 5x - 3x - 15 = 0$

$x(x+5) - 3(x+5) = 0$

$(x+5)(x-3) = 0$

$x + 5 = 0$

$x - 3 = 0$

$x = -5 \quad , x = 3$

c. $2x^2 + 40 = 8x$

$x^2 + 20 = 4x$

$x^2 - 4x + 20 = 0$

$a = 1, b = -4, c = 20$

$x = \frac{-(-4) \pm \sqrt{(-4)^2 - 4 \times 1 \times 20}}{2 \times 1}$

$x = \frac{4 \pm \sqrt{16 - 80}}{2}$

$x = \frac{4 \pm \sqrt{-64}}{2}$

$x \notin \mathbb{R}$

d. $5x^2 - 50x = 0$

$x^2 - 10 = 0$

$a = 1, b = 0, c = -10$

$x = \frac{-0 \pm \sqrt{0^2 - 4 \times 1 \times (-10)}}{2 \times 1}$

$x = \frac{\pm \sqrt{0 + 40}}{2}$

$x = \frac{\pm 2\sqrt{10}}{2}$

$x = -\sqrt{10}, x = \sqrt{10}$

6. ***Cost Functions*** A lumberyard will deliver wood for $4 per board foot plus a delivery charge of $20. Find a function C(x) for the cost of having x board feet of lumber delivered.

$C(x) = 4x + 20$

7. For each function:
   a. Evaluate the given expression.
   b. Find the domain of the function.
   c. Find the range.

   i. $f(x) = \frac{1}{x+4}$ $f(-3)$

$f(-3) = \frac{1}{-3+4}$

$f(x) = \frac{1}{1}$

$f(x) = 1$

$f(x) = \frac{1}{x+4}$

$\frac{1}{x+4}$

$x+4$

$\{-4\}$

ii. $f(x) = \frac{12}{x(x+4)}$ $f(2)$

$f(2) = \frac{12}{2(2+4)}$ $f(2) = 1$

$f(x) = \frac{12}{x(x+4)}$

x

$x + 4$

$\{-4, 0\}$

iii. $f(x) = 4^x$ $f\left(-\frac{1}{2}\right)$

$f\left(-\frac{1}{2}\right) = 4^x$

$f\left(-\frac{1}{2}\right) = \frac{1}{2}$

$f(x) = 4x$

All real numbers

iv. $f(x) = 8^x \quad f\left(-\frac{1}{3}\right)$

$f\left(-\frac{1}{3}\right) = 8^x$

$f\left(-\frac{1}{3}\right) = 2$

$f(x) = 8^x$

All real numbers

8. Solve each equation by factoring.

a. $x^5 + 2x^4 - 3x^3 = 0$

$x^3(x^2 + 2x - 3) = 0$

$x^3(x^2 + 3x - x - 3) = 0$

$x^3[x(x+3) - 1(x+3)] = 0$

$x^3(x+3)(x-1) = 0$

$x = -3, x = 0, x = 1$

b. $5x^3 - 20x = 0$

$5x(x^2 - 4) = 0$

$x(x^2 - 4) = 0$

$x = -2, x = 0, x = 2$

c. $2x^5 - 50x^3 = 0$

$2x^3(x^2 - 25) = 0$

$x^3(x^2 - 25) = 0$

$x = -5, x = 0, x = 5$

d. $3x^{\frac{5}{2}} - 6x^{\frac{3}{2}} = 9x^{\frac{1}{2}}$ $x = 0, x = 3$

9. For each pair of functions f(x) and g(x), find

   a. f(g(x))
   b. g(f(x))
   c. f(f(x))

   i. $f(x) = x^5 \quad g(x) = 7x - 1$

   $f(f(x)) = x^{10}$ $\qquad$ $g(f(x)) = 7(x^5) - 1$

   ii. $f(x) = \frac{1}{x} \quad g(x) = x^2 + 1$

   $f(f(x)) = \frac{1}{\frac{1}{x}}$ $\qquad$ $g(g(x)) = \left(\frac{1}{x}\right)^2 + 1$

10. Find the following limits without using a graphing calculator or making tables.

a. $\lim_{x\to 3}(4x^2 - 10x + 2)$

$4 \cdot 3^2 - 10 \cdot 3 + 2$

$= 8$

b. $\lim_{x\to 5}\left(\frac{3x^2-5x}{7x-10}\right)$

$\frac{3 \times 5^2 - 5 \times 5}{7 \times 5 - 10}$

2

c. $\lim_{x\to 1}\left(\frac{x-1}{x^2+x-2}\right)$

$= \left(\frac{x-1}{x^2+2x-x-2}\right)$

$= \left(\frac{x-1}{x(x+2)-1(x+2)}\right)$

$= \left(\frac{x-1}{(x-1)(x+2)}\right)$

$= \left(\frac{1}{x+2}\right)$

$= \frac{1}{1+2} = \frac{1}{3}$

d. $\lim_{h\to 0}\left(\frac{5x^4h-9xh^2}{h}\right)$

$= \left(\frac{5x^4h-9xh^2}{h}\right)$

$= \left(\frac{5(5x^4h-9xh)}{h}\right)$

$= (5x^4-9xh)$

$= \lim(5x^4) - \lim(9\cdot h)$

$5x^4 - 9x\cdot 0$

$5x^4$

11. Find f'(x) by using the definition of the derivative.

Note: Use the Limit of Definition of Derivative

$$\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$$

a. $f(x)=2x-7$

$f'(x) = \lim_{h\to 0}\left(\frac{2(x+h)-2-(2x-7)}{h}\right)$

$f'(x)=2$

b. $f(x)=x^2+8x-9$

$f'(x) = \lim_{h\to 0}\left(\frac{(x+h)^2+8(x+h)-9-(x^2+8x-9)}{h}\right)$

$f'(x) = 2x+8$

c. $f(x) = x^3 + 2x^2 - 4x + 6$

$$f'(x) = \left(\frac{(x+h)^3 + 2(x+h)^2 - 4(x+h) + 6 - (x^3 + 2x^2 + 4x + 6)}{h} \right)$$

$$f'(x) = 3x^2 + 4x - 4$$

d. $f(x) = \frac{1}{x^2}$

$$f'(x) = \lim_{h \to 0} \left(\frac{\frac{1}{(x+h)^2} - \frac{1}{x^2}}{h} \right)$$

$$f'(x) = \frac{-2}{x^3}$$

12. Find the derivative of the following functions using any method:

a. $f(x) = x^5 \cdot x^{\frac{1}{2}}$

$f'(x) = \frac{d}{dx}\left(x^5 \cdot x^{\frac{1}{2}}\right)$

$f'(x) = \frac{d}{dx}\left(x^{\frac{11}{2}}\right)$

$f'(x) = \frac{11}{2} x^{\frac{9}{2}}$

$f'(x) = \frac{11 x^4 \sqrt{x}}{2}$

b. $f(x) = \frac{(x+1)}{(x+2)}$

$f'(x) = \frac{d}{dx}\left(\frac{x+1}{x+2}\right)$

$f'(x) = \frac{\frac{d}{dx}(x+1)(x+2) - (x+1) \times \frac{d}{dx}(x+2)}{(x+2)^2}$

$f'(x) = \frac{1(x+2) - (x+1) \times 1}{(x+2)^2}$

$f'(x) = \frac{1}{(x+2)^2}$

c. $f(x) = (2x-3)^4$

$f'(x) = \frac{d}{dx}\left((2x-3)^4\right)$

$f'(x) \frac{d}{dg}(g^4) \times \frac{d}{dx}(2x-3)$

$f'(x) = 4g^3 \times 2$

$f'(x) = 4(2x-3)^3 \times 2$

$f'(x) = 8(2x-3)^3$

d. $f(x) = \sqrt[4]{x^7}$

$f'(x) = \frac{d}{dx}\left(\sqrt[4]{x^7}\right)$

$f'(x) = \frac{d}{dx}\left(x^{\frac{7}{4}}\right)$

$f'(x) = \frac{7}{4}x^{\frac{3}{4}}$

$f'(x) = \frac{7}{4} \times \sqrt[4]{x^3}$

13. For each function, find:
    a. $f'(x)$
    b. $f''(x)$
    c. $f'''(x)$
    d. $f''''(x)$

i. $f(x) = x^6 + 2x^4 - x^2$

$f'(x) = \frac{d}{dx}(x^6 + 2x^4 - x^2)$

$f'(x) = (x^6) + (2x^4) - x^2$

$f'(x) = 6x^5 + 2 \cdot 4x^3 - 2x$

$f'(x) = 6x^5 + 8x^3 - 2x$

$f''(x) = 16x^3 + 8x^3 - 2x$

ii. $f(x) = \sqrt[4]{x}$

14. GENERAL Velocity: After t hours a freight train is

$s(t) = 18t^2 - 2t^3$ miles due north of its starting point

$for\ (0 \leq t \leq 9)$

a. Find the velocity at $t = 3$

b. Find the velocity at $t = 7$

c. Find the acceleration at $t = 1$