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MAT 246
Assessment# 1

Assessment# 1 –Chapter 1 & 2

Directions: Provide complete responses to each question.

1. Write an equation of the line satisfying the following conditions. If possible, write your answer in the following form $y = mx + b$:
   a. Passing through the points (5, 3) and (7, 1)

$m = \frac{1-3}{7-5} = \frac{-2}{2} = -1$

plug in:

$\therefore y = -x + 8$

$1 = (-1)(7) + b$

$b = 1 + 7 = 8$

   b. Horizontal and passing through the point (1.5, - 4)

$y = b$

$m = 0$

$\therefore y = -4$

   c. Passing through the point (12, 2) that is (i) parallel to the line $4y - 3x = 5$ and (ii) perpendicular to the line $4y - 3x = 5$

parallel

$m = 3/4$

$y = 3/4 x - 9 + 2$

$y = 3/4 x - 7$

perpendicular

$m = 3/4 \rightarrow -4/3$

$y - 2 = -4/3 (x - 12)$

$y = -4/3 x + 14$

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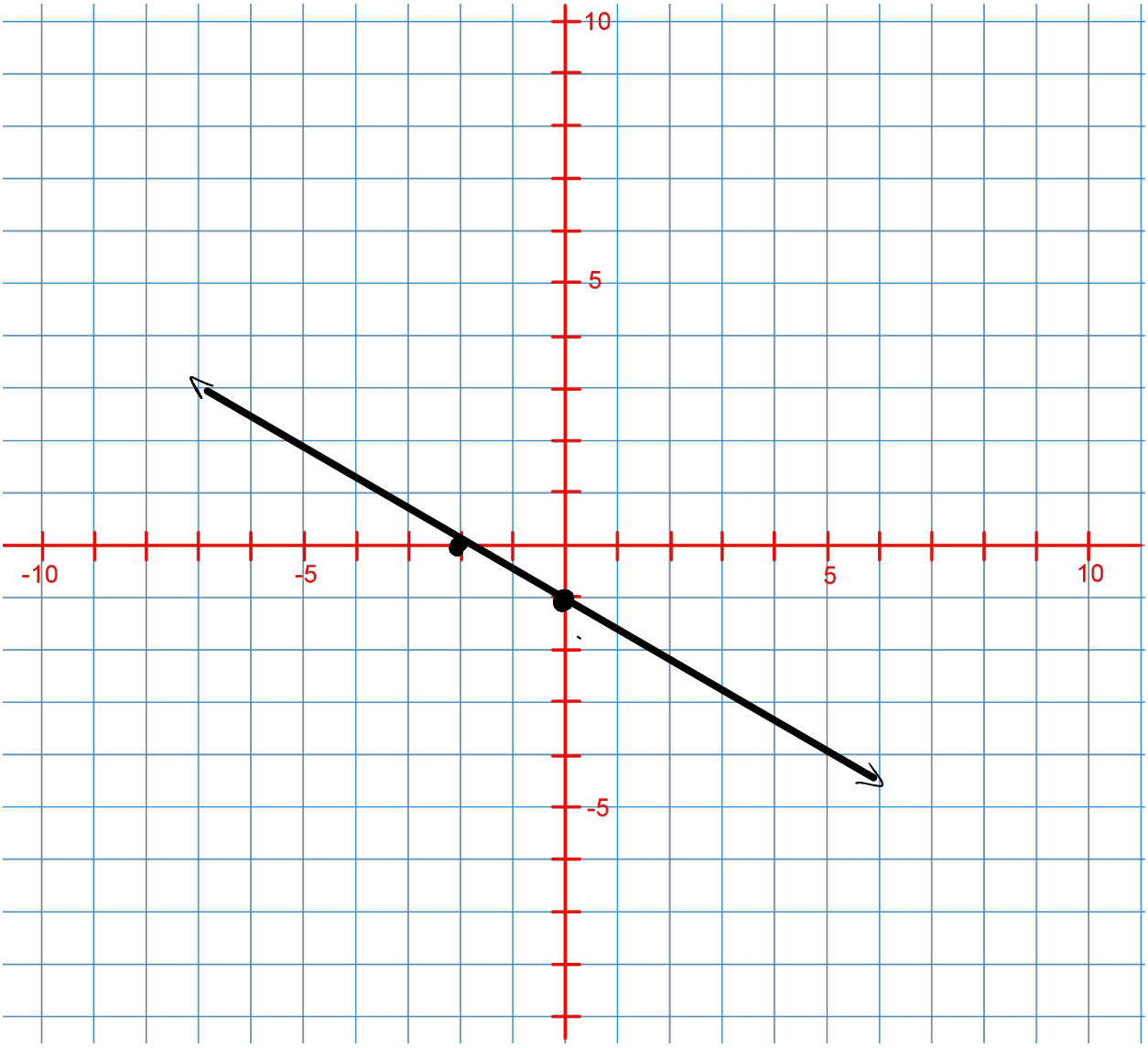
2. Sketch the following linear equation: $x + 2y = -2$



3. Evaluate each expression without using a calculator:

a. $4^{-2} \cdot 2^{-1}$

$= (2^2)^{-2} \cdot 2^{-1}$

$= 2^{-4} \cdot 2^{-1}$

$= 2^{-5} = \frac{1}{2^5} = \frac{1}{32}$

b. $\left[\left(\frac{2}{3}\right)^{-2}\right]^{-4}$

$= \left(\frac{2}{3}\right)^{8} = \frac{256}{6561}$

c. $\left(\frac{1}{32}\right)^{2/5}$

$= \sqrt[5]{\frac{1}{32}^2}$

$= \left(\frac{1}{2}\right)^2 = \frac{1}{4}$

d. $(-8)^{-1/3}$

undefined.

4. Write each expression in power form ax^b for numbers, a and b.

a. $\frac{4}{x^5} = 4x^{-5}$

b. $\frac{4}{\sqrt[3]{8x^4}} = 4\left(8x^{4/3}\right)$

$= 32x^{4/3}$

c. $\frac{24}{\left(2\sqrt{x}\right)^3} = 24\left(2x^{3/2}\right)$

$= 48x^{3/2}$

5. Solve each equation by factoring or the Quadratic Formula, as appropriate.

a. $x^2 - 6x - 7 = 0$

$x^2 + x - 7x - 7 = 0$

$x(x + 1) - 7(x + 1) = 0$

$(x + 1)(x - 7) = 0$

$x + 1 = 0$

$x - 7 = 0$

$x_1 = -1 \quad x_2 = 7$

b. $x^2 + 2x = 15$

$x^2 + 2x - 15 = 0$

$x^2 + 5x - 3x - 15 = 0$

$(x + 5)(x - 3) = 0$

$x_1 = -5 \quad x_2 = 3$

c. $2x^2 + 40 = 8x$

$x^2 - 4x + 20 = 0$

$a = 1 \quad b = -4 \quad c = 20$

$$x = \frac{-(-4) \pm \sqrt{(-4)^2 - 4(1)(20)}}{2(1)}$$

$$x = \frac{4 \pm \sqrt{16 - 80}}{2}$$

$x \notin \mathbb{R}$

d. $5x^2 - 50x = 0$

$5x(x - 10) = 0$

$x = 0 \quad x = 10$

6. ***Cost Functions*** A lumberyard will deliver wood for $4 per board foot plus a delivery charge of $20. Find a function C(x) for the cost of having x board feet of lumber delivered.

$C(x) = 4x + 20$

7. For each function:
   a. Evaluate the given expression.
   b. Find the domain of the function.
   c. Find the range.

   i. $f(x) = \frac{1}{x+4}$ $f(-3)$

$f(-3) = \frac{1}{(-3)+4}$

$f(-3) = \frac{1}{1}$

$f(-3) = 1$

D: $x \in \mathbb{R} \mid \{-4\}$

Range: $(-\infty, \infty)$

ii. $f(x) = \frac{12}{x(x+4)}$ $f(2)$

$f(2) = \frac{12}{2(2+4)}$

$f(2) = \frac{12}{12} = 1$

D: $x \in \mathbb{R} \mid \{-4, 0\}$

Range: $(-\infty, \infty)$

iii. $f(x) = 4^x$ $f\left(-\frac{1}{2}\right)$

$f(-1/2) = 4^{-1/2}$

$f(-1/2) = \frac{1}{4^{1/2}} = \frac{1}{2}$

D: $x \in \mathbb{R}$

Range: $(-\infty, \infty)$

iv. $f(x) = 8^x \quad f\left(-\frac{1}{3}\right)$

$f(-1/3) = 8^{-\frac{1}{3}}$

$f(-1/3) = \frac{1}{8^{\frac{1}{3}}} = \frac{1}{2}$

D: $x \in \mathbb{R}$

Range: $(-\infty, \infty)$

8. Solve each equation by factoring.

a. $x^5 + 2x^4 - 3x^3 = 0$

$x^3(x^2 + 2x - 3) = 0$

$x^3(x+3)(x-1) = 0$

$x = 0 \quad x = -3 \quad x = 1$

b. $5x^3 - 20x = 0$

$5x(x^2 - 4) = 0$

$x(x^2 - 4) = 0$

$x(x+2)(x-2) = 0$

$x = 0 \quad x = -2 \quad x = 2$

c. $2x^5 - 50x^3 = 0$

$2x^3(x^2 - 25) = 0$

$x^3(x^2 - 25) = 0$

$x^3(x + 5)(x - 5) = 0$

$x = 0 \quad x = -5 \quad x = 5$

d. $3x^{\frac{5}{2}} - 6x^{\frac{3}{2}} = 9x^{\frac{1}{2}}$

$3\sqrt{x^5} - 6\sqrt{x^3} = 9\sqrt{x}$

$9x^3 - 36x^4 + 36x^3 = 81x$

$x^5 - 4x^4 + 4x^3 = 9x$

$x^5 - 4x^4 + 4x^3 - 9x = 0$

$x(x+1)(x-3)(x^2 - 2x + 3) = 0$

$x = 0 \quad x = -1 \quad x = 3 \quad x = 1 \pm \sqrt{2}i$

9. For each pair of functions f(x) and g(x), find

a. f(g(x))
b. g(f(x))
c. f(f(x))

i. $f(x) = x^5 \quad g(x) = 7x - 1$

a. $f(g(x)) = (7x-1)^5$

b. $g(f(x)) = 7x^5 - 1$

c. $f(f(x)) = (x^5)^5$

$f(f(x)) = x^{25}$

ii. $f(x) = \frac{1}{x} \quad g(x) = x^2 + 1$

a. $f(g(x)) = \frac{1}{x^2+1}$

b. $g(f(x)) = \left(\frac{1}{x}\right)^2 + 1$

c. $f(f(x)) = \frac{1}{\left(\frac{1}{x}\right)}$

$= \frac{1}{1} \cdot \frac{x}{1}$

$= x$

10. Find the following limits without using a graphing calculator or making tables.

a. $\lim_{x\to 3}(4x^2 - 10x + 2)$

$= 4(3^2) - 10(3) + 2$

$= 4(9) - 30 + 2$

$= 8$

b. $\lim_{x\to 5}\left(\frac{3x^2-5x}{7x-10}\right)$

$= \frac{3(5^2) - 5(5)}{7(5) - 10}$

$= \frac{3(25) - 5}{7 - 10}$

$= 2$

c. $\lim_{x\to 1}\left(\frac{x-1}{x^2+x-2}\right)$

$1-1=0$

$1^2+1-2=0$

$\lim_{x\to} = \frac{0}{0}$

rewrite: $\lim_{x\to 1}\left(\frac{x-1}{x^2-2x-x-2}\right)$

$\lim_{x\to 1}\left(\frac{(x-1)}{(x+2)(x-1)}\right)$

$= \frac{1}{1+2} = \frac{1}{3}$

d. $\lim_{h\to 0}\left(\frac{5x^4h-9xh^2}{h}\right)$

$\lim_{h\to 0}\left(\frac{h(5x^4-9xh)}{h}\right)$

$\lim_{h\to 0}(5x^4-9xh)$

$\lim_{h\to 0}(5x^4) - \lim_{h\to 0}(9xh)$

$= 5x^4 - \lim_{h\to 0}(9xh)$

$= 5x^4 - 9x(\lim_{h\to 0}(h))$

$= 5x^4 - 9x(0) = 5x^4$

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11. Find f'(x) by using the definition of the derivative.

Note: Use the Limit of Definition of Derivative

$$\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$$

a. $f(x)=2x-7$

$$\lim_{h\to 0}\left(\frac{2(x+h)-7-(2x-7)}{h}\right)$$

$$\lim\frac{2x+2h-7-2x+7}{h}$$

$$\lim_{h\to 0}\frac{2h}{h} \qquad \lim_{h\to 0} = 2$$

$$f'(x)=2$$

b. $f(x)=x^2+8x-9$

$$\lim_{h\to 0}(x+h)^2-8(x+h)-9-x^2-8x+9$$

$$\lim_{h\to 0}x^2+2xh+h^2-8x-8h-9-x^2-8x+9$$

$$\lim_{h\to 0}\frac{2xh+h^2-8h}{h}=\frac{h(2x+h-8)}{h}$$

$$\lim_{h\to 0}2x+h-8 = h-8$$

$$f'(x)=2x+8$$

c. $f(x) = x^3 + 2x^2 - 4x + 6$

$$\lim_{h \to 0} \frac{(x+h)^3 + 2(x+h)^2 - 4(x+h) + 6 - x^3 - 2x^2 + 4x - 6}{h}$$

$$\lim_{h \to 0} \frac{3hx^2 + 3h^2x + h^3 + 2x^2 + 4hx + 2h^2 - 4h - 2x^2}{h}$$

$$\lim_{h \to 0} \frac{h(3x^2 + 3hx + h^2 + 4x + 2h - 4)}{h}$$

$$\lim_{h \to 0} 3x^2 + 3hx + h^2 + 4x + 2h - 4$$

$$f'(x) = 3x^2 + 4x - 4$$

d. $f(x) = \frac{1}{x^2}$

$$\lim_{h \to 0} \frac{\frac{1}{(x+h)^2} - \frac{1}{x^2}}{h} \quad ; \quad \lim_{h \to 0} \frac{\frac{1}{x^2 + 2hx + h^2} - \frac{1}{x^2}}{h}$$

$$\lim_{h \to 0} \frac{x^2 - x^2 - 2hx - h^2}{x^4 + 2hx^3 + x^2h^2} \Big/ \frac{h}{1} \quad ; \quad \lim_{h \to 0} \frac{x^2 - x^2 - 2xh - h^2}{x^2h(x+h)^2}$$

$$\lim_{h \to 0} \frac{h(-2x - h)}{(x^2h(x+h))^2} \quad ; \quad \lim_{h \to 0} \left(\frac{-2x - h}{x(x+h)^2} \right)$$

$$f'(x) = -\frac{2}{x^3}$$

12. Find the derivative of the following functions using any method:

a. $f(x) = x^5 \cdot x^{\frac{1}{2}}$

$$f'(x) = \frac{d}{dx}\left(x^5 \cdot x^{1/2}\right)$$

$$f'(x) = \frac{d}{dx}\left(x^{\frac{11}{2}}\right)$$

$$= \frac{11}{2} x^{\frac{9}{2}}$$

$$f'(x) = \frac{11x^4\sqrt{x}}{2}$$

b. $f(x) = \frac{(x+1)}{(x+2)}$

$$f'(x) = \frac{d}{dx}\left(\frac{x+1}{x+2}\right)$$

$$f'(x) = \frac{\frac{d}{dx}(x+1)(x+2) - (x+1)\frac{d}{dx}(x+2)}{(x+2)^2}$$

$$f'(x) = \frac{(x+2)-(x+1)}{(x+2)^2} = \frac{1}{(x+2)^2}$$

c. $f(x) = (2x-3)^4$

$$f'(x) = \frac{d}{dx}\left((2x-3)^4\right)$$

$$f'(x) = \frac{d}{dg}(g^4) \times \frac{d}{dx}(2x-3)$$

$$f'(x) = 4g^3 \times 2$$

$$f'(x) = 4(2x-3)^3 \times 2$$

$$f'(x) = 8(2x-3)^3$$

d. $f(x) = \sqrt[4]{x^7}$

$$f'(x) = \frac{d}{dx}\left(\sqrt[4]{x^7}\right)$$

$$f'(x) = \frac{d}{dx}\left(x^{\frac{7}{4}}\right)$$

$$f'(x) = \frac{7}{4}x^{\frac{3}{4}}$$

$$f'(x) = \frac{7}{4}\left(\sqrt[4]{x^3}\right)$$

13. For each function, find:
    a. $f'(x)$
    b. $f''(x)$
    c. $f'''(x)$
    d. $f''''(x)$

i. $f(x) = x^6 + 2x^4 - x^2$

a. $f'(x) = 6x^5 + 8x^3 - 2x$

b. $f''(x) = 30x^4 + 24x^2 - 2$

c. $f'''(x) = 120x^3 + 48x$

d. $f''''(x) = 360x^2 + 48$

ii. $f(x) = \sqrt[4]{x}$

a. $f'(x) = \frac{1}{4x^{3/4}}$

b. $f''(x) = -\frac{3}{16x^{7/4}}$

c. $f'''(x) = \frac{21}{64x^{11/4}}$

d. $f''''(x) = -\frac{231}{256x^{15/4}}$

14. GENERAL Velocity: After t hours a freight train is

$s(t) = 18t^2 - 2t^3$ miles due north of its starting point

$for\ (0 \leq t \leq 9)$

a. Find the velocity at $t = 3$

$S(t) = 18t^2 - 2t^3$

$S(3) = 108$

b. Find the velocity at $t = 7$

$S(7) = 196$

c. Find the acceleration at $t = 1$

$S(1) = 16$