

AS1 (Assignment 1, Unit 3): Z-scores and Probability

Please type your answers in red 😊

1. What information is provided by the sign (+/-) of a z-score? **The positive sign indicates that the x-score is above the mean.**
The negative sign indicates that the x-score is below the mean.

2. What information is provided by the numerical value of the z-score? **The distance from the mean**

3. A distribution has a standard deviation of $\sigma = 10$. Find the z-score for each of the following locations in the distribution.

a. Above the mean by 15 points

Answer: 1.5

b. Above the mean by 25 points.

Answer: 2.5

c. Below the mean by 20 points.

Answer: -2

d. Below the mean by 5 points.

Answer: -1

4. For a distribution with a standard deviation of $\sigma = 12$, describe the location of each of the following z-scores in terms of its position relative to the mean. For example, $+1.00$ is a location that is 12 points above the mean.

a. $z = +2.00$ Answer: 24

b. $z = +.50$ Answer: 6

c. $z = -1.00$ Answer: -12

d. $z = -0.25$ Answer: -0.25

5. For a population with $\mu = 70$ and $\sigma = 10$,

- a. Find the z-score for each of the following X values. (Note: You should be able to find these values using the definition of a z-score. You should not need to use a formula or do any serious calculations.)

X = 75: z = 0.5 X = 100: z = 3 X = 60: z = -1

X = 95: z = 2.5 X = 50: z = -2 X = 85: z = 1.5

- b. Find the score (X value) that corresponds to each of the following z-scores. (Again, you should not need a formula or any serious calculations.)

z = 1.00: X = 80 z = 0.20: X = 72 z = 1.50: X = 55

z = -0.50: X = -65 z = -2.00: X = -50 z = -1.50: X = 1.5

6. For a population with a mean of $\mu = 40$ and $\sigma = 12$, find the z-score for each of the following X values. (Note: You probably will need to use a formula and a calculator to find these values)

X = 45: z = 0.41 X = 52: z = 1 X = 41: z = 0.08

X = 30: z = -0.83 X = 25: z = -1.25 X = 38: z = -0.16

7. For a population with a mean of $\mu = 100$ and a standard deviation of $\sigma = 10$,

- a. Find the z-score for each of the following X values.

X = 108: z = 0.8 X = 115: z = 1.5 X = 130: z = 3

X = 90: z = -1 X = 88: z = -1.2 X = 95: z = -0.5

- b. Find the score (X value) that corresponds to each of the following z-scores.

z = -0.40: X = 96 z = -0.50: X = 95 z = 1.80: X = 118

z = 0.75: X = 107.5 z = 1.50: X = 115 z = -1.25: X = 87.5

8. A population has a mean of $\mu = 70$ and a standard deviation of $\sigma = 12$.

a. For the population, find the z-score for each of the following X values.

$$X = 69: z = \underline{-0.08}$$

$$X = 84: z = \underline{1.16}$$

$$X = 63: z = \underline{-0.58}$$

$$X = 54: z = \underline{-1.33}$$

$$X = 48: z = \underline{-1.83}$$

$$X = 45: z = \underline{-2.08}$$

b. For the same population, find the score (X value) that corresponds to each of the following z-scores.

$$z = 0.50: X = \underline{76}$$

$$z = 1.50: X = \underline{88}$$

$$z = -2.50: X = \underline{40}$$

$$z = -0.25: X = \underline{67}$$

$$z = -0.50: X = \underline{64}$$

$$z = 1.25: X = \underline{85}$$

9. A sample has a mean of $M = 30$ and a standard deviation of $s = 7$. Find the z-score of for each of the following X values from this sample.

$$X = 32: z = \underline{0.28}$$

$$X = 34: z = \underline{0.57}$$

$$X = 36: z = \underline{0.85}$$

$$X = 28: z = \underline{-0.28}$$

$$X = 20: z = \underline{-1.42}$$

$$X = 18: z = \underline{-1.71}$$

10. A sample has a mean of $M = 35$ and a standard deviation of $s = 5$. For this sample, find the X value corresponding to each of the following z-scores.

$$z = 0.40: X = \underline{37} \quad z = 1.20: X = \underline{41} \quad z = 2.00: X = \underline{45}$$

$$z = -0.80: X = \underline{31} \quad z = -0.60: X = \underline{32} \quad z = -1.40: X = \underline{28}$$

11. Find the z-score corresponding to a score of $X = 45$ of the following distributions.

a. $\mu = 40$ and $\sigma = 20$

$$\text{Answer: } \underline{0.25}$$

b. $\mu = 40$ and $\sigma = 10$

$$\text{Answer: } \underline{0.5}$$

c. $\mu = 40$ and $\sigma = 5$

$$\text{Answer: } \underline{1}$$

d. $\mu = 40$ and $\sigma = 2$

$$\text{Answer: } \underline{2.5}$$

Find the X value corresponding to $z = 0.30$ for each of the following distributions.

e. $\mu = 40$ and $\sigma = 4$Answer: 41.2

f. $\mu = 40$ and $\sigma = 8$Answer: 42.4

g. $\mu = 40$ and $\sigma = 16$Answer: 44.8

h. $\mu = 40$ and $\sigma = 32$Answer: 49.6

12. For each of the following populations, would a score of $X = 50$ be considered a central score (near the middle of the distribution) or an extreme score (far out in the tail of the distribution)?

a. $\mu = 45$ and $\sigma = 10$Answer: Central Score

b. $\mu = 45$ and $\sigma = 2$Answer: Extreme Score

c. $\mu = 90$ and $\sigma = 20$Answer: Extreme Score

d. $\mu = 60$ and $\sigma = 20$Answer: Central Score

13. A distribution of exam scores has a mean of $\mu = 78$.

a. If your score is $X = 70$, which standard deviation would give you a better grade: $\sigma = 4$ or $\sigma = 8$?

Answer: 8

b. If your score is $X = 80$, which standard deviation would give you a better grade: $\sigma = 4$ or $\sigma = 8$?

Answer: 8

14. For each of the following, identify the exam score that should lead to the better grade.

a. A score of $X = 74$ on an exam with $M = 82$ and $\sigma = 8$; or a score of $X = 40$ on an exam with $\mu = 50$ and $\sigma = 20$.

Answer: 40

b. A score of $X = 51$ on an exam with $\mu = 45$ and $\sigma = 2$; or a score of $X = 90$ on an exam with $\mu = 70$ and $\sigma = 20$.

Answer: 51

- c. A score of $X = 62$ on an exam with $\mu = 50$ and $\sigma = 8$; or a score of $X = 23$ on an exam with $\mu = 20$ and $\sigma = 2$ Answer: they both have the same Z score 1.5, Maybe the mean because its exam score 23 because its closer to the mean,

15. A population consists of the following $N = 5$ scores: 0, 5, 4, 4, and 12.

- a. Compute μ and σ for the population

Mean = 5 Standard deviation = 3.89

- b. Find the z-score for each score in the population

$X=0, z = -1.28$ $X=5, z = 0.$ $X=7, z = 0.51$ $X=3, z = -0.51$ $X=12, z = 1.79$

15. A sample consists of the following $n = 7$ scores: 5, 0, 4, 5, 1, 2, and 4.

- a. Compute the mean and standard deviation for the sample.

Mean = 7 Standard deviation = 2

- b. Find the z-score for each score in the sample.

$X=5, z = 1$ $X=0, z = -3.5$ $X=4, z = -1.5$ $X=5, z = 1$ $X=1, z = -3$

$X=2, z = -2.5$ $X=4, z = -1.5$

16. There are 40 black marbles, 20 blue marbles, and 4 red marbles in a jar.

- a. What is the probability of selecting one red marble? 4/64

- b. What is the probability of selecting one black marble? 40/64

- c. What is the probability of selecting one blue marble?
20/64

- d. Which has the highest probability of being selected? black marbles

- e. Which has the lowest probability of being selected? Red marbles

17. Probability values range from 1/16 to 6.

18. Please describe at least three characteristics of the NORMAL DISTRIBUTION

1. All measures of central tendency have the same value and are located in the center.
2. More scores are located near the mean than score that move away from the mean.
3. The shape is symmetrical.

19. In a normal population of IQ scores, what percent of people have “average” IQ’s?

Answer 68.26%

20. In a normal distribution, what percentage of people would be located at or below 2 standard deviations from the mean?

Answer

2.28%

21. Answer the following questions based on a distribution with a $\mu = 25$ and $\sigma = 4$:

- a. What range of scores is considered “average”? 21 to 29
- b. What percentage of people has an average score? 68.26%
- c. What percentage of people has extremely high or extremely low scores?
2.28%
- d. What *range of scores* (requires numbers to be noted in the blank spaces) have the highest probability of being selected? 21 to 29

Sorry to write on the exam. I really struggle with this unit, especially the probabilities. Can you please explain the ones I may have gotten wrong so I can see where I went wrong and go over it and learn the right way to do it.

Thank you 😊