AS1 (Assignment 1, Unit 3): Z-scores and Probability

Please type your answers in red ☺

1. What information is provided by the sign (+/-) of a *z*-score?
   If the sign of the z-score is positive, then we know that the location of the x-score is above the mean and if the sign of the z-score is negative, then we know that the location of the x-score is below the mean.

2. What information is provided by the numerical value of the *z*-score?
   The numerical value of the z-score tells us the distance from the mean at which the x-score is located in standard deviations.

3. A distribution has a standard deviation of $\sigma = 10$. Find the *z*-score for each of the following locations in the distribution.

   a. Above the mean by 15 points.
      Answer: 1.5

   b. Above the mean by 25 points.
      Answer: 2.5

   c. Below the mean by 20 points.
      Answer: -2

   d. Below the mean by 5 points.
      Answer: -0.5

4. For a distribution with a standard deviation of $\sigma = 12$, describe the location of each of the following *z*-scores in terms of its position relative to the mean. For example, = +1.00 is a location that is 12 points above the mean.

   a. $z = +2.00$ Answer:24 points above the mean

   b. $z = +.50$ Answer: 6 points above the mean

   c. $z = -1.00$ Answer:-12 points below the mean

   d.

   e. $z = -0.25$ Answer:-3 points below the mean

5. For a population with $\mu = 70$ and $\sigma = 10$,

a. Find the *z*-score for each of the following X values. (Note: You should be able to find these values using the definition of a *z*-score. You should not need to use a formula or do any serious calculations.)

X = 75: z =0.5 X = 100: z = 3 X = 60: z = -1

X = 95: z=2.5 X = 50: z =-2 X = 85 z = 1.5

b. Find the score (X value) that corresponds to each of the following *z*-scores. (Again, you should not need a formula or any serious calculations.)

z = 1.00: X= 80 *z* = 0.20: X=72 *z* = 1.50: X=85

z = -0.50: X=65 *z* = -2.00: X=50 *z* = -1.50: X=55

6. For a population with a mean of $\mu = 40$ and $\sigma = 12$, find the *z*–score for each of the following X values. (Note: You probably will need to use a formula and a calculator to find these values)

X = 45: z =0.41 X = 52: z =1 X = 41: z =0.08

X = 30: z =-0.83 X = 25: z =-1.25 X = 38: z =-0.16

7. For a population with a mean of $\mu = 100$ and a standard deviation of $\sigma = 10$,

a. Find the z-score for each of the following X values.

X = 108: z =0.8 X = 115: z =1.5 X = 130: z =3

X = 90: z =-1 X = 88: z =-1.2 X = 95: z =-0.5

b. Find the score (X value) that corresponds to each of the following z-scores.

z = -0.40: X=96 *z* = -0.50: X=95 *z* = 1.80: X=118

z = 0.75: X=107.5 *z* = 1.50: X=115 *z* = -1.25: X=87.5

8. A population has a mean of $\mu = 70$ and a standard deviation of $\sigma = 12$.

   a. For the population, find the z-score for each of the following X values.

   X = 69: z =-0.08 X = 84: z =1.16 X = 63: z =-0.58

   X = 54: z =-1.33 X = 48: z =-1.83 X = 45: z =-2.08

   b. For the same population, find the score (X value) that corresponds to each of the following z-scores.

   z = 0.50: X=76 z = 1.50: X=88 z = -2.50: X=40

   z = -0.25: X=67 z = -0.50: X=64 z = 1.25: X=85

9. A sample has a mean of M = 30 and a standard deviation of s = 7. Find the z-score of for each of the following X values from this sample.

   X = 32: z =0.28 X = 34: z =0.57 X = 36: z =0.85

   X = 28: z =-0.28 X = 20: z =-1.42 X = 18: z =-1.71

10. A sample has a mean of M = 35 and a standard deviation of $s = 5$. For this sample, find the X value corresponding to each of the following z-scores.

    z = 0.40: X=37 z = 1.20: X=41 z = 2.00: X=45

    z = -0.80: X=31 z = -0.60: X=32 z = -1.40: X=28

11. Find the z-score corresponding to a score of X = 45 of the following distributions.

    a. $\mu = 40$ and $\sigma = 20$
       Answer: *0.25*

    b. $\mu = 40$ and $\sigma = 10$
       Answer: *0.5*

    c. $\mu = 40$ and $\sigma = 5$
       Answer: *1*

    d. $\mu = 40$ and $\sigma = 2$
       Answer: 2.5

Find the X value corresponding to z = 0.30 for each of the following distributions.

e. μ = 40 and σ = 4…….Answer: 41.2

f. μ = 40 and σ = 8…….Answer: 42.4

g. μ = 40 and σ = 16……..Answer: 44.8

h. μ = 40 and σ = 32……Answer: 49.6

12. For each of the following populations, would a score of X = 50 be considered a central score (near the middle of the distribution) or an extreme score (far out in the tail of the distribution)?

a. μ = 45 and σ = 10……..Answer: Central score

b. μ = 45 and σ = 2……..Answer: Extreme score

c. μ = 90 and σ = 20……..Answer: Extreme score

d. μ = 60 and σ = 20……..Answer: Central score

13. A distribution of exam scores has a mean of μ = 78.

a. If your score is X = 70, which standard deviation would give you a better grade: σ = 4 or σ = 8?

Answer: $\sigma = 8$

b. If your score is X = 80, which standard deviation would give you a better grade: σ = 4 or σ = 8?

Answer: $\sigma = 4$

14. For each of the following, identify the exam score that should lead to the better grade.

a. A score of X = 74 on an exam with M = 82 and σ = 8; or a score of X = 40 on an exam with μ = 50 and σ = 20.

Answer: X=40

b. A score of X = 51 on an exam with μ = 45 and σ = 2; or a score of X = 90 on an exam with μ = 70 and σ = 20.

Answer: X=51

c. A score of X = 62 on an exam with μ = 50 and σ = 8; or a score of X = 23 on an exam with μ = 20 and σ = 2 Answer: Both X=62 & X=23

15. A population consists of the following N = 5 scores: 0, 5, 4, 4, and 12.

a. Compute μ and σ for the population

Mean =5 Standard deviation=3.89

b. Find the *z*-score for each score in the population

X=0, z =-1.28 X=5, z=0 X=7, z=0.51 X=3, z=-0.51 X=12, z=1.79

15. A sample consists of the following n = 7 scores: 5, 0, 4, 5, 1, 2, and 4.

a. Compute the mean and standard deviation for the sample.

Mean =3 Standard deviation =2

b. Find the *z*-score for each score in the sample.

X=5, z=1 X=0, z=-1.50 X=4, z=0.5 X=5, z=1 X=1, z=-1

X=2, z=-0.5 X=4, z=0.5

16. There are 40 black marbles, 20 blue marbles, and 4 red marbles in a jar.

a. What is the probability of selecting one red marble? $\frac{4}{64}$ 6%

b. What is the probability of selecting one black marble? $\frac{40}{64}$ 62%

c. What is the probability of selecting one blue marble? $\frac{20}{64}$ 31%

d. Which has the highest probability of being selected? Black

e. Which has the lowest probability of being selected? Red

17. Probability values range from 0 to 1.

18. Please describe at least three characteristics of the NORMAL DISTRIBUTION

1. Mean, Median, and mode are all equal.
2. The curve is symmetrical
3. The curve is always a bell shaped curve

19. In a normal population of IQ scores, what percent of people have “average” IQ’s?

    Answer 68%

20. In a normal distribution, what percentage of people would be located at or below 2 standard deviations from the mean?

    Answer 5%

21. Answer the following questions based on a distribution with a $\mu = 25$ and $\sigma = 4$:

    a. What range of scores is considered “average”? 25 to 30

    b. What percentage of people has an average score? 68%

    c. What percentage of people has extremely high or extremely low scores? 5%

    d. What *range of scores* (requires numbers to be noted in the blank spaces) have the highest probability of being selected? 20 to 40