

## AS1 (Assignment 1, Unit 3): Z-scores and Probability

Please type your answers in red

- What information is provided by the sign (+/-) of a z-score?  
If the z-score is positive, it indicated that the x-score will be above the mean. And, if it is negative then the x-score will be below the mean.
- What information is provided by the numerical value of the z-score?  
It tells you the distance the x-score is located from the mean in units of standard deviation.
- A distribution has a standard deviation of  $\sigma = 10$ . Find the z-score for each of the following locations in the distribution.
  - Above the mean by 15 points.  
Answer: 1.5
  - Above the mean by 25 points.  
Answer: 2.5
  - Below the mean by 20 points.  
Answer: -2
  - Below the mean by 5 points.  
Answer: -0.5
- For a distribution with a standard deviation of  $\sigma = 12$ , describe the location of each of the following z-scores in terms of its position relative to the mean. For example,  $+1.00$  is a location that is 12 points above the mean.
  - $z = +2.00$     Answer: 24
  - $z = +.50$     Answer: 6
  - $z = -1.00$     Answer: -12
  - $z = -0.25$     Answer: -3
- For a population with  $\mu = 70$  and  $\sigma = 10$ ,

a. Find the z-score for each of the following X values. (Note: You should be able to find these values using the definition of a z-score. You should not need to use a formula or do any serious calculations.)

$$X = 75: z = \underline{0.5} \quad X = 100: z = \underline{3} \quad X = 60: z = \underline{-1}$$

$$X = 95: z = \underline{2.5} \quad X = 50: z = \underline{-2} \quad X = 85: z = \underline{1.5}$$

- Find the score (X value) that corresponds to each of the following z-scores. (Again, you should not need a formula or any serious calculations.)

$$z = 1.00: X = \underline{80} \quad z = 0.20: X = \underline{72} \quad z = 1.50: X = \underline{85}$$

$$z = -0.50: X = \underline{65} \quad z = -2.00: X = \underline{50} \quad z = -1.50: X = \underline{55}$$

- For a population with a mean of  $\mu = 40$  and  $\sigma = 12$ , find the z-score for each of the following X values. (Note: You probably will need to use a formula and a calculator to find these values)

$$X = 45: z = \underline{0.42} \quad X = 52: z = \underline{1} \quad X = 41: z = \underline{0.08}$$

$$X = 30: z = \underline{-0.83} \quad X = 25: z = \underline{-1.25} \quad X = 38: z = \underline{-0.17}$$

- For a population with a mean of  $\mu = 100$  and a standard deviation of  $\sigma = 10$ ,

- Find the z-score for each of the following X values.

$$X = 108: z = \underline{0.8} \quad X = 115: z = \underline{1.5} \quad X = 130: z = \underline{3}$$

$$X = 90: z = \underline{-1} \quad X = 88: z = \underline{-1.2} \quad X = 95: z = \underline{-0.5}$$

- Find the score (X value) that corresponds to each of the following z-scores.

$$z = -0.40: X = \underline{96} \quad z = -0.50: X = \underline{95} \quad z = 1.80: X = \underline{118}$$

$$z = 0.75: X = \underline{107.5} \quad z = 1.50: X = \underline{115} \quad z = -1.25: X = \underline{87.5}$$

- A population has a mean of  $\mu = 70$  and a standard deviation of  $\sigma = 12$ .
  - For the population, find the z-score for each of the following X values.

$$X = 69: z = \underline{-0.08} \quad X = 84: z = \underline{1.17} \quad X = 63: z = \underline{-0.58}$$

$X = 54: z = \underline{-1.3}$                        $X = 48: z = \underline{-1.18}$                        $X = 45: z = \underline{-2.08}$

• For the same population, find the score (X value) that corresponds to each of the following z-scores.

$z = 0.50: X = \underline{76}$                        $z = 1.50: X = \underline{88}$                        $z = -2.50: X = \underline{40}$   
 $z = -0.25: X = \underline{67}$                        $z = -0.50: X = \underline{64}$                        $z = 1.25: X = \underline{85}$

• A sample has a mean of  $M = 30$  and a standard deviation of  $s = 7$ . Find the z-score of for each of the following X values from this sample.

$X = 32: z = \underline{0.29}$                        $X = 34: z = \underline{0.57}$                        $X = 36: z = \underline{0.86}$   
 $X = 28: z = \underline{-0.29}$                        $X = 20: z = \underline{-1.43}$                        $X = 18: z = \underline{-1.71}$

• A sample has a mean of  $M = 35$  and a standard deviation of  $s = 5$ . For this sample, find the X value corresponding to each of the following z-scores.

$z = 0.40: X = \underline{37}$                        $z = 1.20: X = \underline{41}$                        $z = 2.00: X = \underline{45}$   
 $z = -0.80: X = \underline{31}$                        $z = -0.60: X = \underline{32}$                        $z = -1.40: X = \underline{28}$

• Find the z-score corresponding to a score of  $X = 45$  of the following distributions.

- $\mu = 40$  and  $\sigma = 20$   
Answer: 0.25
- $\mu = 40$  and  $\sigma = 10$   
Answer: 0.5
- $\mu = 40$  and  $\sigma = 5$   
Answer: 1
- $\mu = 40$  and  $\sigma = 2$   
Answer: 2.5

Find the X value corresponding to  $z = 0.30$  for each of the following distributions.

- $\mu = 40$  and  $\sigma = 4$ .....Answer:

\_\_\_\_\_ 41.2 \_\_\_\_\_

- $\mu = 40$  and  $\sigma = 8$ .....Answer:

\_\_\_\_\_ 42.4 \_\_\_\_\_

- $\mu = 40$  and  $\sigma = 16$ .....Answer:

\_\_\_\_\_ 44.8 \_\_\_\_\_

- $\mu = 40$  and  $\sigma = 32$ .....Answer:

\_\_\_\_\_ 49.6 \_\_\_\_\_

• For each of the following populations, would a score of  $X = 50$  be considered a central score (near the middle of the distribution) or an extreme score (far out in the tail of the distribution)?

- $\mu = 45$  and  $\sigma = 10$ .....Answer:

\_\_\_\_\_ central \_\_\_\_\_

- $\mu = 45$  and  $\sigma = 2$ .....Answer:

\_\_\_\_\_ extreme \_\_\_\_\_

- $\mu = 90$  and  $\sigma = 20$ .....Answer:

\_\_\_\_\_ extreme \_\_\_\_\_

- $\mu = 60$  and  $\sigma = 20$ .....Answer:

\_\_\_\_\_ central \_\_\_\_\_

- A distribution of exam scores has a mean of  $\mu = 78$ .

- If your score is  $X = 70$ , which standard deviation would give you a better grade:  $\sigma = 4$  or  $\sigma = 8$ ?

Answer: \_\_\_\_\_ 8 \_\_\_\_\_

- If your score is  $X = 80$ , which standard deviation would give you a better grade:  $\sigma = 4$  or  $\sigma = 8$ ?

Answer: \_\_\_\_\_ 4 \_\_\_\_\_

• For each of the following, identify the exam score that should lead to the better grade.

- A score of  $X = 74$  on an exam with  $M = 82$  and  $\sigma = 8$ ; or a score of  $X = 40$  on an exam with  $\mu = 50$  and  $\sigma = 20$ .

Answer: a score of  $X = 40$  on an exam with  $\mu = 50$  and  $\sigma = 20$ .

- A score of  $X = 51$  on an exam with  $\mu = 45$  and  $\sigma = 2$ ; or a score of  $X = 90$  on an exam with  $\mu = 70$  and  $\sigma = 20$ .

Answer: A score of  $X = 51$  on an exam with  $\mu = 45$  and  $\sigma = 2$

- A score of  $X = 62$  on an exam with  $\mu = 50$  and  $\sigma = 8$ ; or a score of  $X = 23$  on an exam with  $\mu = 20$  and  $\sigma = 2$  Answer: Both have a  $z$  score of 1.5

15. A population consists of the following  $N = 5$  scores: 0, 5, 4, 4, and 12.

a. Compute  $\mu$  and  $\sigma$  for the population

Mean = 5 Standard deviation = 4

b. Find the  $z$ -score for each score in the population

$X=0, z = \underline{-1.25}$   $X=5, z = \underline{0}$   $X=7, z = \underline{0.5}$   $X=3, z = \underline{-0.5}$   $X=12, z = \underline{1.75}$

- A sample consists of the following  $n = 7$  scores: 5, 0, 4, 5, 1, 2, and 4.
  - Compute the mean and standard deviation for the sample.

Mean = 3 Standard deviation = 2

- Find the  $z$ -score for each score in the sample.

$X=5, z = \underline{1}$   $X=0, z = \underline{-1.5}$   $X=4, z = \underline{0.5}$   $X=5, z = \underline{1}$   $X=1, z = \underline{-1}$   
 $X=2, z = \underline{-0.5}$   $X=4, z = \underline{0.5}$

- There are 40 black marbles, 20 blue marbles, and 4 red marbles in a jar.

- What is the probability of selecting one red marble?  
 \_\_\_\_\_ 4/64 \_\_\_\_\_

- What is the probability of selecting one black marble?  
 \_\_\_\_\_ 40/64 \_\_\_\_\_

- What is the probability of selecting one blue marble?  
 \_\_\_\_\_ 20/64 \_\_\_\_\_

- Which has the highest probability of being selected?  
 \_\_\_\_\_ black \_\_\_\_\_

- Which has the lowest probability of being selected?  
 \_\_\_\_\_ red \_\_\_\_\_

- Probability values range from \_\_\_\_\_ 0 (0%) \_\_\_\_\_ to \_\_\_\_\_ 1 \_\_\_\_\_  
(100%) \_\_\_\_\_.

- Please describe at least three characteristics of the NORMAL DISTRIBUTION

- Shape is symmetrical
- mean, median, and mode are all located in the center, all equal
- More scores located in the center and less and less out to the sides.
- In a normal population of IQ scores, what percent of people have “average” IQ’s?

Answer 68%

- In a normal distribution, what percentage of people would be located at or below 2 standard deviations from the mean?

Answer 2.28%

- Answer the following questions based on a distribution with a  $\mu = 25$  and  $\sigma = 4$ :

- What range of scores is considered “average”? \_\_\_\_\_ 21 \_\_\_\_\_  
 to \_\_\_\_\_ 29 \_\_\_\_\_

- What percentage of people has an average score?

\_\_\_\_\_ **68%** \_\_\_\_\_

• What percentage of people has extremely high or extremely low scores? \_\_\_\_\_ **2.28%** \_\_\_\_\_

• What *range of scores* (requires numbers to be noted in the blank spaces) have the highest probability of being selected? \_\_\_\_\_ **21** \_\_\_\_\_ to \_\_\_\_\_ **29** \_\_\_\_\_