

AS1 (Assignment 1, Unit 3): Z-scores and Probability

Please type your answers in red 😊

1. What information is provided by the sign (+/-) of a z-score? + of a z-score tells us that it is above the mean, while - tells us that it is below the mean of a distribution
2. What information is provided by the numerical value of the z-score? a it tells us that how many standard deviation to its mean, that is its distance from the mean
3. A distribution has a standard deviation of $\sigma = 10$. Find the z-score for each of the following locations in the distribution.
 - a. Above the mean by 15 points.
Answer: 1.5
 - b. Above the mean by 25 points.
Answer: 2.5
 - c. Below the mean by 20 points.
Answer: -2
 - d. Below the mean by 5 points.
Answer: -0.5
4. For a distribution with a standard deviation of $\sigma = 12$, describe the location of each of the following z-scores in terms of its position relative to the mean. For example, $+1.00$ is a location that is 12 points above the mean.
 - a. $z = +2.00$ Answer: 24 point above mean
 - b. $z = +.50$ Answer: 60 point above mean
 - c. $z = -1.00$ Answer: 12 point below mean
 - d. $z = -0.25$ Answer: 3 point below mean

5. For a population with $\mu = 70$ and $\sigma = 10$,

- a. Find the z -score for each of the following X values. (Note: You should be able to find these values using the definition of a z -score. You should not need to use a formula or do any serious calculations.)

$$X = 75: z = \underline{0.5} \quad X = 100: z = \underline{3} \quad X = 60: z = \underline{-1}$$

$$X = 95: z = \underline{2.5} \quad X = 50: z = \underline{-2} \quad X = 85: z = \underline{1.5}$$

- b. Find the score (X value) that corresponds to each of the following z -scores. (Again, you should not need a formula or any serious calculations.)

$$z = 1.00: X = \underline{80} \quad z = 0.20: X = \underline{72} \quad z = 1.50: X = \underline{85}$$

$$z = -0.50: X = \underline{65} \quad z = -2.00: X = \underline{50} \quad z = -1.50: X = \underline{55}$$

6. For a population with a mean of $\mu = 40$ and $\sigma = 12$, find the z -score for each of the following X values. (Note: You probably will need to use a formula and a calculator to find these values)

$$X = 45: z = \underline{0.41} \quad X = 52: z = \underline{1} \quad X = 41: z = \underline{0.08}$$

$$X = 30: z = \underline{-0.83} \quad X = 25: z = \underline{-1.23} \quad X = 38: z = \underline{-0.16}$$

7. For a population with a mean of $\mu = 100$ and a standard deviation of $\sigma = 10$,

- a. Find the z -score for each of the following X values.

$$X = 108: z = \underline{0.8} \quad X = 115: z = \underline{1.5} \quad X = 130: z = \underline{3}$$

$$X = 90: z = \underline{-1} \quad X = 88: z = \underline{-1.2} \quad X = 95: z = \underline{-0.5}$$

- b. Find the score (X value) that corresponds to each of the following z -scores.

$$z = -0.40: X = \underline{96} \quad z = -0.50: X = \underline{95} \quad z = 1.80: X = \underline{118}$$

$$z = 0.75: X = \underline{107.5} \quad z = 1.50: X = \underline{115} \quad z = -1.25: X = \underline{87.5}$$

8. A population has a mean of $\mu = 70$ and a standard deviation of $\sigma = 12$.

a. For the population, find the z-score for each of the following X values.

X = 69: z = -0.08

X = 84: z = 1.16

X = 63: z = -0.58

X = 54: z = -1.33

X = 48: z = -1.83

X = 45: z = -2.08

b. For the same population, find the score (X value) that corresponds to each of the following z-scores.

z = 0.50: X = 76

z = 1.50: X = 88

z = -2.50: X = 40

z = -0.25: X = 67

z = -0.50: X = 64

z = 1.25: X = 85

9. A sample has a mean of $M = 30$ and a standard deviation of $s = 7$. Find the z-score of for each of the following X values from this sample.

X = 32: z = 0.28

X = 34: z = 0.57

X = 36: z = 0.85

X = 28: z = -0.28

X = 20: z = -1.42

X = 18: z = -1.71

10. A sample has a mean of $M = 35$ and a standard deviation of $s = 5$. For this sample, find the X value corresponding to each of the following z-scores.

z = 0.40: X = 37 z = 1.20: X = 41 z = 2.00: X = 45

z = -0.80: X = 31 z = -0.60: X = 32 z = -1.40: X = 28

11. Find the z-score corresponding to a score of $X = 45$ of the following distributions.

a. $\mu = 40$ and $\sigma = 20$

Answer: 0.25

b. $\mu = 40$ and $\sigma = 10$

Answer: 0.5

c. $\mu = 40$ and $\sigma = 5$

Answer: 1

d. $\mu = 40$ and $\sigma = 2$

Answer: 2.5

Find the X value corresponding to $z = 0.30$ for each of the following distributions.

e. $\mu = 40$ and $\sigma = 4$Answer: 41.2

f. $\mu = 40$ and $\sigma = 8$Answer: 42.4

g. $\mu = 40$ and $\sigma = 16$Answer: 44.8

h. $\mu = 40$ and $\sigma = 32$Answer: 49.6

12. For each of the following populations, would a score of $X = 50$ be considered a central score (near the middle of the distribution) or an extreme score (far out in the tail of the distribution)?

a. $\mu = 45$ and $\sigma = 10$Answer: central score

b. $\mu = 45$ and $\sigma = 2$Answer: extreme score

c. $\mu = 90$ and $\sigma = 20$Answer: extreme score

d. $\mu = 60$ and $\sigma = 20$Answer: central score

13. A distribution of exam scores has a mean of $\mu = 78$.

a. If your score is $X = 70$, which standard deviation would give you a better grade: $\sigma = 4$ or $\sigma = 8$?

Answer: $\sigma = 4$

b. If your score is $X = 80$, which standard deviation would give you a better grade: $\sigma = 4$ or $\sigma = 8$?

Answer: $\sigma = 8$

14. For each of the following, identify the exam score that should lead to the better grade.

a. A score of $X = 74$ on an exam with $M = 82$ and $\sigma = 8$; or a score of $X = 40$ on an exam with $\mu = 50$ and $\sigma = 20$.

Answer: a score of $X = 40$

- b. A score of $X = 51$ on an exam with $\mu = 45$ and $\sigma = 2$; or a score of $X = 90$ on an exam with $\mu = 70$ and $\sigma = 20$.

Answer: ___ a score of $X = 90$ _____

- c. A score of $X = 62$ on an exam with $\mu = 50$ and $\sigma = 8$; or a score of $X = 23$ on an exam with $\mu = 20$ and $\sigma = 2$ Answer: ___ both the same grade _____

15. A population consists of the following $N = 5$ scores: 0, 5, 4, 4, and 12.

- a. Compute μ and σ for the population

Mean = ___ 5 _____ Standard deviation = ___ 3.89 _____

- b. Find the z -score for each score in the population

$X=0, z = \underline{-1.28}$ $X=5, z = \underline{0}$ $X=7, z = \underline{0.51}$ $X=3, z = \underline{-0.51}$
 $X=12, z = \underline{1.79}$

15. A sample consists of the following $n = 7$ scores: 5, 0, 4, 5, 1, 2, and 4.

- a. Compute the mean and standard deviation for the sample.

Mean = ___ 3 _____ Standard deviation = ___ 2 _____

- b. Find the z -score for each score in the sample.

$X=5, z = \underline{1}$ $X=0, z = \underline{-1.5}$ $X=4, z = \underline{0.5}$ $X=5, z = \underline{1}$

$X=1, z = \underline{-1}$

$X=2, z = \underline{-0.5}$ $X=4, z = \underline{0.5}$

16. There are 40 black marbles, 20 blue marbles, and 4 red marbles in a jar.
- What is the probability of selecting one red marble? 0.0625
 - What is the probability of selecting one black marble? 0.625
 - What is the probability of selecting one blue marble? 0.3125
 - Which has the highest probability of being selected? black
 - Which has the lowest probability of being selected? blue

17. Probability values range from 0 (0%) to 1 (100%).

18. Please describe at least three characteristics of the NORMAL DISTRIBUTION

- its shape is symmetrical
- Its central tendency measurement is right at center, and all have the same value (mean, median, mode)
- most scores located near the mean

19. In a normal population of IQ scores, what percent of people have “average” IQ’s?

Answer 68%: the average means people IQ score within one standard deviation from the mean. That is the range fall within the 1 standard deviation above and below the mean

20. In a normal distribution, what percentage of people would be located at or below 2 standard deviations from the mean?

Answer 5%: the percentage that are located or below 2 standard deviation are considered extremely low, that since 95% are within the 2 standard deviation from the mean.

21. Answer the following questions based on a distribution with a $\mu = 25$ and $\sigma = 4$:

- What range of scores is considered “average”? 21 to 29
- What percentage of people has an average score? 68%
- What percentage of people has extremely high or extremely low scores? 5%

- d. What *range of scores* (requires numbers to be noted in the blank spaces) have the highest probability of being selected? ___17___ to ___33___