

## AS1 (Assignment 1, Unit 3): Z-scores and Probability

Please type your answers in red 😊

- What information is provided by the sign (+/-) of a z-score?  
**The sign of a z-score let me know the location of an x-score is above or below the mean. For example, -2 z-score is below the mean, and +3 z-score is above the mean.**
- What information is provided by the numerical value of the z-score?  
**The numerical value of an z-score informs me the location of the x-score in units of the standard deviation and how far the distance of the x-score is from the mean.**
- A distribution has a standard deviation of  $\sigma = 10$ . Find the z-score for each of the following locations in the distribution.
  - Above the mean by 15 points.  
Answer: 1.5
  - Above the mean by 25 points.  
Answer: 2.5
  - Below the mean by 20 points.  
Answer: -2
  - Below the mean by 5 points.  
Answer: -0.5
- For a distribution with a standard deviation of  $\sigma = 12$ , describe the location of each of the following z-scores in terms of its position relative to the mean. For example,  $+1.00$  is a location that is 12 points above the mean.
  - $z = +2.00$     Answer: 6 points
  - $z = +.50$     Answer: 24 points
  - $z = -1.00$     Answer: -12 points
  - $z = -0.25$     Answer: -48 points
- For a population with  $\mu = 70$  and  $\sigma = 10$ ,

- a. Find the z-score for each of the following X values. (Note: You should be able to find these values using the definition of a z-score. You should not need to use a formula or do any serious calculations.)

$$X = 75: z = \underline{0.5} \quad X = 100: z = \underline{3} \quad X = 60: z = \underline{-1}$$

$$X = 95: z = \underline{2.5} \quad X = 50: z = \underline{-2} \quad X = 85: z = \underline{1.5}$$

- b. Find the score (X value) that corresponds to each of the following z-scores. (Again, you should not need a formula or any serious calculations.)

$$z = 1.00: X = \underline{80} \quad z = 0.20: X = \underline{72} \quad z = 1.50: X = \underline{85}$$

$$z = -0.50: X = \underline{75} \quad z = -2.00: X = \underline{50} \quad z = -1.50: X = \underline{55}$$

6. For a population with a mean of  $\mu = 40$  and  $\sigma = 12$ , find the z-score for each of the following X values. (Note: You probably will need to use a formula and a calculator to find these values)

$$X = 45: z = \underline{0.41} \quad X = 52: z = \underline{1} \quad X = 41: z = \underline{0.08}$$

$$X = 30: z = \underline{-0.83} \quad X = 25: z = \underline{-1.25} \quad X = 38: z = \underline{-0.16}$$

7. For a population with a mean of  $\mu = 100$  and a standard deviation of  $\sigma = 10$ ,

- a. Find the z-score for each of the following X values.

$$X = 108: z = \underline{0.8} \quad X = 115: z = \underline{1.5} \quad X = 130: z = \underline{2.5}$$

$$X = 90: z = \underline{-1} \quad X = 88: z = \underline{-1.2} \quad X = 95: z = \underline{-0.5}$$

- b. Find the score (X value) that corresponds to each of the following z-scores.

$$z = -0.40: X = \underline{96} \quad z = -0.50: X = \underline{95} \quad z = 1.80: X = \underline{118}$$

$$z = 0.75: X = \underline{107.50} \quad z = 1.50: X = \underline{115} \quad z = -1.25: X = \underline{87.50}$$

8. A population has a mean of  $\mu = 70$  and a standard deviation of  $\sigma = 12$ .
- a. For the population, find the z-score for each of the following X values.

X = 69: z = **-0.08**      X = 84: z = **1.16**    X = 63: z = **-0.58**

X = 54: z = **-1.33**      X = 48: z = **-1.83**      X = 45: z = **-2.08**

- b. For the same population, find the score (X value) that corresponds to each of the following z-scores.

z = 0.50: X = **76**      z = 1.50: X = **88**      z = -2.50: X = **40**

z = -0.25: X = **67**      z = -0.50: X = **64**      z = 1.25: X = **85**

9. A sample has a mean of  $M = 30$  and a standard deviation of  $s = 7$ . Find the z-score of for each of the following X values from this sample.

X = 32: z = **-0.6**      X = 34: z = **0.57**    X = 36: z = **0.85**

X = 28: z = **-0.28**      X = 20: z = **-1.42**    X = 18: z = **-1.71**

10. A sample has a mean of  $M = 35$  and a standard deviation of  $s = 5$ . For this sample, find the X value corresponding to each of the following z-scores.

z = 0.40: X = **37**      z = 1.20: X = **41**      z = 2.00: X = **45**

z = -0.80: X = **31**      z = -0.60: X = **32**      z = -1.40: X = **28**

11. Find the z-score corresponding to a score of  $X = 45$  of the following distributions.

- a.  $\mu = 40$  and  $\sigma = 20$

Answer **0.25**

- b.  $\mu = 40$  and  $\sigma = 10$

Answer: **0.5**

- c.  $\mu = 40$  and  $\sigma = 5$

Answer: **1**

- d.  $\mu = 40$  and  $\sigma = 2$

Answer: **2.5**

Find the X value corresponding to  $z = 0.30$  for each of the following distributions.

- e.  $\mu = 40$  and  $\sigma = 4$ .....Answer: **41.20**
- f.  $\mu = 40$  and  $\sigma = 8$ .....Answer: **42.40**
- g.  $\mu = 40$  and  $\sigma = 16$ .....Answer: **44.80**
- h.  $\mu = 40$  and  $\sigma = 32$ .....Answer: **49.60**

12. For each of the following populations, would a score of  $X = 50$  be considered a central score (near the middle of the distribution) or an extreme score (far out in the tail of the distribution)?

- a.  $\mu = 45$  and  $\sigma = 10$ .....Answer: **central score**
- b.  $\mu = 45$  and  $\sigma = 2$ .....Answer: **extreme score**
- c.  $\mu = 90$  and  $\sigma = 20$ .....Answer: **extreme score**
- d.  $\mu = 60$  and  $\sigma = 20$ .....Answer: **central score**

13. A distribution of exam scores has a mean of  $\mu = 78$ .

- a. If your score is  $X = 70$ , which standard deviation would give you a better grade:  $\sigma = 4$  or  $\sigma = 8$ ?

Answer:  **$\sigma = 8$**

- b. If your score is  $X = 80$ , which standard deviation would give you a better grade:  $\sigma = 4$  or  $\sigma = 8$ ?

Answer:  **$\sigma = 4$**

14. For each of the following, identify the exam score that should lead to the better grade.

- a. A score of  $X = 74$  on an exam with  $M = 82$  and  $\sigma = 8$ ; or a score of  $X = 40$  on an exam with  $\mu = 50$  and  $\sigma = 20$ .

Answer: **score of  $X = 40$  on an exam with  $\mu = 50$  and  $\sigma = 20$**

- b. A score of  $X = 51$  on an exam with  $\mu = 45$  and  $\sigma = 2$ ; or a score of  $X = 90$  on an exam with  $\mu = 70$  and  $\sigma = 20$ .

Answer: **score of  $X = 90$  on an exam with  $\mu = 70$  and  $\sigma = 20$ .**

c. A score of  $X = 62$  on an exam with  $\mu = 50$  and  $\sigma = 8$ ; or a score of  $X = 23$  on an exam with  $\mu = 20$  and  $\sigma = 2$     Answer: **Both**

15. A population consists of the following  $N = 5$  scores: 0, 5, 4, 4, and 12.

a. Compute  $\mu$  and  $\sigma$  for the population

Mean = **5**    Standard deviation = **3.87**

b. Find the z-score for each score in the population

$X=0, z = -1.29$     $X=5, z = **0**$     $X=7, z = **-0.51**$     $X=3, z = **-0.51**$     $X=12, z = **1.80**$

15. A sample consists of the following  $n = 7$  scores: 5, 0, 4, 5, 1, 2, and 4.

a. Compute the mean and standard deviation for the sample.

Mean = **3**    Standard deviation = **2**

b. Find the z-score for each score in the sample.

$X=5, z = **1**$     $X=0, z = **-1.5**$     $X=4, z = **0.5**$     $X=5, z = **1**$     $X=1, z = **-2**$

$X=2, z = **-0.5**$     $X=4, z = **0.5**$

16. There are 40 black marbles, 20 blue marbles, and 4 red marbles in a jar.

a. What is the probability of selecting one red marble? **4/64**

b. What is the probability of selecting one black marble? **40/64**

c. What is the probability of selecting one blue marble? **20/64**

d. Which has the highest probability of being selected? **Black**

e. Which has the lowest probability of being selected? **Red**

17. Probability values range from **0%** to **100%**.

18. Please describe at least three characteristics of the NORMAL DISTRIBUTION

1. **Symmetrical shape**
2. **All measures of central tendency are located dead in the center, and all have the same value.**
3. **A distribution is normal if and only if it has specific proportions.**

19. In a normal population of IQ scores, what percent of people have “average” IQ’s?

Answer **68.26%**

20. In a normal distribution, what percentage of people would be located at or below 2 standard deviations from the mean?

Answer **2.28%**

21. Answer the following questions based on a distribution with a  $\mu = 25$  and  $\sigma = 4$ :

- a. What range of scores is considered “average”? **21 to 29**
- b. What percentage of people has an average score? **68.26%**
- c. What percentage of people has extremely high or extremely low scores? **2.28%**
- d. What *range of scores* (requires numbers to be noted in the blank spaces) have the highest probability of being selected? **21 to 29**