AS1 (Assignment 1, Unit 3): Z-scores and Probability

Please type your answers in red ☺

1. What information is provided by the sign (+/-) of a *z*-score?

   If the sign of the z-score is positive, then you would know that the location of the X-score is ABOVE mean, and if the sign of the z-score is negative, then you would know that the location of the X-score is BELOW the mean.

2. What information is provided by the numerical value of the *z*-score?

The **numerical** value of the z-score tells you about the distance from the mean at which the X-score is located, in units of **standard deviations.**

3. A distribution has a standard deviation of $\sigma = 10$. Find the *z*-score for each of the following locations in the distribution.

   a. Above the mean by 15 points.

   Answer: 1.5

   b. Above the mean by 25 points.

   Answer: 2.5

   c. Below the mean by 20 points.

   Answer: -2

   d. Below the mean by 5 points.

   Answer: -0.5

4. For a distribution with a standard deviation of $\sigma = 12$, describe the location of each of the following *z*-scores in terms of its position relative to the mean. For example, = +1.00 is a location that is 12 points above the mean.

   a. *z* = +2.00   Answer: 24 points above the mean

   b. *z* = +.50   Answer: 6 points above the mean

   c. *z* = -1.00   Answer: 12 points below the mean

   d. *z* = -0.25   Answer: 3 points below the mean

5. For a population with $\mu = 70$ and $\sigma = 10$,

   a. Find the z-score for each of the following X values. (Note: You should be able to find these values using the definition of a z-score. You should not need to use a formula or do any serious calculations.)

   | X = 75: | z = +5 | X = 100: z = +3.0 | X = 60: z = - 1 |

   X = 75:   z = +5   X = 100: z = +3.0   X = 60: z = - 1

   X = 95:   z= +2.5   X = 50: z = -2   X = 85 z = +1.5

   b. Find the score (X value) that corresponds to each of the following z-scores. (Again, you should not need a formula or any serious calculations.)

   z = 1.00: X= 80   z = 0.20: X= 72   z = 1.50: X= 85

   z = -0.50: X= 65   z = -2.00: X= 50   z = -1.50: X= 55

6. For a population with a mean of $\mu = 40$ and $\sigma = 12$, find the z–score for each of the following X values. (Note: You probably will need to use a formula and a calculator to find these values)

   X = 45: z = 0.41   X = 52: z = 1   X = 41: z = 0.08

   X = 30: z = -0.83   X = 25: z = -1.25   X = 38: z = -0.16

7. For a population with a mean of $\mu = 100$ and a standard deviation of $\sigma = 10$,

   a. Find the z-score for each of the following X values.

   X = 108: z = 0.8   X = 115: z = 1.5   X = 130: z = 3

   X = 90: z = -1   X = 88: z = -1.2   X = 95: z = -0.5

   b. Find the score (X value) that corresponds to each of the following z-scores.

   z = -0.40: X= 96   z = -0.50: X= 95   z = 1.80: X= 118

   z = 0.75: X= 107.5   z = 1.50: X= 115   z = -1.25: X= 87.5

8. A population has a mean of $\mu = 70$ and a standard deviation of $\sigma = 12$.

   a. For the population, find the z-score for each of the following X values.

   X = 69: z = -0.08   X = 84: z = 1.16   X = 63: z = -0.58

   X = 54: z = -1.33   X = 48: z = -1.83   X = 45: z = -2.08

   b. For the same population, find the score (X value) that corresponds to each of the following z-scores.

   z = 0.50: X= 76   z = 1.50: X= 88   z = -2.50: X= 40

   z = -0.25: X= 67   z = -0.50: X= 64   z = 1.25: X= 85

9. A sample has a mean of M = 30 and a standard deviation of s = 7. Find the z-score of for each of the following X values from this sample.

   X = 32: z = 0.28   X = 34: z = 0.57   X = 36: z = 0.85

   X = 28: z = -0.28   X = 20: z = -1.42   X = 18: z = -1.71

10. A sample has a mean of M = 35 and a standard deviation of $s = 5$. For this sample, find the X value corresponding to each of the following z-scores.

   z = 0.40: X= 37   z = 1.20: X= 41   z = 2.00: X= 45

   z = -0.80: X= 31   z = -0.60: X= 32   z = -1.40: X= 28

11. Find the z-score corresponding to a score of X = 45 of the following distributions.

   a. $\mu = 40$ and $\sigma = 20$

   Answer: *0.25*

   b. $\mu = 40$ and $\sigma = 10$

   Answer: *0.5*

   c. $\mu = 40$ and $\sigma = 5$

   Answer: *1*

   d. $\mu = 40$ and $\sigma = 2$

   Answer: 2.5

Find the X value corresponding to z = 0.30 for each of the following distributions.

    e. μ = 40 an*d* σ = 4………………………Answer: 41.2

    f. μ = 40 and σ = 8…………………….Answer: 42.4

    g. μ = 40 a*nd* σ = 16…………………..Answer: 44.8

    h. μ = 40 and σ = 32……………………Answer: 49.6

12. For each of the following populations, would a score of X = 50 be considered a central score (near the middle of the distribution) or an extreme score (far out in the tail of the distribution)?

    a. μ = 45 and σ = 10……………………..Answer: central score

    b. μ = 45 and σ = 2……………………..Answer: an extreme

    c. μ = 90 and σ = 20……………………..Answer: an extreme

    d. μ = 60 and σ = 20……………………..Answer: central score

13. A distribution of exam scores has a mean of μ = 78.

    a. If your score is X = 70, which standard deviation would give you a better grade: σ = 4 or σ = 8?

                    Answer: σ 8

    b. If your score is X = 80, which standard deviation would give you a better grade: σ = 4 or σ = 8?

                    Answer: σ 4

14. For each of the following, identify the exam score that should lead to the better grade.

    a. A score of X = 74 on an exam with M = 82 and σ = 8; or a score of X = 40 on an exam with μ = 50 and σ = 20.

                    Answer: first score

    b. A score of X = 51 on an exam with μ = 45 and σ = 2; or a score of X = 90 on an exam with μ = 70 and σ = 20.

                    Answer: first score

    c. A score of X = 62 on an exam with μ = 50 and σ = 8; or a score of X = 23 on an exam with μ = 20 and σ = 2   Answer: first score

15. A population consists of the following N = 5 scores: 0, 5, 4, 4, and 12.

a. Compute μ and σ for the population

Mean = 5   Standard deviation= 3.89

b. Find the *z*-score for each score in the population

X=0, z = -1.28   X=5, z= 0   X=7, z= 0.51   X=3, z= -0.51   X=12, z= 1.79

15. A sample consists of the following *n* = 7 scores: 5, 0, 4, 5, 1, 2, and 4.
a. Compute the mean and standard deviation for the sample.

Mean = 3   Standard deviation = 2

b. Find the *z*-score for each score in the sample.

X=5, z= 1   X=0, z= -1.5   X=4, z= 0.5   X=5, z= 1   X=1, z= -1

X=2, z= -0.5   X=4, z= 0.5

16. There are 40 black marbles, 20 blue marbles, and 4 red marbles in a jar.

a.   What is the probability of selecting one red marble? 4/64

b.   What is the probability of selecting one black marble?   40/64

c.   What is the probability of selecting one blue marble? 20/64

d.   Which has the highest probability of being selected? black

e.   Which has the lowest probability of being selected? red

17. Probability values range from 0 (0%) to 1 (100%)

18. Please describe at least three characteristics of the NORMAL DISTRIBUTION

1. symmetrical in shape

2. All measures of central tendency are located dead center, and all have the same value.

3. There are more scores located near the mean (the center of the distribution…that "average" range) than there are scores as we move away from the mean.

19. In a normal population of IQ scores, what percent of people have "average" IQ's?

Answer 68.26%

20. In a normal distribution, what percentage of people would be located at or below 2 standard deviations from the mean?

Answer 95%

21. Answer the following questions based on a distribution with a $\mu = 25$ and $\sigma = 4$:

    a.   What range of scores is considered "average"? 26 to 28

    b.   What percentage of people has an average score? 25 %

    c.   What percentage of people has extremely high or extremely low scores? 50%

    d.   What *range of scores* (requires numbers to be noted in the blank spaces) have the highest probability of being selected? 25 to 29