

AS1 (Assignment 1, Unit 3): Z-scores and Probability

Please type your answers in red 😊

1. What information is provided by the sign (+/-) of a z-score? _
The sign (+/-) of a z score is that the + sign means the score above the mean. The – sign means the score below the mean.
2. What information is provided by the numerical value of the z-score?
_The numerical value of the z score tells us the distance from the mean in standard deviations.
3. A distribution has a standard deviation of $\sigma = 10$. Find the z-score for each of the following locations in the distribution.
 - a. Above the mean by 15 points.
Answer: 1.5
 - b. Above the mean by 25 points.
Answer: 2.5
 - c. Below the mean by 20 points.
Answer: -2
 - d. Below the mean by 5 points.
Answer: -0.5
4. For a distribution with a standard deviation of $\sigma = 12$, describe the location of each of the following z-scores in terms of its position relative to the mean. For example, $+1.00$ is a location that is 12 points above the mean.
 - a. $z = +2.00$ Answer: 24
 - b. $z = +.50$ Answer: .50
 - c. $z = -1.00$ Answer: -12
 - d. $z = -0.25$ Answer: -0.75
5. For a population with $\mu = 70$ and $\sigma = 10$,

- a. Find the z -score for each of the following X values. (Note: You should be able to find these values using the definition of a z -score. You should not need to use a formula or do any serious calculations.)

$$X = 75: z = \underline{.5} \quad X = 100: z = \underline{3} \quad X = 60: z = \underline{-1}$$

$$X = 95: z = \underline{2.5} \quad X = 50: z = \underline{-2} \quad X = 85: z = \underline{1.5}$$

- b. Find the score (X value) that corresponds to each of the following z -scores. (Again, you should not need a formula or any serious calculations.)

$$z = 1.00: X = \underline{80} \quad z = 0.20: X = \underline{72} \quad z = 1.50: X = \underline{85}$$

$$z = -0.50: X = \underline{65} \quad z = -2.00: X = \underline{50} \quad z = -1.50: X = \underline{55}$$

6. For a population with a mean of $\mu = 40$ and $\sigma = 12$, find the z -score for each of the following X values. (Note: You probably will need to use a formula and a calculator to find these values)

$$X = 45: z = \underline{0.416} \quad X = 52: z = \underline{1} \quad X = 41: z = \underline{-0.083}$$

$$X = 30: z = \underline{-0.83} \quad X = 25: z = \underline{-1.25} \quad X = 38: z = \underline{-0.16}$$

7. For a population with a mean of $\mu = 100$ and a standard deviation of $\sigma = 10$,

- a. Find the z -score for each of the following X values.

$$X = 108: z = \underline{0.8} \quad X = 115: z = \underline{1.5} \quad X = 130: z = \underline{3}$$

$$X = 90: z = \underline{-1} \quad X = 88: z = \underline{-1.2} \quad X = 95: z = \underline{-0.5}$$

- b. Find the score (X value) that corresponds to each of the following z -scores.

$$z = -0.40: X = \underline{96} \quad z = -0.50: X = \underline{95} \quad z = 1.80: X = \underline{118}$$

$$z = 0.75: X = \underline{107.5} \quad z = 1.50: X = \underline{115} \quad z = -1.25: X = \underline{87.5}$$

8. A population has a mean of $\mu = 70$ and a standard deviation of $\sigma = 12$.

a. For the population, find the z-score for each of the following X values.

$X = 69: z = \underline{-0.83}$ $X = 84: z = \underline{1.16}$ $X = 63: z = \underline{-0.58}$

$X = 54: z = \underline{-1.3}$ $X = 48: z = \underline{-1.83}$ $X = 45: z = \underline{-2.08}$

b. For the same population, find the score (X value) that corresponds to each of the following z-scores.

$z = 0.50: X = \underline{76}$ $z = 1.50: X = \underline{88}$ $z = -2.50: X = \underline{40}$

$z = -0.25: X = \underline{67}$ $z = -0.50: X = \underline{64}$ $z = 1.25: X = \underline{85}$

9. A sample has a mean of $M = 30$ and a standard deviation of $s = 7$. Find the z-score of for each of the following X values from this sample.

$X = 32: z = \underline{0.28}$ $X = 34: z = \underline{0.57}$ $X = 36: z = \underline{0.85}$

$X = 28: z = \underline{-0.28}$ $X = 20: z = \underline{-1.42}$ $X = 18: z = \underline{-1.42}$

10. A sample has a mean of $M = 35$ and a standard deviation of $s = 5$. For this sample, find the X value corresponding to each of the following z-scores.

$z = 0.40: X = \underline{37}$ $z = 1.20: X = \underline{41}$ $z = 2.00: X = \underline{45}$

$z = -0.80: X = \underline{31}$ $z = -0.60: X = \underline{32}$ $z = -1.40: X = \underline{28}$

11. Find the z-score corresponding to a score of $X = 45$ of the following distributions.

a. $\mu = 40$ and $\sigma = 20$

Answer: $\underline{0.25}$

b. $\mu = 40$ and $\sigma = 10$

Answer: $\underline{0.5}$

c. $\mu = 40$ and $\sigma = 5$

Answer: $\underline{1}$

d. $\mu = 40$ and $\sigma = 2$

Answer: $\underline{2.5}$

Find the X value corresponding to $z = 0.30$ for each of the following distributions.

e. $\mu = 40$ and $\sigma = 4$Answer: 41.2

f. $\mu = 40$ and $\sigma = 8$Answer: 42.4

g. $\mu = 40$ and $\sigma = 16$Answer: 44.8

h. $\mu = 40$ and $\sigma = 32$Answer: 49.6

12. For each of the following populations, would a score of $X = 50$ be considered a central score (near the middle of the distribution) or an extreme score (far out in the tail of the distribution)?

a. $\mu = 45$ and $\sigma = 10$Answer: central

b. $\mu = 45$ and $\sigma = 2$Answer: extreme

c. $\mu = 90$ and $\sigma = 20$Answer: extreme

d. $\mu = 60$ and $\sigma = 20$Answer: central

13. A distribution of exam scores has a mean of $\mu = 78$.

a. If your score is $X = 70$, which standard deviation would give you a better grade: $\sigma = 4$ or $\sigma = 8$?

Answer: $\sigma = 8$

b. If your score is $X = 80$, which standard deviation would give you a better grade: $\sigma = 4$ or $\sigma = 8$?

Answer: $\sigma = 8$

14. For each of the following, identify the exam score that should lead to the better grade.

a. A score of $X = 74$ on an exam with $M = 82$ and $\sigma = 8$; or a score of $X = 40$ on an exam with $\mu = 50$ and $\sigma = 20$.

Answer: $X = 40$ $\mu = 50$ and $\sigma = 20$

b. A score of $X = 51$ on an exam with $\mu = 45$ and $\sigma = 2$; or a score of $X = 90$ on an exam with $\mu = 70$ and $\sigma = 20$.

Answer: $X = 90$ $\mu = 70$ and $\sigma = 20$

c. A score of $X = 62$ on an exam with $\mu = 50$ and $\sigma = 8$; or a score of $X = 23$ on an exam with $\mu = 20$ and $\sigma = 2$ Answer: Both are equal

15. A population consists of the following $N = 5$ scores: 0, 5, 4, 4, and 12.

a. Compute μ and σ for the population

$$\text{Mean} = \underline{5} \quad \text{Standard deviation} = \underline{3.89}$$

b. Find the z -score for each score in the population

$$X=0, z = -1.28 \quad X=5, z = 0 \quad X=7, z = 0.514 \quad X=3, z = -0.514 \quad X=12, z = 1.799$$

15. A sample consists of the following $n = 7$ scores: 5, 0, 4, 5, 1, 2, and 4.

a. Compute the mean and standard deviation for the sample.

$$\text{Mean} = \underline{3} \quad \text{Standard deviation} = \underline{2}$$

b. Find the z -score for each score in the sample.

$$X=5, z = 1 \quad X=0, z = -1.5 \quad X=4, z = 0.5 \quad X=5, z = 1 \quad X=1, z = -1$$

$$X=2, z = -0.5 \quad X=4, z = 0.5$$

16. There are 40 black marbles, 20 blue marbles, and 4 red marbles in a jar.

a. What is the probability of selecting one red marble? $\underline{4/64}$

b. What is the probability of selecting one black marble? $\underline{40/64}$

c. What is the probability of selecting one blue marble? $\underline{20/64}$

d. Which has the highest probability of being selected? $\underline{\text{black}}$

e. Which has the lowest probability of being selected? $\underline{\text{red}}$

17. Probability values range from $\underline{0 (0\%)}$ to $\underline{1 (100\%)}$.

18. Please describe at least three characteristics of the NORMAL DISTRIBUTION

1. $\underline{\text{It is symmetrical in shape.}}$

2. $\underline{\text{All measures of central tendency are located in center \& all have the same value.}}$

3. $\underline{\text{More scores are located near the mean, than scores that are away from the mean.}}$

19. In a normal population of IQ scores, what percent of people have “average” IQ’s?

$$\text{Answer } \underline{68.13\%}$$

20. In a normal distribution, what percentage of people would be located at or below 2 standard deviations from the mean?

Answer 2.28%

21. Answer the following questions based on a distribution with a $\mu = 25$ and $\sigma = 4$:

- a. What range of scores is considered “average”? 21 to 29
- b. What percentage of people has an average score? 68.26%
- c. What percentage of people has extremely high or extremely low scores? 2.28%
- d. What *range of scores* (requires numbers to be noted in the blank spaces) have the highest probability of being selected? 50 to 50