

AS1 (Assignment 1, Unit 3): Z-scores and Probability

Please type your answers in red 😊

1. What information is provided by the sign (+/-) of a z-score? positive z-score means that the x-score is located above the mean, negative means the location below the mean.

2. What information is provided by the numerical value of the z-score? the distance from the mean at which the x-score is located in units of standard deviations.

3. A distribution has a standard deviation of $\sigma = 10$. Find the z-score for each of the following locations in the distribution.

- a. Above the mean by 15 points.

Answer: 1.5

- b. Above the mean by 25 points.

Answer: 2.5

- c. Below the mean by 20 points.

Answer: -2

- d. Below the mean by 5 points.

Answer: -0.5

4. For a distribution with a standard deviation of $\sigma = 12$, describe the location of each of the following z-scores in terms of its position relative to the mean. For example, $+1.00$ is a location that is 12 points above the mean.

- a. $z = +2.00$ Answer: 24 points above the mean

- b. $z = +.50$ Answer: 6 points above the mean

- c. $z = -1.00$ Answer: 12 points below the mean

- d. $z = -0.25$ Answer: 3 points below the mean

5. For a population with $\mu = 70$ and $\sigma = 10$,

- a. Find the z-score for each of the following X values. (Note: You should be able to find these values using the definition of a z-score. You should not need to use a formula or do any serious calculations.)

X = 75: z = 0.5 X = 100: z = 3 X = 60: z = -1

X = 95: z = 2.5 X = 50: z = -2 X = 85: z = 1.5

- b. Find the score (X value) that corresponds to each of the following z-scores. (Again, you should not need a formula or any serious calculations.)

z = 1.00: X = 80 z = 0.20: X = 72 z = 1.50: X = 85

z = -0.50: X = 65 z = -2.00: X = 50 z = -1.50: X = 55

6. For a population with a mean of $\mu = 40$ and $\sigma = 12$, find the z-score for each of the following X values. (Note: You probably will need to use a formula and a calculator to find these values)

X = 45: z = 0.41

X = 52: z = 1

X = 41: z = 0.083

X = 30: z = -0.83

X = 25: z = -1.25

X = 38: z = -0.16

7. For a population with a mean of $\mu = 100$ and a standard deviation of $\sigma = 10$,

- a. Find the z-score for each of the following X values.

X = 108: z = 0.8 X = 115: z = 1.5 X = 130: z = 3

X = 90: z = -1 X = 88: z = -1.2 X = 95: z = -0.5

- b. Find the score (X value) that corresponds to each of the following z-scores.

z = -0.40: X = 96 z = -0.50: X = 95

z = 1.80: X = 118

z = 0.75: X = 107.5 z = 1.50: X = 115

z = -1.25: X = 87.5

8. A population has a mean of $\mu = 70$ and a standard deviation of $\sigma = 12$.
- a. For the population, find the z-score for each of the following X values.

X = 69: z = -0.08 X = 84: z = 1.16
X = 63: z = -0.58

X = 54: z = -1.33 X = 48: z = -1.83
X = 45: z = -2.08

- b. For the same population, find the score (X value) that corresponds to each of the following z-scores.

z = 0.50: X = 76 z = 1.50: X = 88 z = -2.50: X = 40

z = -0.25: X = 67 z = -0.50: X = 64 z = 1.25: X = 85

9. A sample has a mean of $M = 30$ and a standard deviation of $s = 7$. Find the z-score of for each of the following X values from this sample.

X = 32: z = 0.28 X = 34: z = 0.57 X = 36: z = 0.85

X = 28: z = -0.28 X = 20: z = -1.42
X = 18: z = -1.71

10. A sample has a mean of $M = 35$ and a standard deviation of $s = 5$. For this sample, find the X value corresponding to each of the following z-scores.

z = 0.40: X = 37 z = 1.20: X = 41 z = 2.00: X = 45

z = -0.80: X = 31 z = -0.60: X = 32 z = -1.40: X = 28

11. Find the z-score corresponding to a score of $X = 45$ of the following distributions.

a. $\mu = 40$ and $\sigma = 20$
Answer: 0.25

b. $\mu = 40$ and $\sigma = 10$
Answer: 0.5

c. $\mu = 40$ and $\sigma = 5$
Answer: 1

d. $\mu = 40$ and $\sigma = 2$
Answer: 2.5

Find the X value corresponding to $z = 0.30$ for each of the following distributions.

e. $\mu = 40$ and $\sigma = 4$Answer: 41.2

f. $\mu = 40$ and $\sigma = 8$Answer: 42.4

g. $\mu = 40$ and $\sigma = 16$Answer: 44.8

h. $\mu = 40$ and $\sigma = 32$Answer: 49.6

12. For each of the following populations, would a score of $X = 50$ be considered a central score (near the middle of the distribution) or an extreme score (far out in the tail of the distribution)?

a. $\mu = 45$ and $\sigma = 10$Answer: 0.5 central

b. $\mu = 45$ and $\sigma = 2$Answer: 2.5 extreme

c. $\mu = 90$ and $\sigma = 20$Answer: -2 extreme

d. $\mu = 60$ and $\sigma = 20$Answer: -0.5 central

13. A distribution of exam scores has a mean of $\mu = 78$.

a. If your score is $X = 70$, which standard deviation would give you a better grade: $\sigma = 4$ or $\sigma = 8$?

Answer: 8

b. If your score is $X = 80$, which standard deviation would give you a better grade: $\sigma = 4$ or $\sigma = 8$?

Answer: 4

14. For each of the following, identify the exam score that should lead to the better grade.

a. A score of $X = 74$ on an exam with $M = 82$ and $\sigma = 8$; or a score of $X = 40$ on an exam with $\mu = 50$ and $\sigma = 20$.

Answer: a score of $X = 40$ on an exam with $\mu = 50$ and $\sigma = 20$.

b. A score of $X = 51$ on an exam with $\mu = 45$ and $\sigma = 2$; or a score of $X = 90$ on an exam with $\mu = 70$ and $\sigma = 20$.

Answer: A score of $X = 51$ on an exam with $\mu = 45$ and $\sigma = 2$.

- c. A score of $X = 62$ on an exam with $\mu = 50$ and $\sigma = 8$; or a score of $X = 23$ on an exam with $\mu = 20$ and $\sigma = 2$ Answer: equal z-score gives equal grade

15. A population consists of the following $N = 5$ scores: 0, 5, 4, 4, and 12.

- a. Compute μ and σ for the population

Mean = 5 Standard deviation = 3.89

- b. Find the z-score for each score in the population

$X=0, z = -1.28$ $X=5, z=0$ $X=7, z=0.51$
 $X=3, z=-0.51$ $X=12, z=-1.79$

15. A sample consists of the following $n = 7$ scores: 5, 0, 4, 5, 1, 2, and 4.

- a. Compute the mean and standard deviation for the sample.

Mean = 3 Standard deviation = 2

- b. Find the z-score for each score in the sample.

$X=5, z=1$ $X=0, z=-1.5$ $X=4, z=0.5$ $X=5, z=1$
 $X=1, z=-1$
 $X=2, z=-0.5$ $X=4, z=0.5$

16. There are 40 black marbles, 20 blue marbles, and 4 red marbles in a jar.

- a. What is the probability of selecting one red marble?

$\frac{4}{64} = 0.0625$

- b. What is the probability of selecting one black marble? $\frac{40}{64} = 0.625$

- c. What is the probability of selecting one blue marble? $\frac{20}{64} =$

$= 0.3125$

d. Which has the highest probability of being selected? ____ 0.625 for black marbles _____

e. Which has the lowest probability of being selected? ____ 0.0625 for red marbles _____

17. Probability values range from ____ 0 (0%) _____ to ____ 1(100%) _____.

18. Please describe at least three characteristics of the NORMAL DISTRIBUTION

1. ____ symmetrical shape _____

2. ____ mean, median and mode have same value _____

3. ____ more scores located near the mean _____

19. In a normal population of IQ scores, what percent of people have “average” IQ’s?

Answer

68.26% _____

20. In a normal distribution, what percentage of people would be located at or below 2 standard deviations from the mean?

Answer

2.28% _____

21. Answer the following questions based on a distribution with a $\mu = 25$ and $\sigma = 4$:

a. What range of scores is considered “average”? ____ 21 _____ to ____ 29 _____

b. What percentage of people has an average score? ____ 68.26% _____

c. What percentage of people has extremely high or extremely low scores? ____ 4.56% _____

d. What *range of scores* (requires numbers to be noted in the blank spaces) have the highest probability of being selected? ____ 25 _____ to ____ 29 _____