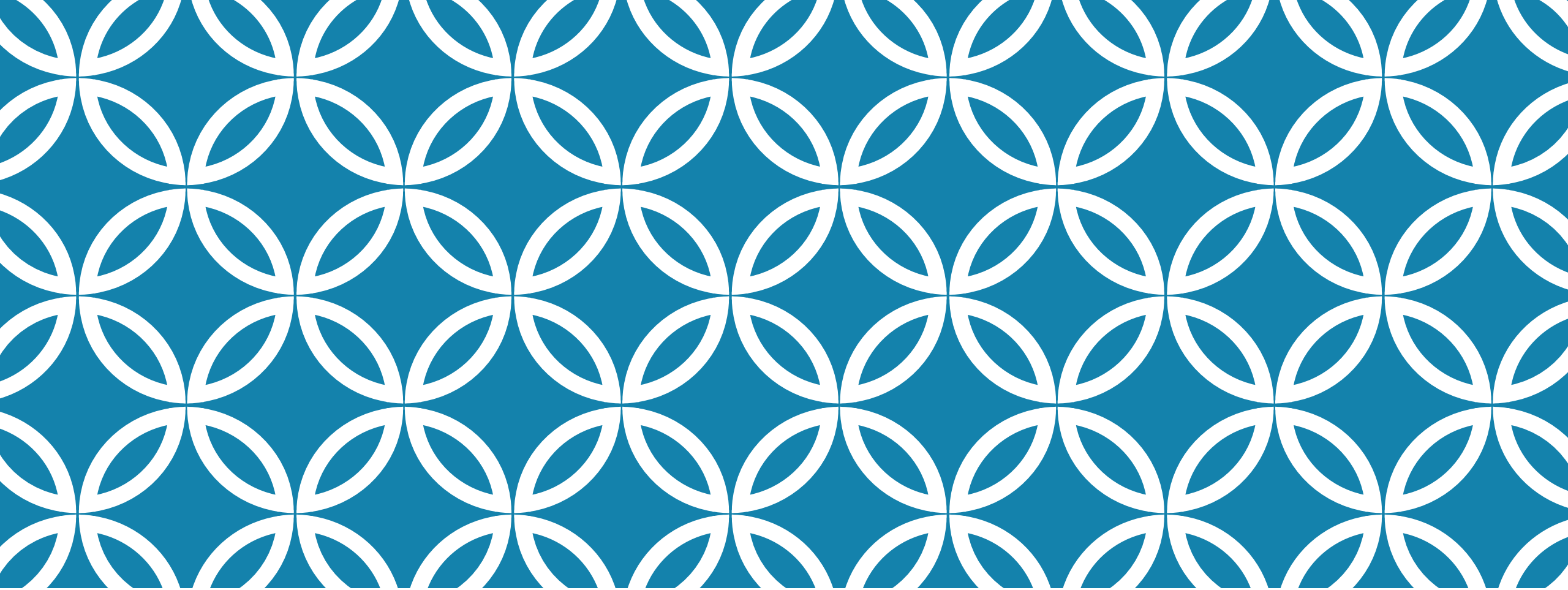

# MAT 246-CALCULUS

## *DIAGNOSTIC TEST*

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## *OVERVIEW*

*Are you ready to study calculus?*

*Math is the language in which we express the ideas of calculus. Therefore, to understand calculus and express its ideas with precision, you need to know some math.*

*If you are comfortable with the math covered in the following problems, you are ready to begin your study of calculus. If not, turn to the Math Appendix beginning on page A.xxx and review the Complete Solutions to these problems, and continue reading the other parts of the Appendix that cover anything that you do not know.*

# PROBLEM#1

True or False?

$$\frac{1}{2} < -3$$

False.

$0.5 < -3$

# PROBLEM#2

Express $\{x \mid -4 < x \leq 5\}$ in interval notation.

$$x = \{-3, -2, -1, 0, 1, 2, 3, 4, 5\}$$

# PROBLEM#3

What is the slope of the line through the points (6,−7) and (9, 8)?

$\Delta y = 8 - (-7) = 15$

$\Delta x = 9 - 6 = 3$

$m = 15/3 = 5$

# PROBLEM#4

On the line y = 3x + 4, what value of $\Delta y$ corresponds $\Delta x = 2$?

$y_1 = 3(2) + 4$

$y_1 = 10$

$y_2 = 3(4) + 4$

$y_2 = 16$

$\Delta y = y_2 - y_1 = 16 - 10 -$

$\Delta y = 6$

$\Delta x = 2$

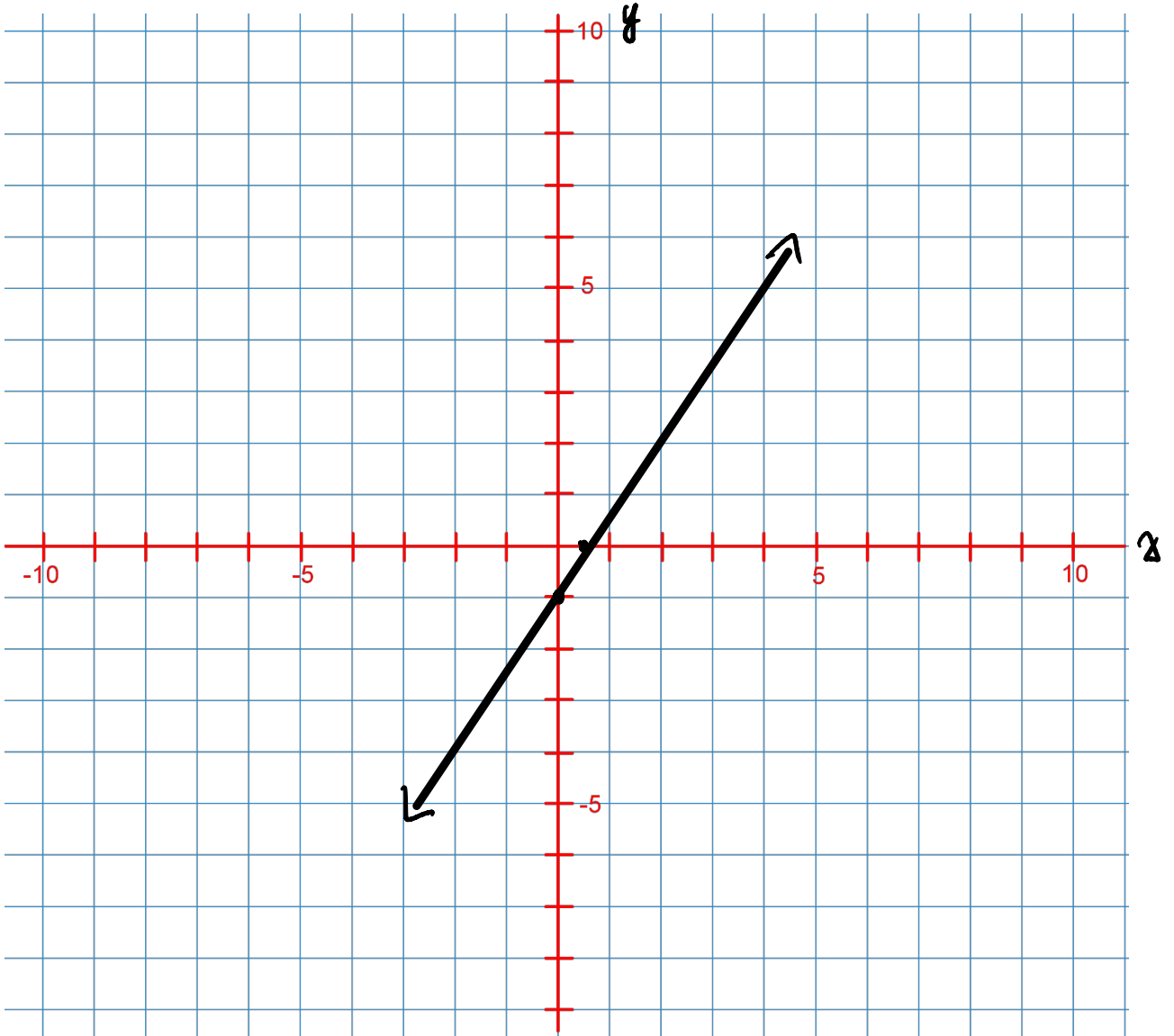
$= x_2 - x_1$

$4 - 2$

$(2; y_1)(4; y_2)$

# PROBLEM#5

Sketch the graph of the line $y = 2x - 1$?




$(0, -1)$

$y = 2(0) - 1$

$y = -1$

$(\frac{1}{2}, 0)$

$0 = 2x - 1$

$2x = 1$

$x = \frac{1}{2}$

# PROBLEM#6

$True\ or\ False$: $\frac{y^2}{x} = \left(\frac{\sqrt{x}}{y}\right)^{-2}$

True

$$\frac{y^2}{x} = \frac{y^2}{\sqrt{x}^2}$$

# PROBLEM#7

Find the zeros of the function: $f(x) = 9x^2 - 6x - 1$

$$0 = 9x^2 - 6x - 1 \quad \div 9$$

$$0 = x^2 - \frac{2x}{3} - \frac{1}{9}$$

$$\frac{1}{9} = x^2 - \frac{2x}{3}$$

$$\frac{2}{9} = x^2 - \frac{2x}{3} + \frac{1}{9}$$

$$\frac{2}{9} = \left(x - \frac{1}{3}\right)^2$$

$$x = \frac{\sqrt{2}}{3} - \frac{1}{3}$$

$$x = \frac{\sqrt{2}}{3} - \frac{1}{3}$$

# PROBLEM#8

Expand and simplify $x(8 - x) - (3x + 7)$.

$$8x - x^2 - 3x - 7$$

$$-x^2 + 5x - 7 = 0$$

# PROBLEM#9

What is the domain of: $\frac{x^2-3x-2}{x^3+x^2-6x}$

$$= \frac{x^2-3x-2}{x(x+2)(x-3)}$$

$x=0 \quad x=-2 \quad x=3$

Domain $= \{x \mid x \neq 0, x \neq -2, x \neq 3\}$

# PROBLEM#10

For $f(x) = x^2 - 5x$, find the difference quotient: $\frac{f(x+h)-f(x)}{h}$

$$= \frac{\left((x+h)^2 - 5(x+h) - (x^2 - 5x)\right)}{h}$$

$$= h + 2x - 5$$

# PROBLEM#11

Complete the following table with the appropriate values:

| | 0° | 30° | 45° | 60° | 90° |
|---|---|---|---|---|---|
| $\sin \emptyset$ | $0$ | $\frac{1}{2}$ | $\frac{\sqrt{2}}{2}$ | $\frac{\sqrt{3}}{2}$ | $1$ |
| $\cos \emptyset$ | $1$ | $\frac{\sqrt{3}}{2}$ | $\frac{\sqrt{2}}{2}$ | $\frac{1}{2}$ | $0$ |
| $\tan \emptyset$ | $0$ | $\frac{\sqrt{3}}{3}$ | $1$ | $\sqrt{3}$ | $\infty$ |

# PROBLEM#12

Prove the following:

$$\frac{\csc \emptyset}{\sin \emptyset} - \cot^2 \emptyset = 1$$

$$\frac{\frac{1}{\sin x}}{\sin x} - \left(\frac{\cos x}{\sin x}\right)^2 = 1$$

$$\frac{1}{\sin x^2} - \frac{\cos x^2}{\sin x^2} = 1$$

$$\frac{1 - \cos x^2}{\sin x^2} = 1$$

$$\frac{\sin x^2}{\sin x^2} = 1$$

# PROBLEM#13

Prove the following:

$$\sin \emptyset + \frac{cos^2 \emptyset}{1+\sin \emptyset} = 1$$

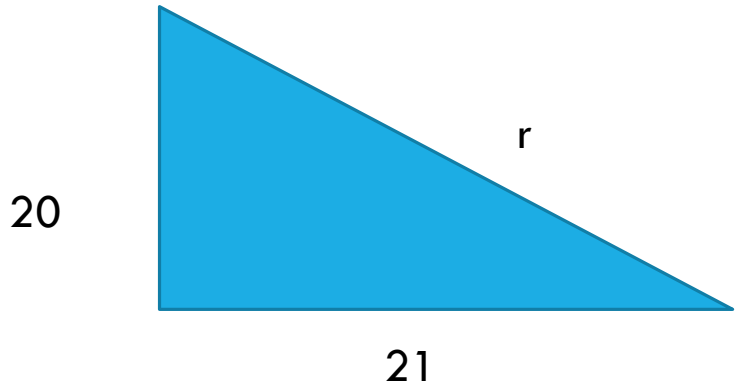
$$\sin x + \frac{1-\sin x^2}{1+\sin x}$$

$$\sin x + \frac{(1-\sin x)(1+\sin x)}{1+\sin x}$$

$$\sin x + 1 - \sin x$$

$$1 = 1$$

# PROBLEM#14




FIND r:

$$r^2 = 20^2 + 21^2$$

$$r^2 = 400 + 441$$

$$r^2 = 841$$

$$r = 29$$

# PROBLEM#15

FIND ALL THE POSSIBLE VALUES FOR $\emptyset$ $BETWEEN$ $0° \leq \emptyset \leq 360°$

$$csc^2\emptyset - 3\csc\emptyset - 4 = 0$$

$t^2 - 3t - 4 = 0$

$t = -1 \quad t = 4$

$\csc\theta = -1 \quad \csc\theta = 4$

$\theta = 270° + 360° \quad \theta = 180° + 360°$

# PROBLEM#16

Complete the following table with the appropriate values:

| | 0° | 30° | 45° | 60° | 90° |
|---|---|---|---|---|---|
| $\sec \emptyset$ | 1 | $2/\sqrt{3}$ | $\sqrt{2}$ | 2 | not defined |
| $\csc \emptyset$ | not defined | 2 | $\sqrt{2}$ | $2/\sqrt{3}$ | 1 |
| $\cot \emptyset$ | not defined | $\sqrt{3}$ | 1 | $1/\sqrt{3}$ | 0 |

# PROBLEM#17

Simplify

$$\frac{2^{5x+1}}{2 \cdot 2^{-x}}$$

$$(5x+1) - (-x+1) =$$

$$5x + x + 1 - 1 =$$

$$> \frac{2^{5x+1}}{(2^1) \cdot (2^{-x})} = \frac{2^{5x+1}}{2^{-x+1}} = 2^{6x}$$

# PROBLEM#18

Simplify

$$\left(\frac{1}{64}\right)^{\frac{x}{2}} = \frac{1}{64^{\frac{1}{2}}}$$

# PROBLEM#19

Solve for x

$$(2-3x)\cdot 5^x + 4\cdot 5^x = 0$$

$$5^x(2-3x+4) = 0$$

$$5^x(6-3x) = 0$$

$$5^x = 0 \qquad 6-3x = 0$$

$$\emptyset \qquad x = 2$$

# PROBLEM#20

Solve for x

$$5\ln(2x) = 8$$

$$\ln(2x) = 8/5$$

$$2x = e^{8/5}$$

$$2x = \sqrt[5]{e^8}$$

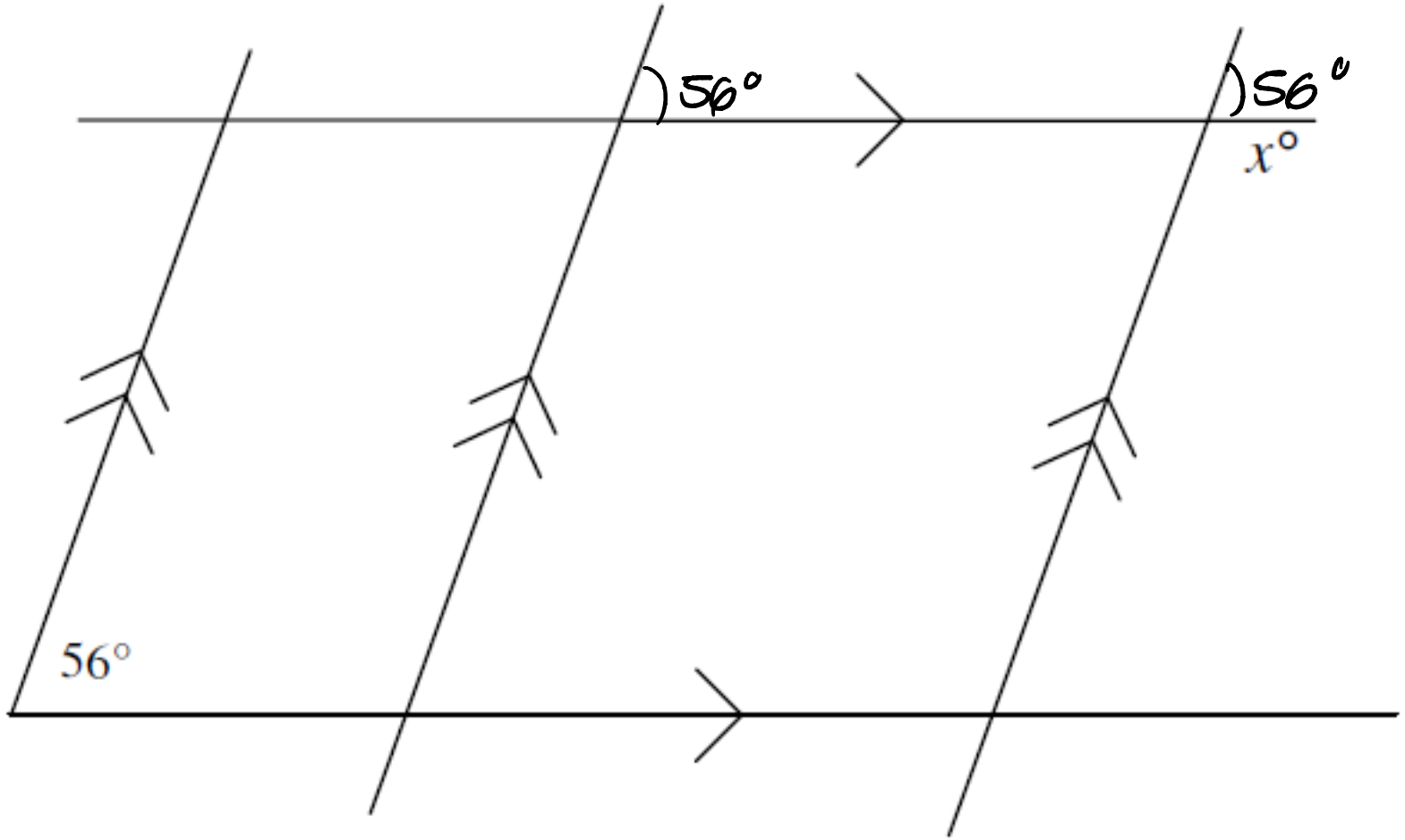
$$x = \frac{\sqrt[5]{e^3}}{2}$$

# PROBLEM#21

Solve for x

# PROBLEM#22

Solve for x



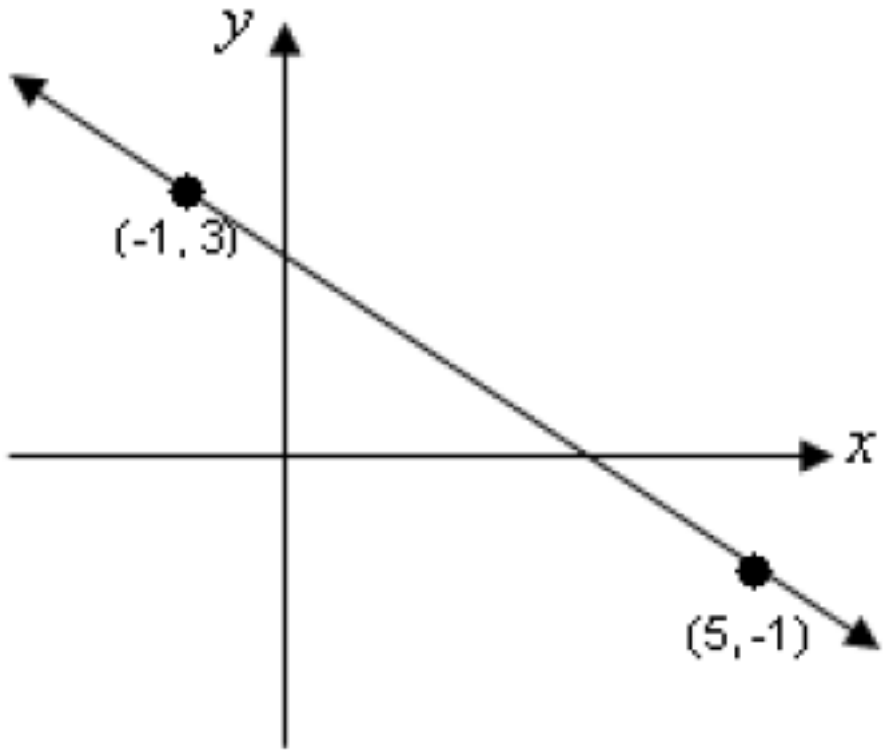

$56 + x = 180°$

$x = 180° - 56°$

$x = 175°$

# PROBLEM#23

The gradient of the line is



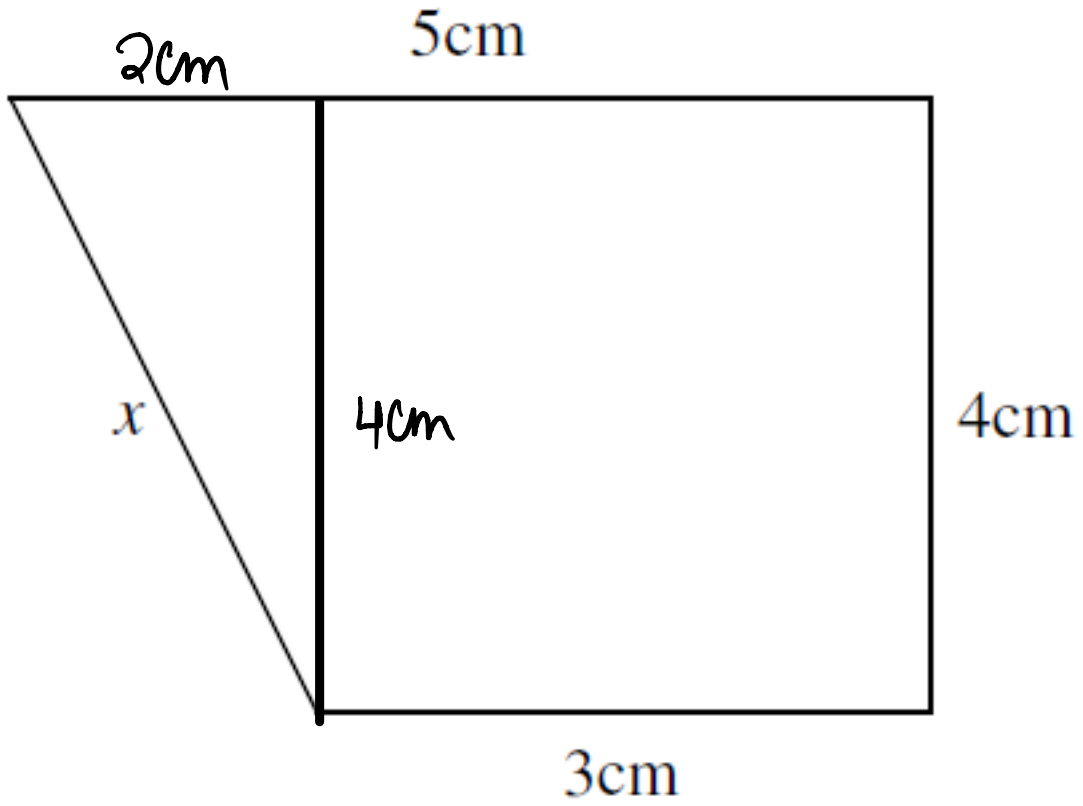

$$\tan\theta = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m = \frac{5 - (-1)}{-1 - (3)}$$

$$m = \frac{6}{-4} = -\frac{3}{2}$$

# PROBLEM#24

The length of $x$ equals



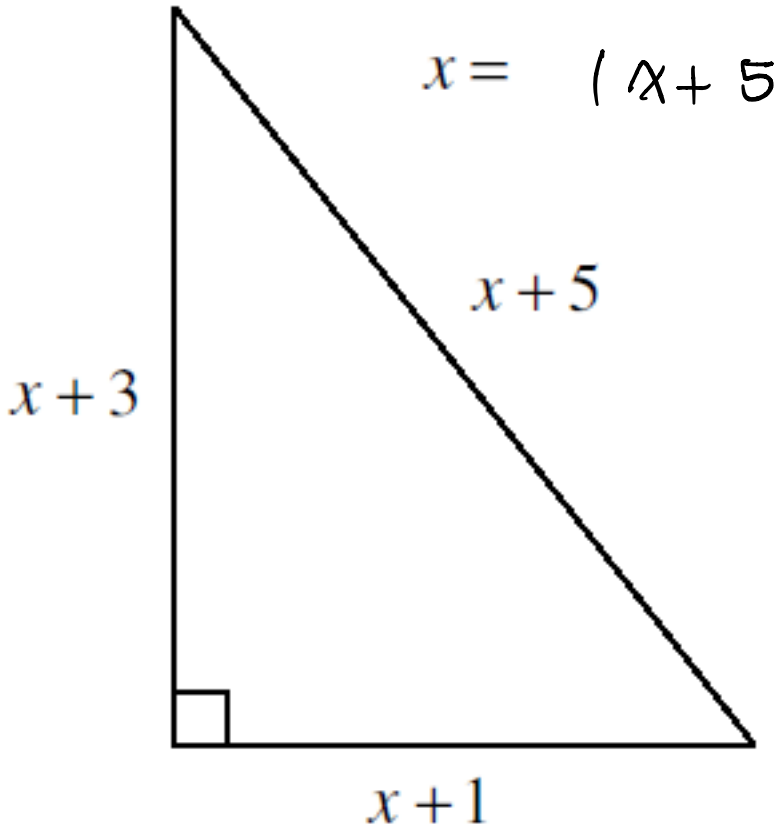

$x^2 = 2^2 + 4^2$

$x^2 = 4 + 16$

$x^2 = 20$

$x = \sqrt{20}$

# PROBLEM#25




$x = (x+5)^2 = (x+3)^2 + (x+1)^2$

$x^2 + 10x + 25 = x^2 + 6x + 9 + x^2 + 2x + 1$

$10x + 25 = 8x + 10 + x^2$

$x^2 - 2x - 15 = 0$

$x^2 + 3x - 5x - 15 = 0$

$(x+3)(x-5) = 0$

$x_1 = -3 \qquad x_2 = 5$

# PROBLEM#26

*Solve for x*: $\frac{4x-3}{5} - \frac{2x-3}{2} = -2$ $\times 10$

$2(4x-3) - 5(2x-3) = -20$

$-2x + 9 = -20$

$-2x = -20-9$

$-2x = -29$

$x = \frac{29}{2}$

# PROBLEM#27

*Determine the average rate of change on the interval* $[1,5]$ *for* $f(x) = \dfrac{5x-3}{2}$

$$\frac{f(b)-f(a)}{b-a} = \frac{f(5)-f(1)}{5-1}$$

$$= \frac{\frac{5(5)-3}{2} - \frac{5(1)-3}{2}}{4} = \frac{\frac{22}{2} - \frac{2}{2}}{4}$$

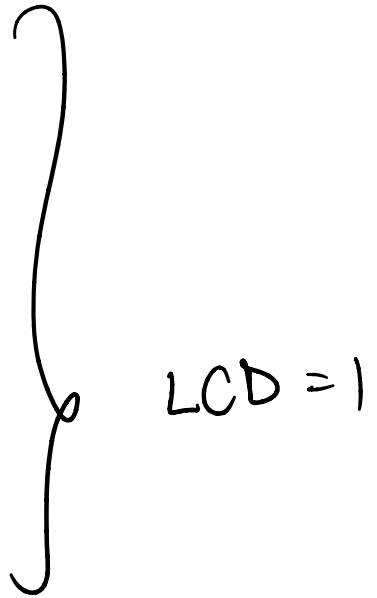
$$\frac{11-1}{4} = \frac{10}{4}$$

# PROBLEM#28

Find the prime factorization of 240 and 144; then determine the least common multiple (LCD) for the pair of numbers.

| | |
|---|---|
| 240 | 2 |
| 120 | 2 |
| 60 | 2 |
| 30 | 2 |
| 15 | 3 |
| 5 | 5 |
| 1 | |

| | |
|---|---|
| 144 | 2 |
| 72 | 2 |
| 36 | 2 |
| 18 | 2 |
| 9 | 3 |
| 3 | 3 |
| 1 | |



# PROBLEM#29

$(h, k)$ $r$

Find an equation of the circle with center at (2, –5) and radius 4.

$$(x-h)^2+(y-k)^2=r^2$$

$$(x-2)^2+(y-k)^2=4$$

# PROBLEM#30

*Solve the system using any method*:

$$\begin{cases} 3x-2y=4 \\ 2x+y=5 \quad \times 2 \end{cases}$$

$$\begin{array}{r} 3x-2y=4 \\ 4x+2y=10 \\ \hline 7x = 14 \end{array} \quad +$$

$$x = \frac{14}{7} = 2$$

plug in

$$2(2)+y=5$$

$$y=5-4$$

$$y=1$$

$$\therefore \quad y=1$$

$$x=2$$