AIM

SWBAT define a permutation & use permutations to solve problems.

Do Now

(1) What does 6! equal ?

(2) There are four people in front of the class. If they all shake hands with each other once, how many handshakes will there be ?

Lesson-Permutations

A permutation is an arrangement of objects in specific order.

Note: The order of the arrangement is important. If the order does not matter, it is a Combination.

Example 1

You want to visit the homes of three friends Alex ("a"), Betty ("b") and Chandra ("c"), but haven't decided in what order. What choices do you have?

Answer: {a,b,c} {a,c,b} {b,a,c} {b,c,a} {c,a,b} {c,b,a}

Lesson-Permutations

<u>**Example 2**</u>

Consider, four students walking toward their school entrance. How many different ways could they arrange themselves in this side-by-side pattern?

1,2,3,4	2,1,3,4	3,2,1,4	4,2,3,1
1,2,4,3	2,1,4,3	3,2,4,1	4,2,1,3
1,3,2,4	2,3,1,4	3,1,2,4	4,3,2,1
1,3,4,2	2,3,4,1	3,1,4,2	4,3,1,2
1,4,2,3	2,4,1,3	3,4,2,1	4,1,2,3
1,4,3,2	2,4,3,1	3,4,1,2	4,1,3,2

The number of different arrangements is 24 or 4! = 4 • 3 • 2 • 1.   There are 24 different arrangements, or permutations, of the four students walking side-by-side.

Lesson-Permutations

The notation for a permutation:

$$_n\mathbf{P}_r$$

n is the **total** number of objects

r is the number of objects chosen

Lesson-Permutations

Formula for Permutation

$$_nP_r = \frac{n!}{(n-r)!}$$

Note: If n = r, as it did with the students walking side-by-side, then $_nP_r$ =n! for either formula. Also remember that 0! is equal to 1.

Lesson-Permutations

Example 3

Compute: $_5P_5$ $\color{red}{5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 120}$

$$\frac{5!}{(5-5)!} = \frac{5 \times 4 \times 3 \times 2 \times 1}{1}$$ **Note: 0!=1**

Lesson-Permutations

Example 4

Compute: $_6P_2$ $6 \cdot 5 = 30$

$$\frac{6!}{(6-2)!} = \frac{6 \times 5 \times 4 \times 3 \times 2 \times 1}{4 \times 3 \times 2 \times 1}$$

Lesson-Permutations

Example 5

Find the number of ways to arrange 5 objects that are chosen from a set of 7 different objects.

Compute: $_7P_5$ $7\cdot6\cdot5\cdot4\cdot3 = 2520$

$$\frac{7!}{(7-5)!} = \frac{7\times6\times5\times4\times3\times2\times1}{2\times1}$$

Lesson-Permutations

<u>Permutations with Repetition</u>

Example 6

How many 3 letter words can be formed using the letters **c,a,t** allowing for repetition of the letters?

<u>27 arrangements with Repetition</u>

ccc	aaa	ttt
caa	acc	taa
ctt	att	tcc
cat	act	tac
cta	atc	tca
cac	aca	tct
ctc	ata	tat
cca	aac	ttc
cct	aat	tta

$$3 \cdot 3 \cdot 3 \ = \ 27 \text{ arrangements}$$

Lesson-Permutations

**Permutations with Repetition of Indistinguishable Objects**

The number of different permutations of n objects, where there are n_1 indistinguishable objects of style 1, n_2 indistinguishable objects of style 2, ..., and n_k indistinguishable objects of style k, is:

$$\frac{n!}{n_1! \cdot n_2! \cdot \ldots \cdot n_k!}$$

**In general, repetitions are taken care of by dividing the permutation by the factorial of the number of objects that are identical.**

Lesson-Permutations

Permutations with Repetition of Indistinguishable Objects

Remember that when n = r, $_n P_r$ = n!

So the formula previous can also be seen as:

$$\frac{_n P_r}{n_1 ! \bullet n_2 ! \bullet \ldots \bullet n_k !}$$

Lesson-Permutations

Permutations with Repetition of Indistinguishable Objects

Example 7

How many different 5-letter words can be formed from the word   *APPLE* ?

$$\frac{_5P_5}{2!} \qquad \frac{5!}{2!(5-5)!} = \frac{5\times4\times3\times2\times1}{2\times1\times1}$$

Lesson-Permutations

Permutations with Repetition of Indistinguishable Objects

Example 8

How many different six-digit numerals can be written using all of the following six digits:

4,4,5,5,5,7

$$\frac{_6P_6}{2!\,3!} \qquad \frac{6!}{2!\,3!(6-6)!} = \frac{6 \times 5 \times 4 \times 3 \times 2 \times 1}{(2 \times 1) \times (3 \times 2 \times 1) \times 1} = \frac{720}{12} = 60$$

Guided Practice

(1) How many ways can we arrange the letters in the word *math* ?

Guided Practice

(2) How many ways can we arrange the letters in the *boot* ?

Guided Practice

(3) How many ways can we arrange the letters in the word *BANANA*?

Guided Practice

(4) How many ways can we arrange the letters in the word *MISSISSIPPI* ?