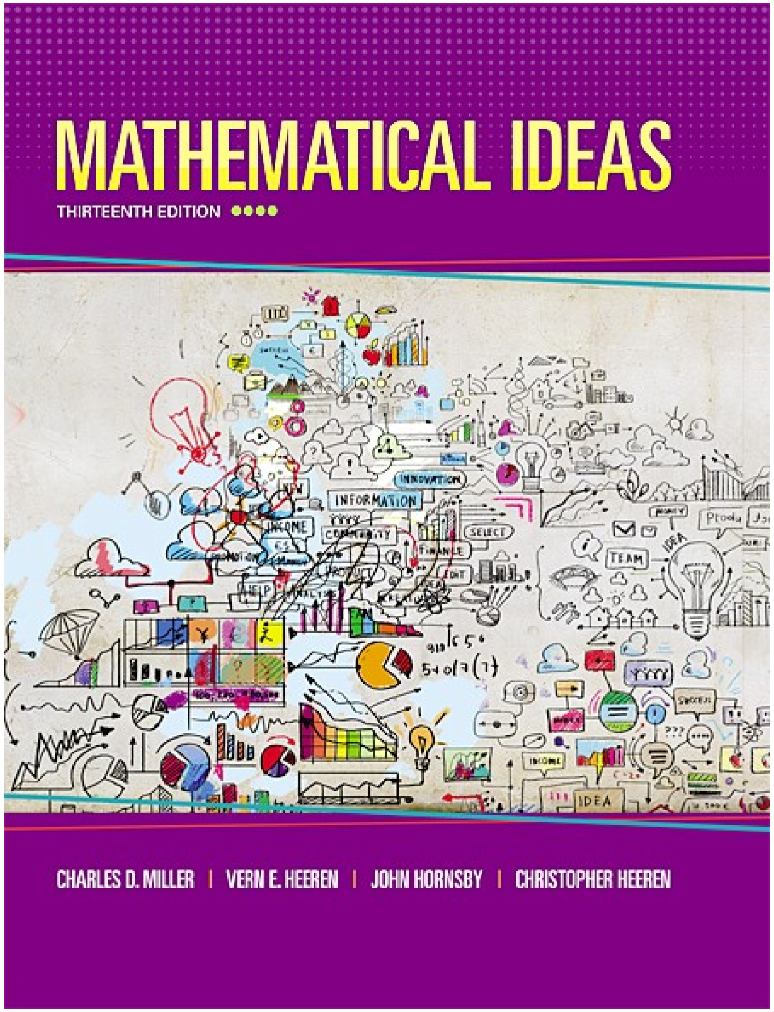
# Chapter 6

## The Real Numbers and Their Representation



# Chapter 6: The Real Numbers and Their Representation

# Section 6-3

Rational Numbers and Decimal Representation

# Rational Numbers and Decimal Representation

- Define and identify rational numbers.
- Write a rational number in lowest terms.
- Add, subtract, multiply, and divide rational numbers in fraction form.
- Solve carpentry problem using operations with fractions.
- Apply the density property and find arithmetic mean.

# Rational Numbers and Decimal Representation

- Convert a rational number in fraction form to a decimal number.
- Convert a terminating or repeating decimal to a rational number in fraction form.

# Definition:  Rational Numbers

**Rational Numbers =**

$\{x \mid x$ is a quotient of two integers, with denominator not equal to $0\}$

# Lowest Terms

A rational number is said to be in **lowest terms** if the greatest common factor of the numerator (top number) and the denominator (bottom number) is 1. Rational numbers are written in lowest terms by using the *fundamental property of rational numbers* (see next slide).

# Fundamental Property of Rational Numbers

If $a$, $b$, and $k$ are integers with $b \neq 0$ and $k \neq 0$, then

$$\frac{a \cdot k}{b \cdot k} = \frac{a}{b}.$$

# Example: Writing a Fraction in Lowest Terms

Write $\dfrac{24}{27}$ in lowest terms.

Solution

$$\frac{24}{27} = \frac{8 \cdot 3}{9 \cdot 3} = \frac{8}{9}.$$

# Cross-Product Test for Equality of Rational Numbers

For rational numbers $\frac{a}{b}$ and $\frac{c}{d}$, $b \neq 0$, $d \neq 0$,

$$\frac{a}{b} = \frac{c}{d} \text{ if and only if } a \cdot d = b \cdot c.$$

*a* and *d* are called the "**extremes**."

*b* and *c* are called the "**means**."

# Adding and Subtracting Rational Numbers

If $\dfrac{a}{b}$ and $\dfrac{c}{d}$ are rational numbers, then

$$\frac{a}{b} + \frac{c}{d} = \frac{ad + bc}{bd}$$

and

$$\frac{a}{b} - \frac{c}{d} = \frac{ad - bc}{bd}.$$

# Adding and Subtracting Rational Numbers

In practical problems involving addition and subtraction of rational numbers, fractions are usually rewritten with the least common multiple of their denominators, called the **least common denominator**.

# Example: Adding and Subtracting Rational Numbers

Perform each operation.

a) $\frac{4}{9}+\frac{2}{15}$  b) $\frac{4}{9}-\frac{2}{15}$

## Solution

a) $\frac{4}{9}+\frac{2}{15} = \frac{20}{45}+\frac{6}{45} = \frac{26}{45}$

b) $\frac{4}{9}-\frac{2}{15} = \frac{20}{45}-\frac{6}{45} = \frac{14}{45}$

# Multiplying Rational Numbers

If $\dfrac{a}{b}$ and $\dfrac{c}{d}$ are rational numbers, then

$$\frac{a}{b}\cdot\frac{c}{d}=\frac{a\cdot c}{b\cdot d}.$$

# Example: Multiplying Rational Numbers

Find the product $\frac{5}{9} \cdot \frac{3}{10}$.

Solution

$$\frac{5}{9} \cdot \frac{3}{10} = \frac{15}{90} = \frac{1 \cdot 15}{6 \cdot 15} = \frac{1}{6}$$

# Definition of Division

If $a$ and $b$ are real numbers with $b \neq 0,$ then

$$\frac{a}{b} = a \cdot \frac{1}{b}.$$

# Dividing Rational Numbers

If $\dfrac{a}{b}$ and $\dfrac{c}{d}$ are rational numbers $\left(\dfrac{c}{d} \neq 0\right)$, then

$$\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \cdot \frac{d}{c} = \frac{ad}{bc}.$$

# Example: Dividing Rational Numbers

Find the quotient $\frac{2}{9} \div \frac{5}{6}$.

Solution

$$\frac{2}{9} \div \frac{5}{6} = \frac{2}{9} \cdot \frac{6}{5} = \frac{12}{45} = \frac{4 \cdot 3}{15 \cdot 3} = \frac{4}{15}$$

# Example: Applying Operations with Fractions to Carpentry

A carpenter has a board 72 inches long that he must cut unto 10 pieces of equal length. This will require 9 cuts.

a) If each cut causes a waste of $\frac{3}{16}$ inch, how many inches of actual board will remain after the cuts?

b) What will the length of each of the resulting pieces be?

Continued

# Example: Applying Operations with Fractions to Carpentry

## Solution

a) We start with 72 inches and must subtract $\frac{3}{16}$ nine times. We represent this by $9\left(\frac{3}{16}\right)$.

$$72 - 9\left(\frac{3}{16}\right) = 72 - \left(\frac{9}{1} \cdot \frac{3}{16}\right)$$

$$= 72 - \frac{27}{16} = \frac{1125}{16} = 70\frac{5}{16}$$

Thus, $70\frac{5}{16}$ inches of actual board will remain.

Continued

# Example: Applying Operations with Fractions to Carpentry

## Solution

b) The $70\frac{5}{16}$ or $\frac{1125}{16}$ inches of board from part (a) will be divided into 10 pieces of equal length.

$$\frac{1125}{16} \div 10 = \frac{1125}{16} \cdot \frac{1}{10}$$

$$= \frac{225}{32} = 7\frac{1}{32}$$

Each piece will measure $7\frac{1}{32}$ inches.

# Density Property of Rational Numbers

If $r$ and $t$ are distinct rational numbers, with $r < t$, then there exists a rational number $s$ such that

$$\boldsymbol{r < s < t}.$$

This leads to the conclusion that there are *infinitely many* rational numbers between two distinct rational numbers.

# Arithmetic Mean

To find the **arithmetic mean**, or **average**, of $n$ numbers, we add the numbers and then divide the sum by $n$. For two numbers, the number that lies halfway between them is their average.

# Example: Finding an Arithmetic Mean (Average)

Find the average of $\frac{3}{5}$ and $\frac{2}{3}$.

## Solution

$$\frac{3}{5}+\frac{2}{3}=\frac{9}{15}+\frac{10}{15}=\frac{19}{15}$$

Add the fractions.

$$\frac{19}{15}\div 2=\frac{19}{15}\cdot\frac{1}{2}=\frac{19}{30}$$

Divide the sum by 2 to get the answer.

# Converting a Rational Number in Fraction Form to a Decimal Number

A rational number in the form a/b can be expressed as a decimal most easily by entering it into a calculator.

For example, to write $\frac{7}{8}$ as a decimal, enter 7, enter the operation of division, enter 8, and then press the equals key to find the following equivalence:

$$\frac{7}{8} = 0.875$$

# Decimal Form of Rational Numbers

Any rational number can be expressed as either a *terminating* (stops) decimal or a *repeating* (reoccurring block of numbers) decimal.

# Criteria for Terminating and Repeating Decimals

A rational number, in lowest terms, results in a **terminating decimal** if the only prime factor of the denominator is 2 or 5 (or both).

A rational number, in lowest terms, results in a **repeating decimal** if a prime other than 2 or 5 appears in the prime factorization of the denominator.

# Example: Determining Whether a Decimal Terminates or Repeats

Without dividing, determine whether the decimal form of the given rational number terminates or repeats.

a) $\dfrac{7}{15}$ b) $\dfrac{15}{16}$

## Solution

a) Repeats; there is a factor of 3 in the denominator.

b) Terminates; the denominator is $2^4$.

# Example: Writing Decimals as Quotients of Integers

Write each decimal as a quotient of integers.

a) 0.759 b) 7.8

## Solution

a) $0.759 = \dfrac{759}{1000}$

b) $7.8 = 7 + \dfrac{8}{10} = \dfrac{78}{10} = \dfrac{39}{5}$