

# The Real Numbers and Their Representation



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# Chapter 6: The Real Numbers and Their Representation

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- 6.1 Real Numbers, Order, and Absolute Value
- 6.2 Operations, Properties, and Applications of Real Numbers
- 6.3 Rational Numbers and Decimal Representation
- 6.4 Irrational Numbers and Decimal Representation
- 6.5 Applications of Decimals and Percents

# Section 6-1

## Real Numbers, Order, and Absolute Value

# Real Numbers, Order, and Absolute Value

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- Represent a number on a number line.
- Identify a number as positive, negative, or zero.
- Identify a number as belonging to one or more sets of numbers.
- Given two numbers,  $a$  and  $b$ , determine whether  $a = b$ ,  $a < b$ , or  $a > b$ .
- Given a number  $a$ , determine its additive inverse, and absolute value.
- Interpret signed numbers in tables of economic and occupations data.

# Sets of Real Numbers

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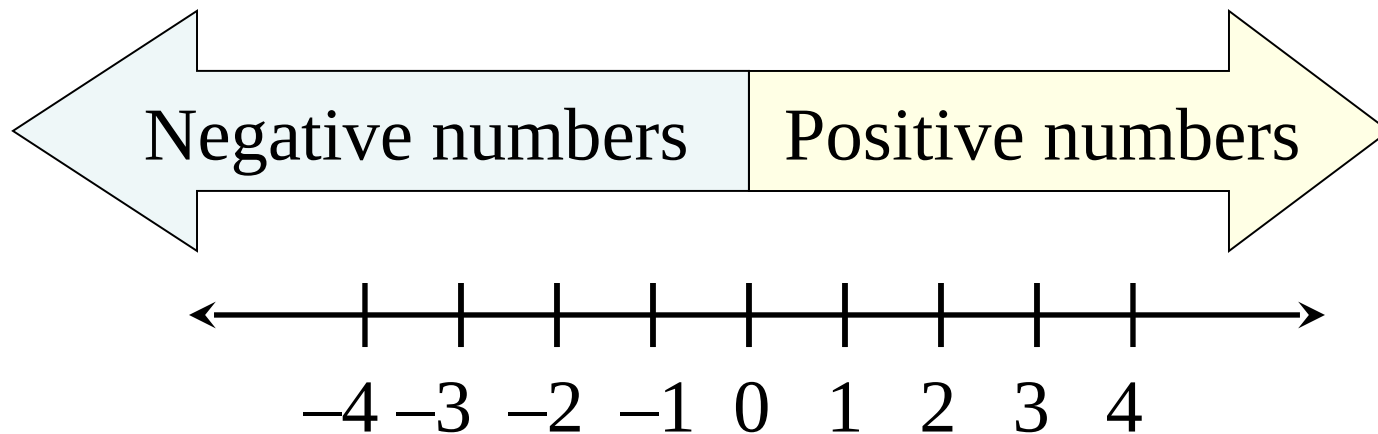
## Natural Numbers

$\{1, 2, 3, 4, \dots\}$  is the set of **natural numbers**.

## Whole Numbers

$\{0, 1, 2, 3, 4, \dots\}$  is the set of **whole numbers**.

# Number Line



Positive and negative numbers are called **signed** numbers.

# Sets of Real Numbers

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The set of numbers marked on the number line on the previous slide, including positive and negative numbers and zero, is part of the set of *integers*.

## Integers

$\{\dots, -3, -2, -1, 0, 1, 2, 3, \dots\}$  is the set of **integers**.

# Sets of Real Numbers

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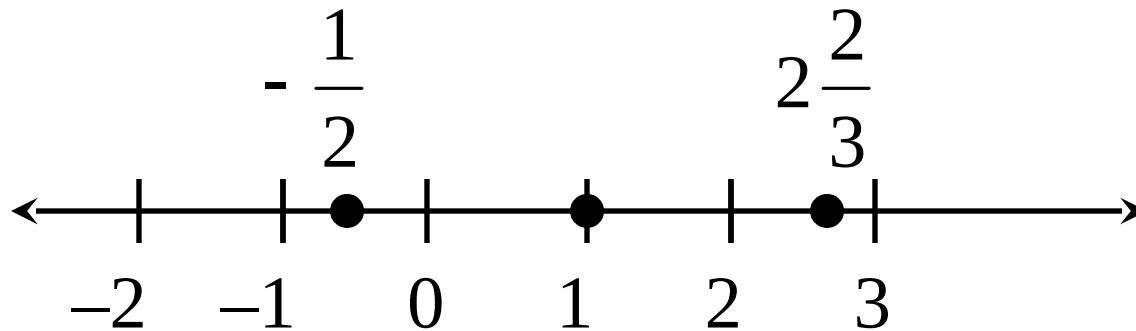
Numbers such as  $\frac{1}{2}$  and  $-1\frac{2}{3}$  are not integers, they are rational numbers.

## Rational Numbers

$\{x \mid x \text{ is a quotient of two integers, with denominator not equal to } 0\}$  is the set of **rational numbers**.

# Sets of Real Numbers

The **graph** of a number is a point on the number line. A few numbers are *graphed* below.



# Sets of Real Numbers

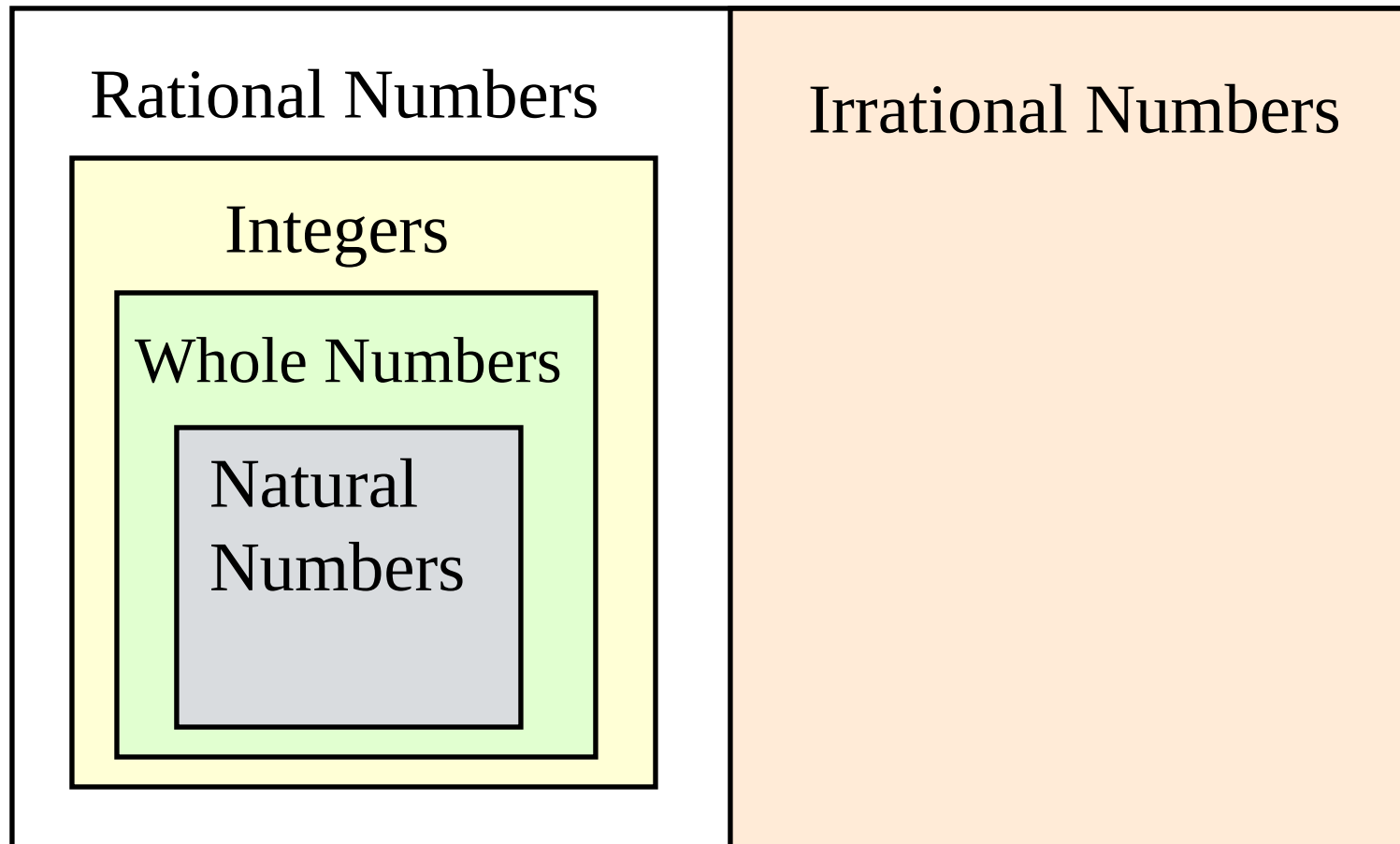
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Not all numbers are rational. For example,  $\sqrt{2}$  can not be written as the quotient of two integers. It is called *irrational*.

## Irrational Numbers

$\{x \mid x \text{ is a number on the number line that is not rational}\}$  is the set of **irrational numbers**.

# The Real Numbers



# Example: Identifying Elements of a Set of Numbers

List the numbers from the set  $\left\{ -3, -\frac{1}{2}, 0, 1, 1.8, \sqrt{7} \right\}$  that are:

- a) natural numbers
- b) rational numbers
- c) real numbers

## Solution

- a) natural numbers: the only natural number is 1.
- b) rational numbers:  $\{-3, -1/2, 0, 1, 1.8\}$
- c) real numbers: all the numbers are real numbers

# Order in the Real Numbers

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Two real numbers may be compared, or ordered, using ideas of equality and inequality.

# Order in the Real Numbers

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The **law of trichotomy** states that, for two numbers  $a$  and  $b$ , one and only one of the following is true.

$a = b$                    $a$  equals  $b$

$a < b$                    $a$  is less than  $b$

$a > b$                    $a$  is greater than  $b$

# Order in the Real Numbers

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The symbol  $\leq$  means “is less than or equal to.”

The statement is true if the  $=$  part or  $<$  part is true.

$5 \leq 7$  is true, as is  $5 \leq 5$ .

The symbol  $\geq$  means “is greater than or equal to.”

The statement is true if the  $=$  part or  $>$  part is true.

# Additive Inverses

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For any real number  $x$  (except 0), there is exactly one number on the number line the same distance from 0 as  $x$  but on the opposite side of 0. For example, 3 and  $-3$  are the same distance from 0 but on opposite sides. These numbers are **additive inverses**, **negatives**, or **opposites** of each other.

# Double Negative Rule

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For any real number  $x$ ,

$$-(-x) = x.$$

# Absolute Value

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The **absolute value** of a real number can be defined as the distance between 0 and the number on the number line. The symbol for the absolute value of  $x$  is  $|x|$ , read “**the absolute value of  $x$ .**”

The absolute value of a number is *never negative*.

# Absolute Value

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For any real number  $x$ ,

$$|x| = \begin{cases} x & \text{if } x \geq 0 \\ -x & \text{if } x < 0. \end{cases}$$

# Example: Using Absolute Value

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Simplify by finding the absolute value.

a)  $|4|$       b)  $|-2|$       c)  $-|-3|$       d)  $|1 - 8|$

## Solution

a) 4

b) 2

c) -3

d) 7

# Applications

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When looking at the amount of *change*, without regard to whether the change is an increase (positive) or decrease (negative), use the absolute value of the number.

# Example: Interpreting Signed Numbers in a Table

Given the temperature readings of the days below, which day had the greatest amount of change?

|       | A.M.<br>Temp. | P.M.<br>Temp. | P.M. –<br>A.M. |
|-------|---------------|---------------|----------------|
| Day 1 | 23° F         | 42° F         | 19° F          |
| Day 2 | 32° F         | 55° F         | 23° F          |
| Day 3 | 40° F         | 13° F         | –27° F         |

# Example: Interpreting Signed Numbers in a Table

|       |       |       |        |
|-------|-------|-------|--------|
| Day 1 | 23° F | 42° F | 19° F  |
| Day 2 | 32° F | 55° F | 23° F  |
| Day 3 | 40° F | 13° F | −27° F |

## Solution

Day 3 had the greatest change at  $|-27| = 27$  degrees.