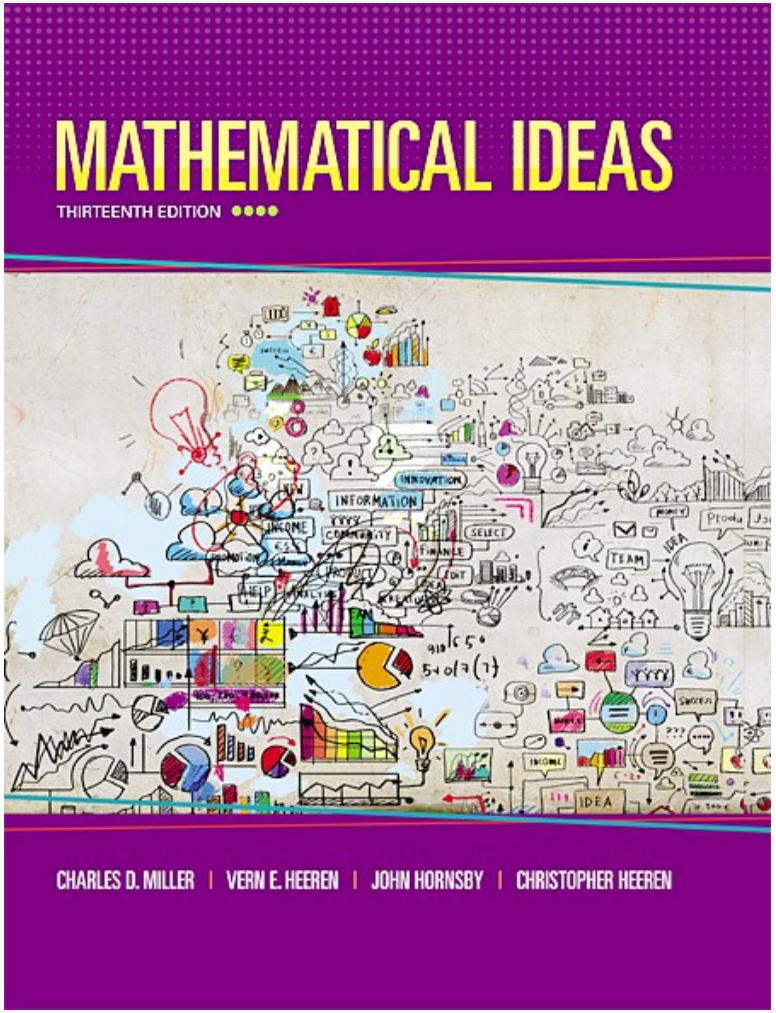
Chapter 6

The Real Numbers and Their Representation



Chapter 6: The Real Numbers and Their Representation

6.1 Real Numbers, Order, and Absolute Value

6.2 Operations, Properties, and Applications of Real Numbers

6.3 Rational Numbers and Decimal Representation

6.4 Irrational Numbers and Decimal Representation

6.5 Applications of Decimals and Percents

Section 6-2

Operations, Properties, and Applications of Real Numbers

Operations, Properties, and Applications of Real Numbers

- Perform the operations of addition, subtraction, multiplication, and division of signed numbers.
- Apply the rules for order of operations.
- Identify and apply properties of addition and multiplication of real numbers.
- Determine change in investment and meteorological data using subtraction and absolute value.

Operations

The addition, subtraction, multiplication and division of signed numbers will be presented.

Adding Real Numbers

The result of adding two numbers is called their **sum**. The numbers being added are called **addends** (or **terms**).

Adding Real Numbers

Like Signs Add two numbers with the *same* sign by adding their absolute values. The sign of the sum is the same as the sign of the two numbers.

Unlike Signs Add two numbers with *different* signs by subtracting the smaller absolute value from the larger to find the absolute value of the sum. The sum is the same sign as the number with the larger absolute value.

Subtracting Real Numbers

The result of subtracting two numbers is called their **difference**. In $a - b$, a is called the **minuend**, and b is called the **subtrahend**.

Definition of Subtraction

For all real numbers a and b,

$$\boldsymbol{a - b = a + (-b).}$$

(Change the sign of the subtrahend and add.)

Multiplying Real Numbers

The result of multiplying two numbers is called their **product**. The two numbers being multiplied are called **factors**.

Multiplying Real Numbers

Like Signs Multiply two numbers with the *same* sign by multiplying their absolute values to find the absolute value of the product. The sign of the product is positive.

Unlike Signs Multiply two numbers with *different* signs by multiplying their absolute values to find the absolute value of the product. The sign of the product is negative.

Dividing Real Numbers

The result of dividing two numbers is called their **quotient**. In the quotient $a \div b$ (or $\frac{a}{b}$), where $b \neq 0$, a is called the **dividend** (or numerator), and b is called the **divisor** (or denominator). For real numbers a, b, and c if

$$\frac{a}{b} = c, \text{ then } a = b \cdot c.$$

Dividing Real Numbers

Like Signs Divide two numbers with the *same* sign by dividing their absolute values to find the absolute value of the quotient. The sign of the quotient is positive.

Unlike Signs Divide two numbers with *different* signs by dividing their absolute values to find the absolute value of the quotient. The sign of the quotient is negative.

Example: Operations With Signed Numbers

Simplify each expression.

a) $11 + (-8)$ b) $3 - (-4)$ c) $\frac{-16}{-4}$

d) $7 \cdot (-4)$

Solution

a) 3

b) 7

c) 4

d) –28

Division with Zero

If 0 is divided by a nonzero number, the quotient is 0. That is,

$$\frac{0}{b} = 0, \text{ for } b \neq 0.$$

Division by 0 is undefined.

Order of Operations

When a problem involves more than one operation, we follow the order of operations.

If parentheses or square brackets are present:

Step 1 Work separately above and below any **fraction bar**.

Step 2 Use the rules on the next slide within each set of **parentheses or square brackets.** Start with the innermost set and work outward.

Order of Operations

If no parentheses or brackets are present:

Step 1 Apply any **exponents.**

Step 2 Do any **multiplications** or **divisions** in the order in which they occur, working left to right.

Step 3 Do any **additions** or **subtractions** in the order in which they occur, working left to right.

Example: Using the Order of Operations

Use the order of operations to simplify the expression below.

$$6 \cdot 2^2 + 5 - 4 \cdot (1 + 2)$$

Solution

$$= 6 \cdot 2^2 + 5 - 4 \cdot (3)$$

$$= 6 \cdot 4 + 5 - 4 \cdot (3)$$

$$= 24 + 5 - 12$$

$$= 17$$

Properties of Addition and Multiplication

For real numbers a, b, and c, the following properties hold.

Closure	$a + b$ and ab are real numbers.
Commutative	$a + b = b + a$ and $ab = ba$.
Associative	$(a + b) + c = a + (b + c)$ $(ab)c = a(bc)$

Properties of Addition and Multiplication

Identity There is a real number 0 such that
$a + 0 = a$ and $0 + a = a$.
There is a real number 1 such that
$a \cdot 1 = a$ and $1 \cdot a = a$.

0 is the **identity element for addition**.
1 is the **identity element for multiplication**.

Properties of Addition and Multiplication

Inverse For each real number a, there is a single real number $-a$ such that $(-a) + a = a + (-a) = 0$.

For each nonzero real number a, there is a single real number $\frac{1}{a}$ such that $a \cdot \frac{1}{a} = \frac{1}{a} \cdot a = 1$.

$-a$ is the **additive inverse** of a.

$\frac{1}{a}$ is the **multiplicative inverse** (*reciprocal*) of a.

Properties of Addition and Multiplication

Distributive Property of Multiplication with Respect to Addition	$a(b + c) = ab + ac$
	$(b + c)a = ba + ca$

Example: Applications of Real Numbers

The record high temperature in a small town was 108°F and its record low was –43°F. How much greater was the highest temperature than the lowest temperature?

Solution

Subtract the lower temperature from the higher temperature.

$$108 - (-43) = 108 + 43 = 151.$$

The difference is 151°F.