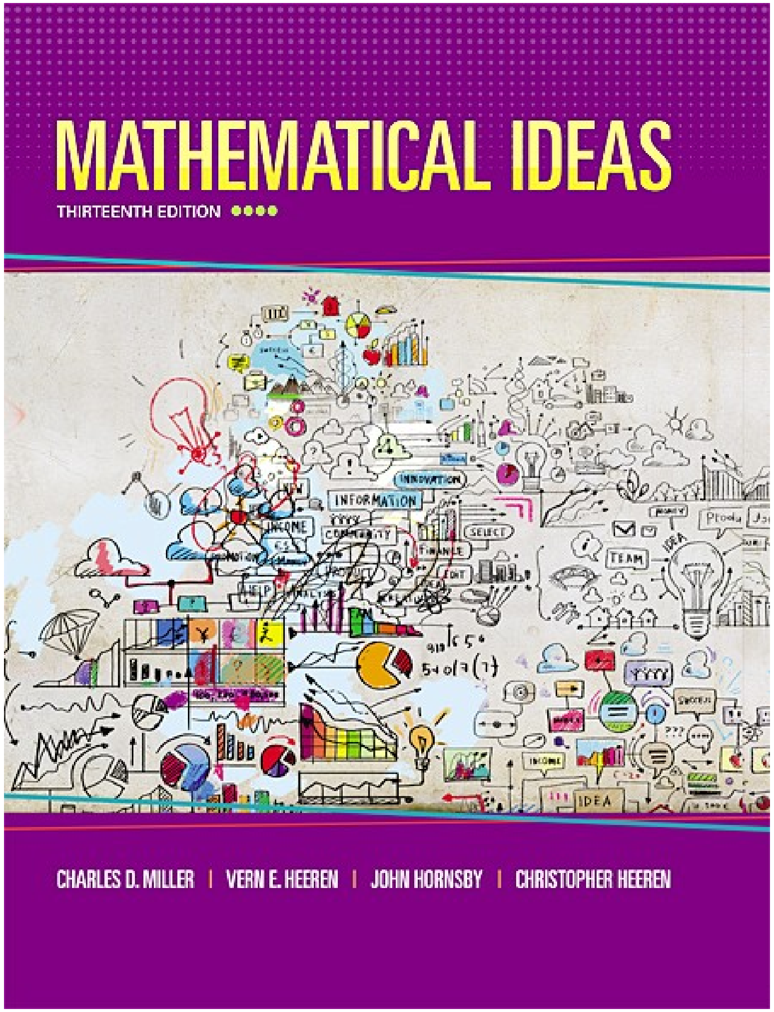
Chapter 5

Number Theory



Chapter 5: Number Theory

Section 5-4

Greatest Common Factor and Least Common Multiple

Greatest Common Factor and Least Common Multiple

- Find the greatest common factor by several methods.
- Find the least common multiple by several methods.

Greatest Common Factor

The **greatest common factor (GCF)** of a group of natural numbers is the largest number that is a factor of all of the numbers in the group.

Finding the Greatest Common Factor (Prime Factors Method)

Step 1 Write the prime factorization of each number.

Step 2 Choose all primes common to *all* factorizations, with each prime raised to the *least* exponent that appears.

Step 3 Form the product of *all* the numbers in Step 2; this product is the greatest common factor.

Example: Greatest Common Factor by Prime Factors Method

Find the greatest common factor of 360 and 1350.

Solution

The prime factorizations are below.

$$360 = 2^3 \cdot 3^2 \cdot 5$$

$$1350 = 2 \cdot 3^3 \cdot 5^2$$

The GCF is $2 \cdot 3^2 \cdot 5 = 90$.

Finding the Greatest Common Factor (Dividing by Prime Factors Method)

Step 1 Write the numbers in a row.

Step 2 Divide each of the numbers by a common prime factor. Try 2, then 3, and so on.

Step 3 Divide the quotients by a common prime factor. Continue until no prime will divide into all the quotients.

Step 4 The product of the primes in steps 2 and 3 is the greatest common factor.

Finding the Greatest Common Factor (Dividing by Prime Factors Method)

Find the greatest common factor of 12, 18, and 30.

Solution

2	12	18	30	Divide by 2
3	6	9	15	Divide by 3
	2	3	5	No common factors

Since there are no common factors in the last row, the GCF is $2 \cdot 3 = 6$.

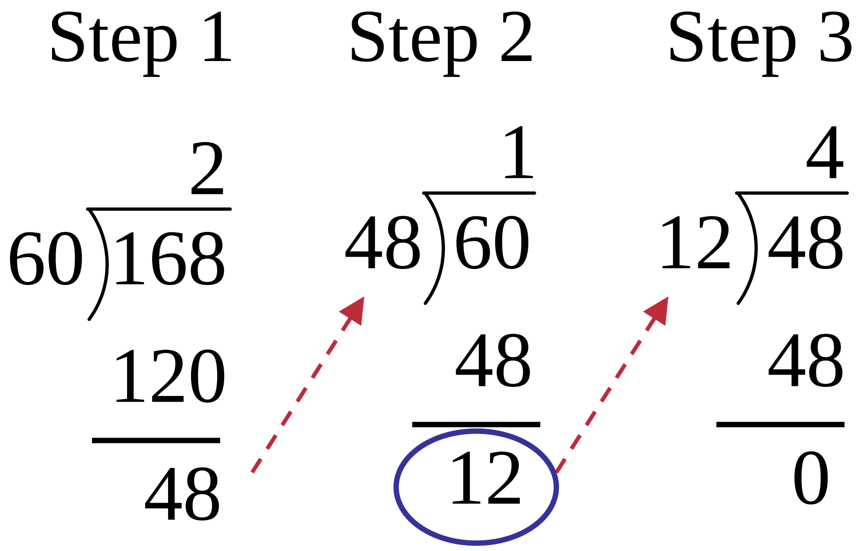
Finding the Greatest Common Factor (Euclidean Algorithm)

To find the greatest common factor of two unequal numbers, divide the larger by the smaller. Note the remainder, and divide the previous divisor by this remainder. Continue the process until a remainder of 0 is obtained. The greatest common factor is the last positive remainder obtained.

Example: Euclidean Algorithm

Use the Euclidean algorithm to find the greatest common factor of 60 and 168.

Solution



Step 1 Step 2 Step 3

$$\begin{array}{r} 2 \\ 60\overline{)168} \\ \underline{120} \\ 48 \end{array} \qquad \begin{array}{r} 1 \\ 48\overline{)60} \\ \underline{48} \\ \boxed{12} \end{array} \qquad \begin{array}{r} 4 \\ 12\overline{)48} \\ \underline{48} \\ 0 \end{array}$$

The GCF is 12.

Least Common Multiple

The **least common multiple (LCM)** of a group of natural numbers is the smallest natural number that is a multiple of all of the numbers in the group.

Finding the Least Common Multiple (Prime Factors Method)

Step 1 Write the prime factorization of each number.

Step 2 Choose all primes belonging to *any* factorization, with each prime raised to the power indicated by the *greatest* exponent that it has in any factorization.

Step 3 Form the product of all the numbers in Step 2; this product is the least common multiple.

Example: Finding the LCM

Find the least common multiple of 360 and 1350.

Solution

The prime factorizations are below.

$$360 = 2^3 \cdot 3^2 \cdot 5$$

$$1350 = 2 \cdot 3^3 \cdot 5^2$$

The LCM is $2^3 \cdot 3^3 \cdot 5^2 = 5400$.

Finding the Least Common Multiple (Dividing by Prime Factors Method)

Step 1 Write the numbers in a row.

Step 2 Divide each of the numbers by a common prime factor. Try 2, then 3, and so on.

Step 3 Divide the quotients by a common prime factor. When no prime will divide all quotients, but a prime will divide some of them, divide where possible and bring any nondivisible quotients down.

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Finding the Least Common Multiple (Dividing by Prime Factors Method)

Step 3 (step 3 continued) Continue until no prime will divide any two quotients.

Step 4 The product of the prime divisors in steps 2 and 3 as well as all remaining quotients is the least common multiple.

Finding the Least Common Multiple (Dividing by Prime Factors Method)

Find the least common multiple of 12, 18, and 30.

Solution

$$\begin{array}{r|rrrl} 2 & 12 & 18 & 30 & \quad\text{Divide by 2} \\ \cline{2-4} 3 & 6 & 9 & 15 & \quad\text{Divide by 3} \\ \cline{2-4} & 2 & 3 & 5 & \quad\text{No common factors} \end{array}$$

The LCM is 2 · 3 · 2 · 3 · 5 = 180.

Finding the Least Common Multiple (Formula)

The least common multiple of m and n is given by

$$\text{LCM} = \frac{m \cdot n}{\text{greatest common factor of } m \text{ and } n}.$$

Example: LCM Formula

Find the LCM of 360 and 1350.

Solution

The GCF is 90.

$$\text{LCM} = \frac{360 \cdot 1350}{90} = \frac{486000}{90} = 5400$$

Example: Finding the Greatest Common Size of Stacks of Cards

Joshua has 450 football cards and 840 baseball cards. He wants to place them in stacks on a table so that each stack has the same number of cards, and no stack has different types of cards within it. What is the greatest number of cards that he can have in each stack?

Solution

Looking for the greatest number that will divide evenly into 450 and 840. Using any method, you will find the GCF is 30.