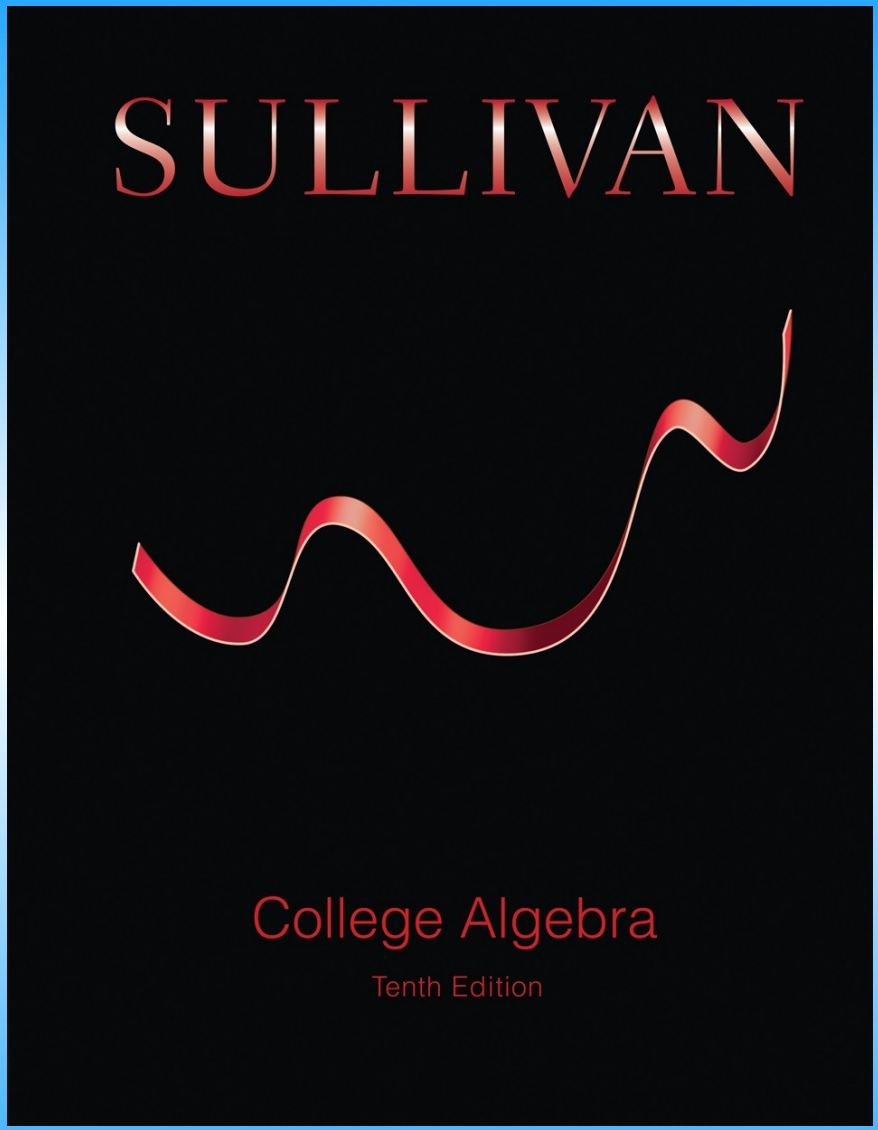
Chapter 2

Section 4



2.4 Circles

PREPARING FOR THIS SECTION *Before getting started, review the following:*

- Completing the Square (Chapter R, Section R.5, p. 56)
- Square Root Method (Section 1.2, pp. 94–95)

Now Work the **'Are You Prepared?'** problems on page 186.

OBJECTIVES

1. Write the Standard Form of the Equation of a Circle (p. 182)
2. Graph a Circle (p. 183)
3. Work with the General Form of the Equation of a Circle (p. 184)

Write the Standard Form of the Equation of a Circle

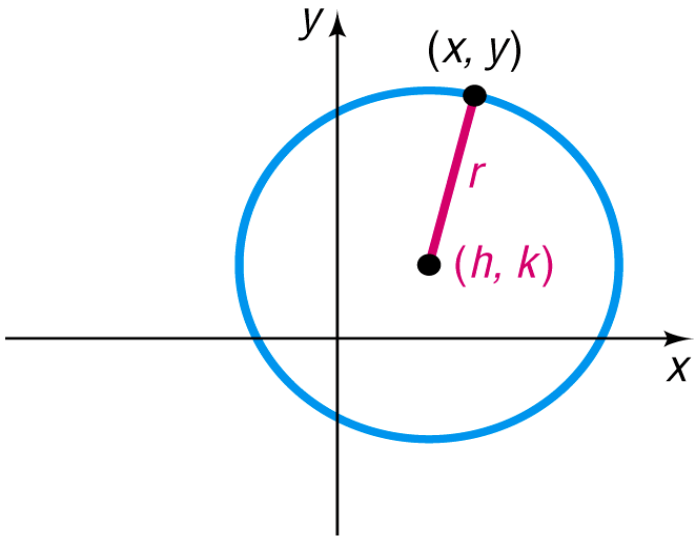
Definition

A **circle** is a set of points in the xy-plane that are a fixed distance r from a fixed point (h, k). The fixed distance r is called the **radius**, and the fixed point (h, k) is called the **center** of the circle.

Definition

The **standard form of an equation of a circle** with radius r and center (h, k) is

$$(x - h)^2 + (y - k)^2 = r^2 \qquad \textbf{(1)}$$



Theorem

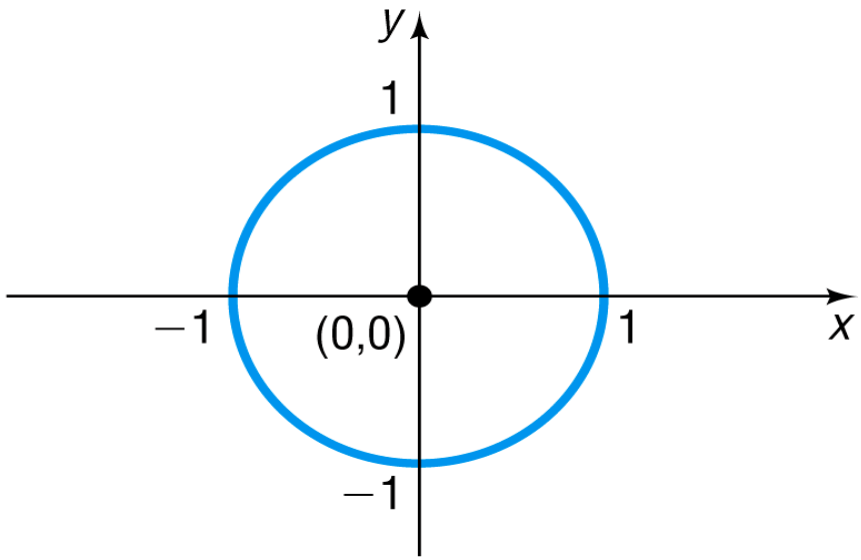
The standard form of an equation of a circle of radius r with center at the origin $(0, 0)$ is

$$x^2 + y^2 = r^2$$

Definition

If the radius $r = 1$, the circle whose center is at the origin is called the **unit circle** and has the equation

$$x^2 + y^2 = 1$$



Example

Writing the Standard Form of the Equation of a Circle

Write the standard form of the equation of the circle with radius 5 and center $(-3, 6)$.

Solution

Substitute the values $r = 5, h = -3$, and $k = 6$ into equation (1).

$$(x - h)^2 + (y - k)^2 = r^2$$

$$(x + 3)^2 + (y - 6)^2 = 25$$

Graph a Circle

Example

Graphing a Circle

Graph the equation: $(x + 3)^2 + (y - 2)^2 = 16$

Solution

Since the equation is in the form of equation (1), its graph is a circle. To graph the equation, compare the given equation to the standard form of the equation of a circle. The comparison yields information about the circle.

$$(x+3)^2+(y-2)^2=16$$

$$(x-(-3))^2+(y-2)^2=4^2$$

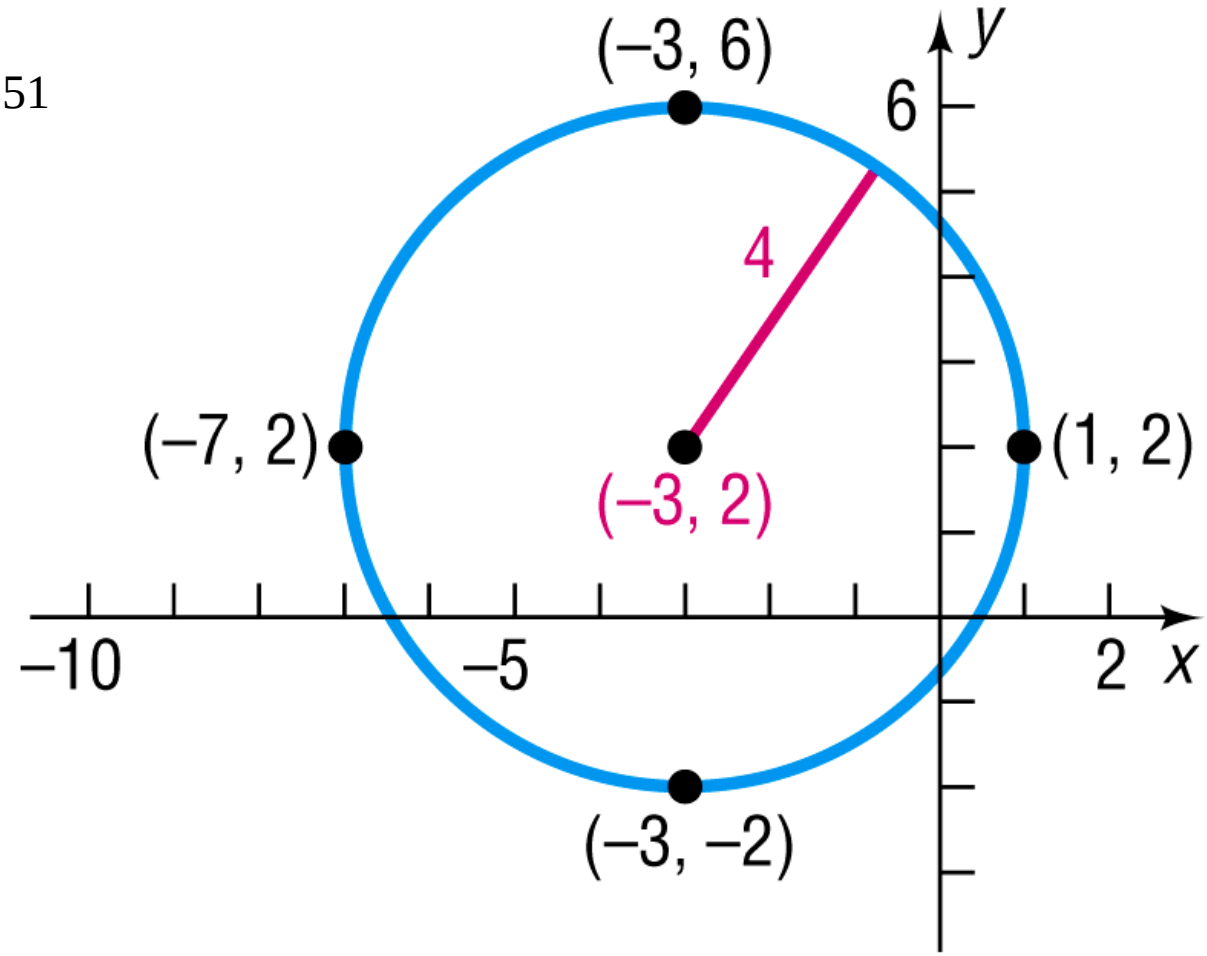
$$\uparrow \qquad \uparrow \qquad \uparrow$$

$$(x-h)^2+(y-k)^2=r^2$$

We see that $h=-3$, $k=2$, and $r=4$. The circle has center $(-3,2)$ and a radius of 4 units. To graph this circle, first plot the center $(-3,2)$. Since the radius is 4, we can locate four points on the circle by plotting points 4 units to the left, to the right, up, and down from the center. These four points can then be used as guides to obtain the graph. See Figure 51.

Solution continued

Figure 51



Example

Finding the Intercepts of a Circle

For the circle $(x + 3)^2 + (y - 2)^2 = 16$, find the intercepts, if any, of its graph.

Solution

This is the equation discussed and graphed in Example 2. To find the x-intercepts, if any, let $y = 0$. Then

$$(x + 3)^2 + (y - 2)^2 = 16$$
$$(x + 3)^2 + (0 - 2)^2 = 16 \qquad y = 0$$
$$(x + 3)^2 + 4 = 16 \qquad \text{Simplify.}$$
$$(x + 3)^2 = 12 \qquad \text{Simplify.}$$
$$x + 3 = \pm\sqrt{12} \qquad \text{Apply the Square Root Method.}$$
$$x = -3 \pm 2\sqrt{3} \qquad \text{Solve for x.}$$

The x-intercepts are $-3 - 2\sqrt{3} \approx -6.46$ and $-3 + 2\sqrt{3} \approx 0.46$.

To find the y-intercepts, if any, let $x = 0$. Then

$$(x + 3)^2 + (y - 2)^2 = 16$$
$$(0 + 3)^2 + (y - 2)^2 = 16$$
$$9 + (y - 2)^2 = 16$$
$$(y - 2)^2 = 7$$
$$y - 2 = \pm\sqrt{7}$$
$$y = 2 \pm \sqrt{7}$$

The y-intercepts are $2 - \sqrt{7} \approx -0.65$ and $2 + \sqrt{7} \approx 4.65$.

Look back at Figure 51 to verify the approximate locations of the intercepts.

Work with the General Form of the Equation of a Circle

Definition

When its graph is a circle, the equation

$$x^2 + y^2 + ax + by + c = 0$$

is the **general form of the equation of a circle.**

Example

Graphing a Circle Whose Equation Is in General Form

Graph the equation: $x^2 + y^2 + 4x - 6y + 12 = 0$

Solution

Group the terms involving x, group the terms involving y, and put the constant on the right side of the equation. The result is

$$(x^2 + 4x) + (y^2 - 6y) = -12$$

Next, complete the square of each expression in parentheses. Remember that any number added on the left side of the equation must also be added on the right.

$$(x^2 + 4x + 4) + (y^2 - 6y + 9) = -12 + 4 + 9$$

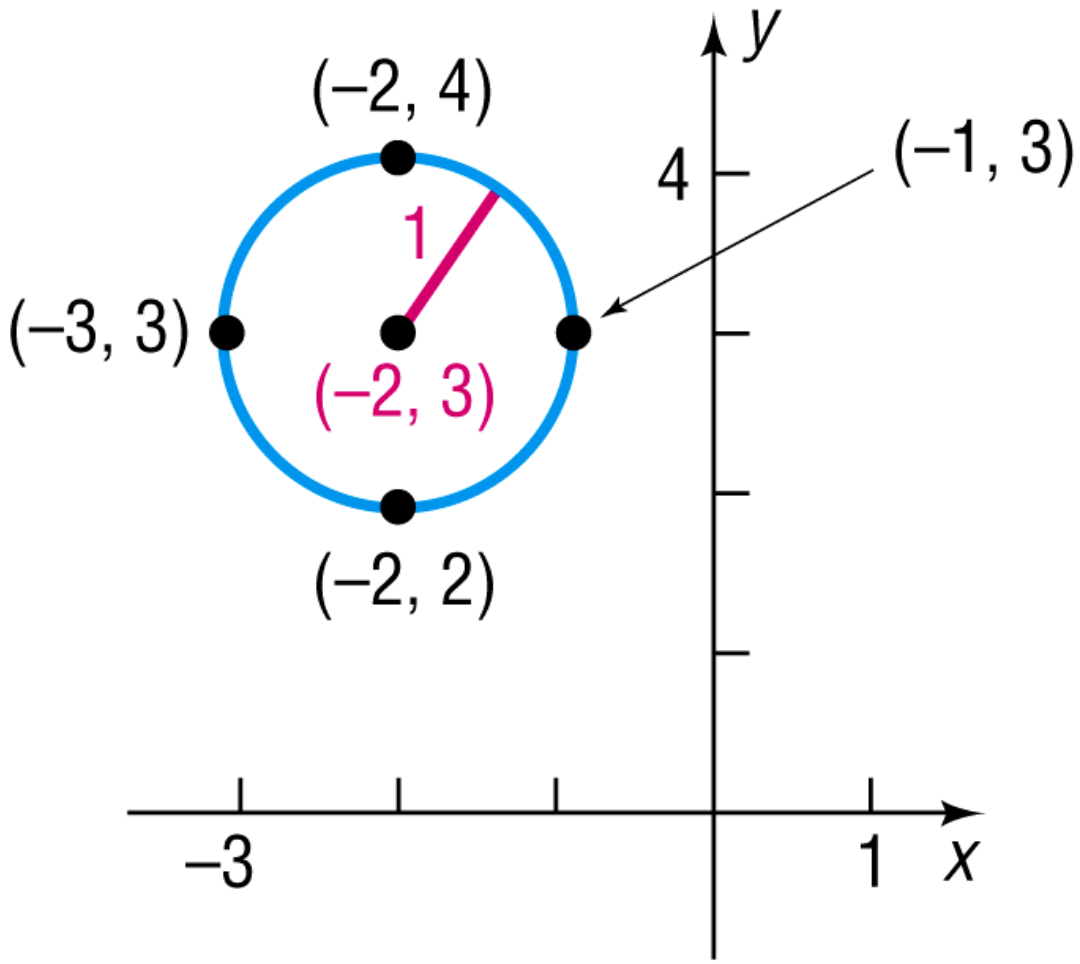
$\left(\frac{4}{2}\right)^2 = 4$ $\qquad$ $\left(\frac{-6}{2}\right)^2 = 9$

$$(x + 2)^2 + (y - 3)^2 = 1 \quad \text{Factor.}$$

This equation is the standard form of the equation of a circle with radius 1 and center $(-2, 3)$. To graph the equation, use the center $(-2, 3)$ and the radius 1. See Figure 52.

Solution continued

Figure 52



Using a Graphing Utility to Graph a Circle