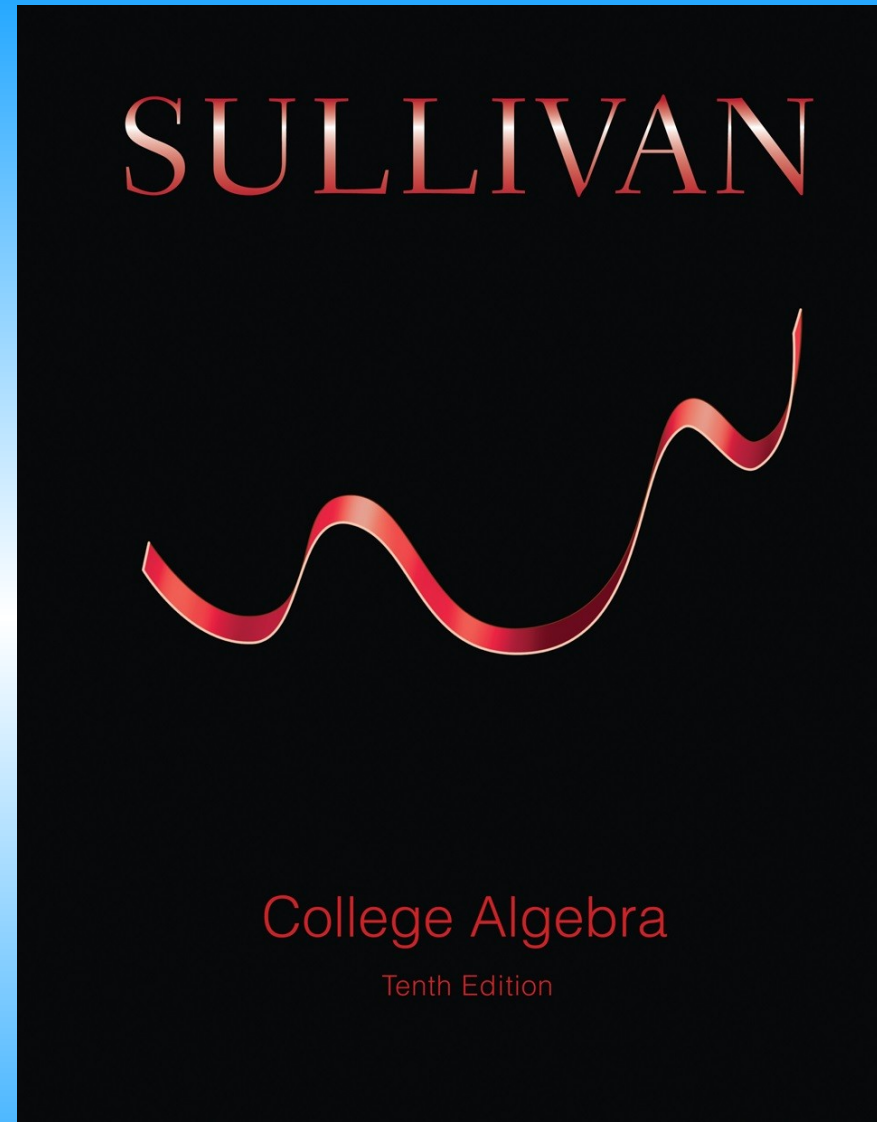


Chapter 2

Section 1



2.1 The Distance and Midpoint Formulas

PREPARING FOR THIS SECTION Before getting started, review the following:

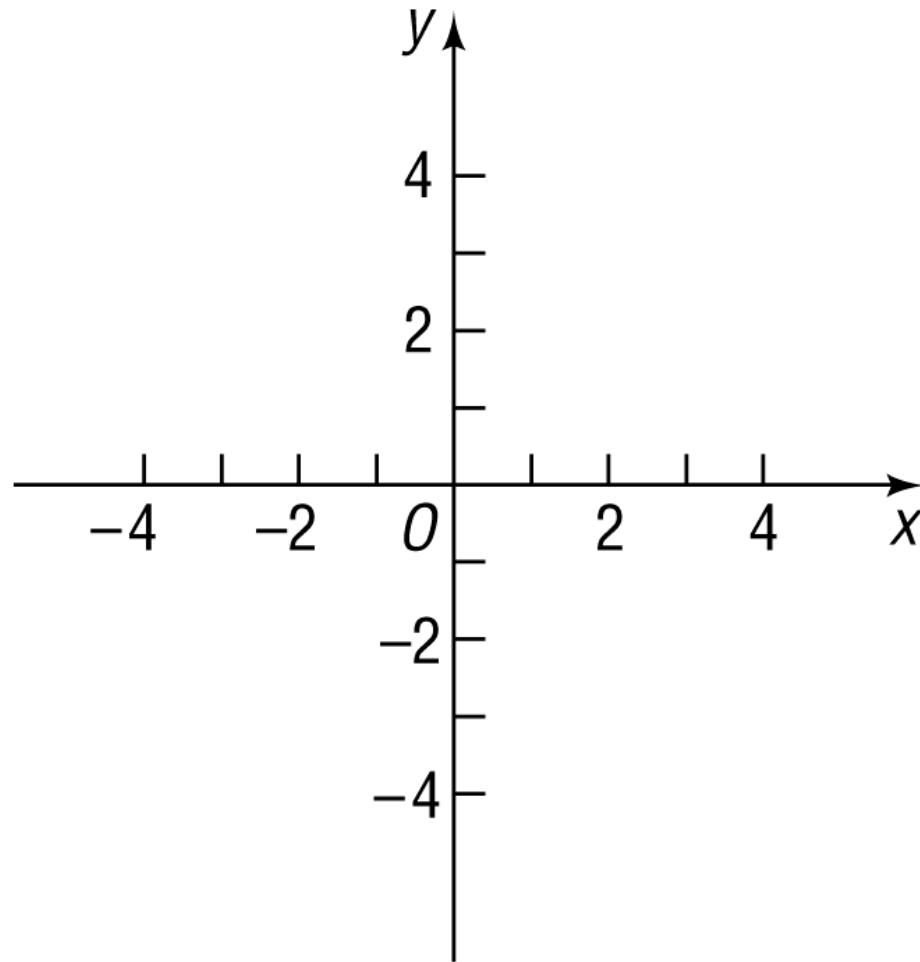
- Algebra Essentials (Chapter R, Section R.2, pp. 17–26)
- Geometry Essentials (Chapter R, Section R.3, pp. 30–35)



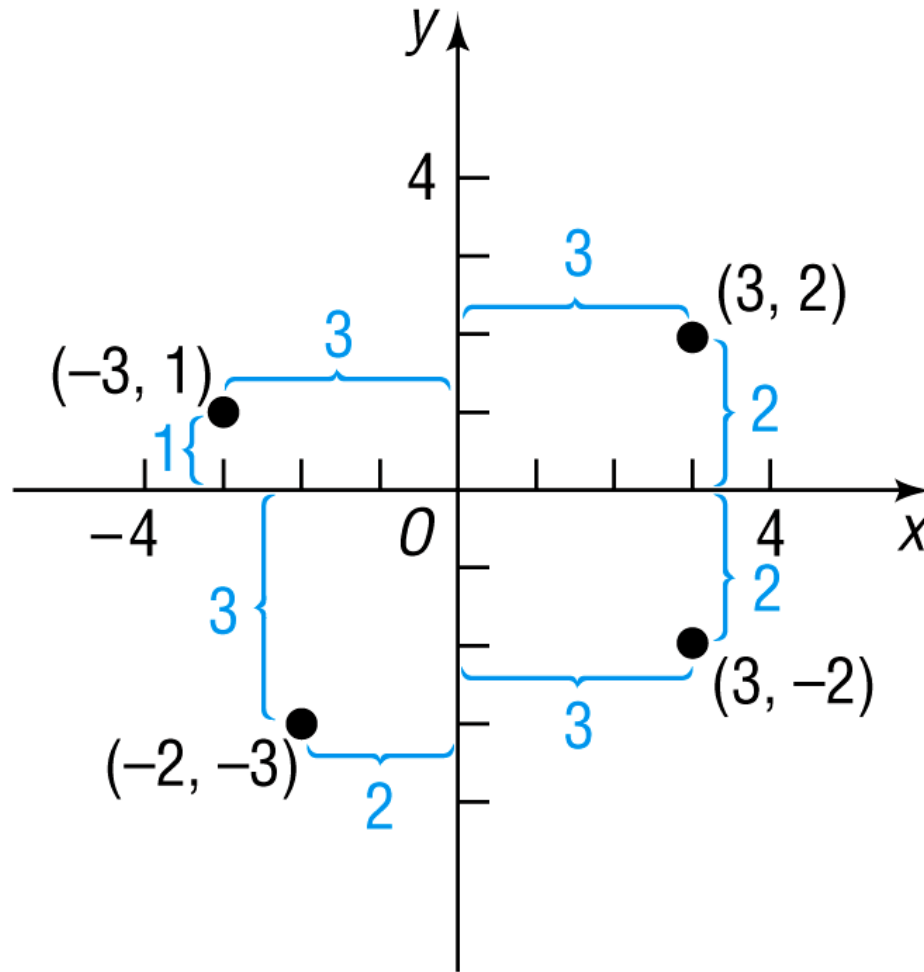
Now Work the 'Are You Prepared?' problems on page 154.

- OBJECTIVES**
- 1** Use the Distance Formula (p. 151)
 - 2** Use the Midpoint Formula (p. 153)

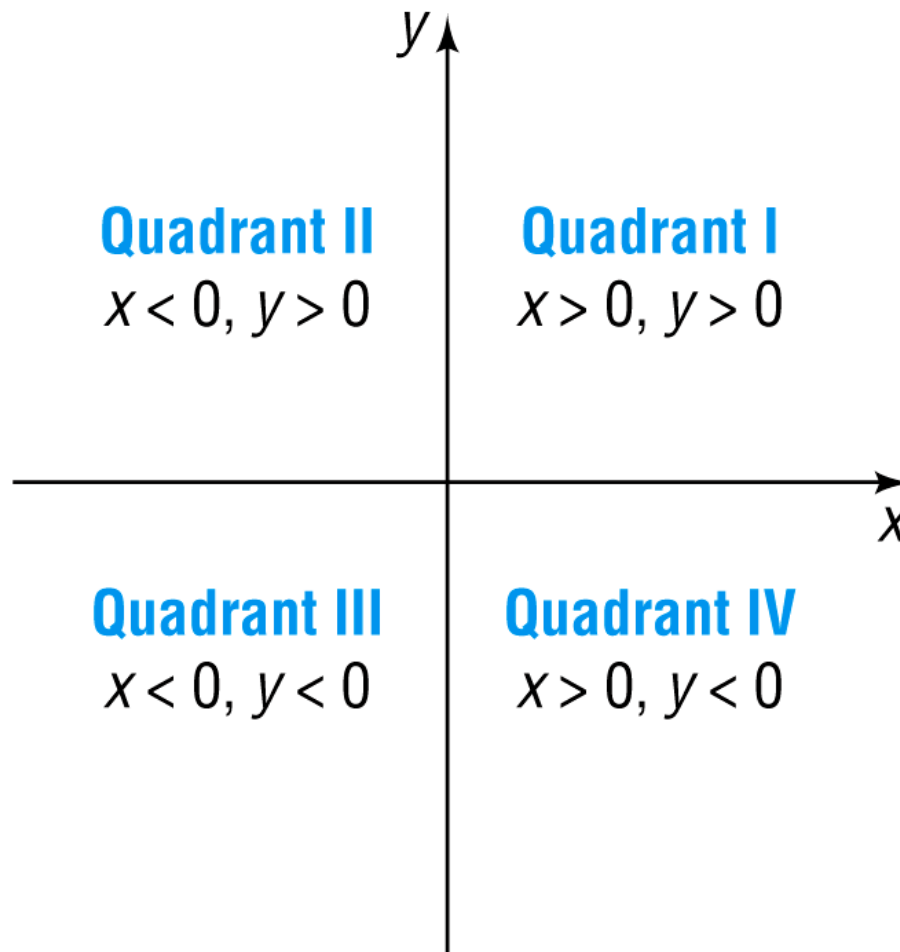
xy-Plane



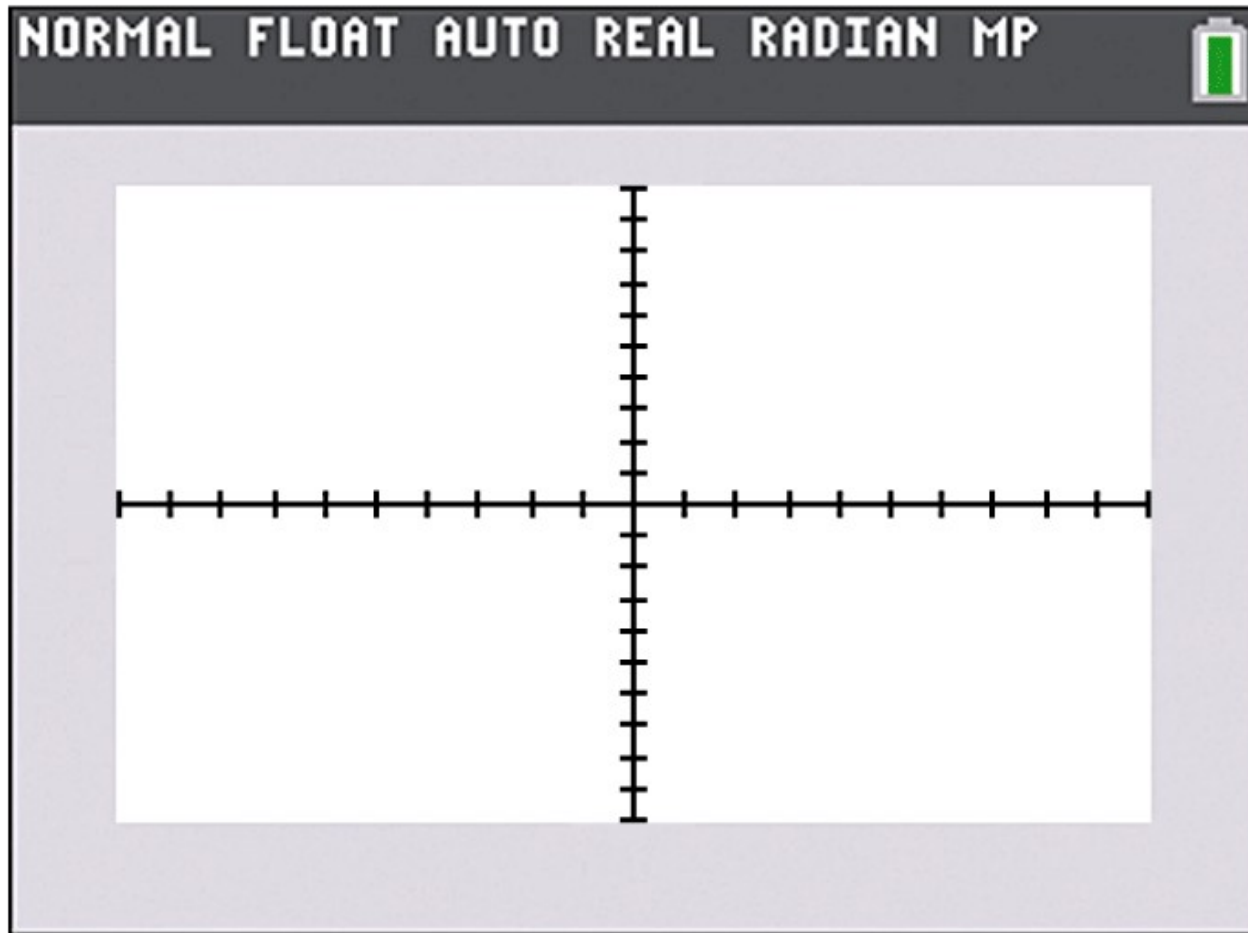
Some Plotted Points



Quadrants

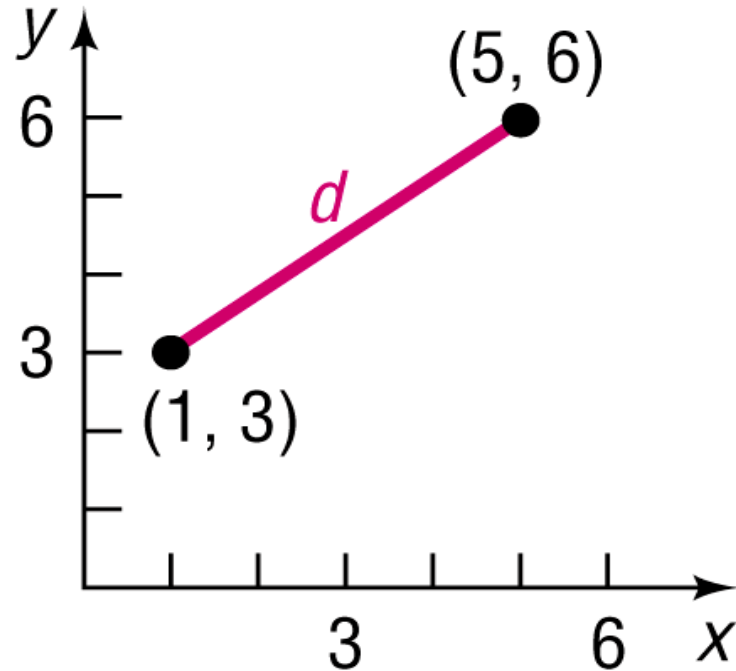


TI-84 Plus C Standard Viewing Rectangle

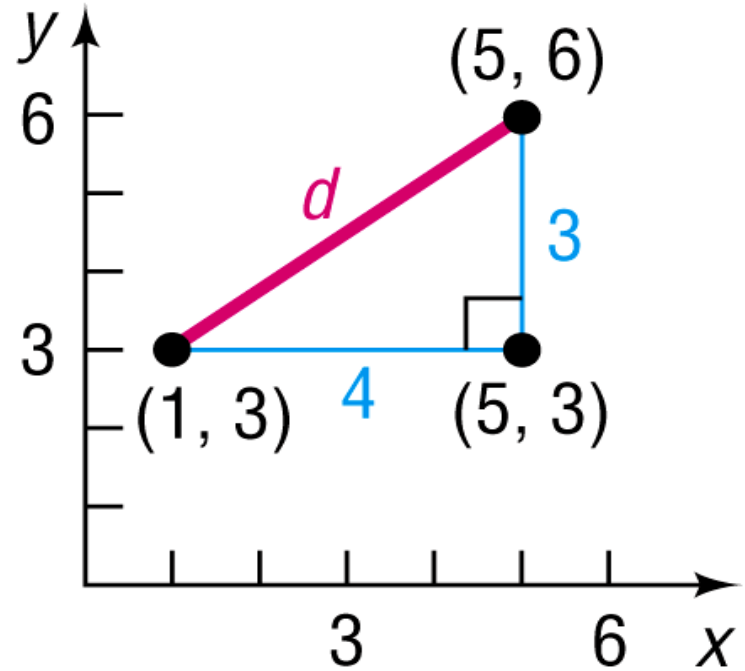


Use the Distance Formula

Distance Between Two Points



(a)



(b)

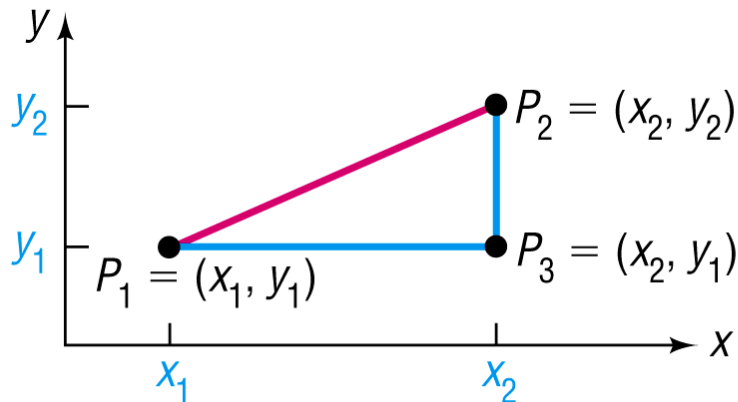
Theorem

Distance Formula

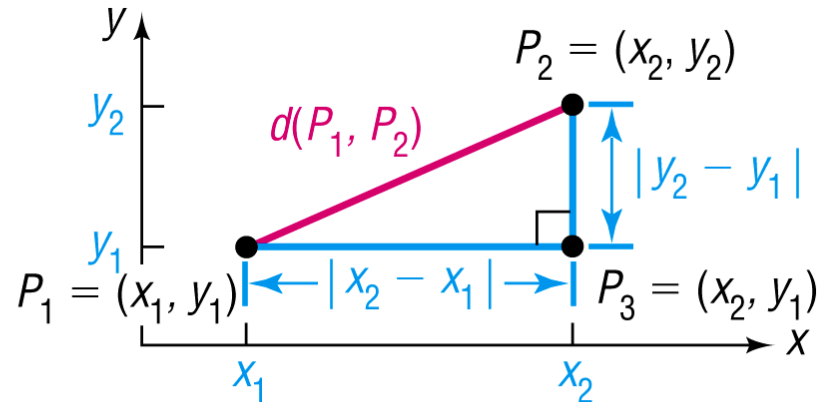
The distance between two points $P_1 = (x_1, y_1)$ and $P_2 = (x_2, y_2)$, denoted by $d(P_1, P_2)$, is

$$d(P_1, P_2) = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \quad (1)$$

Proof of Distance Formula

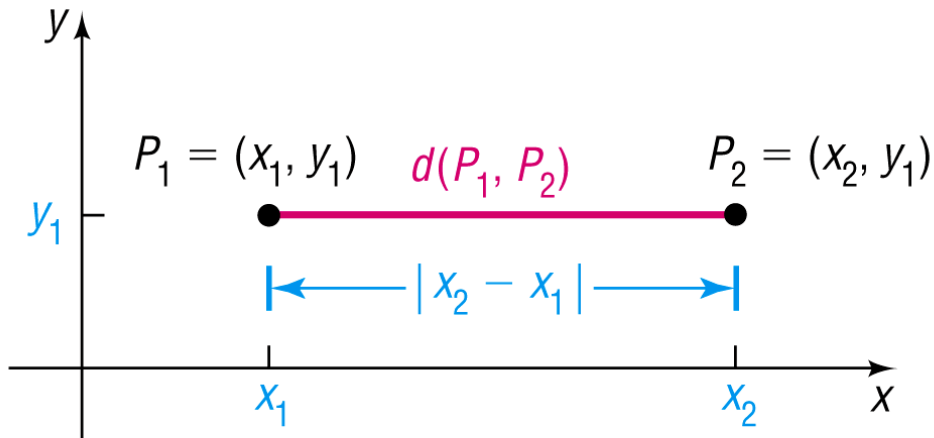


(a)

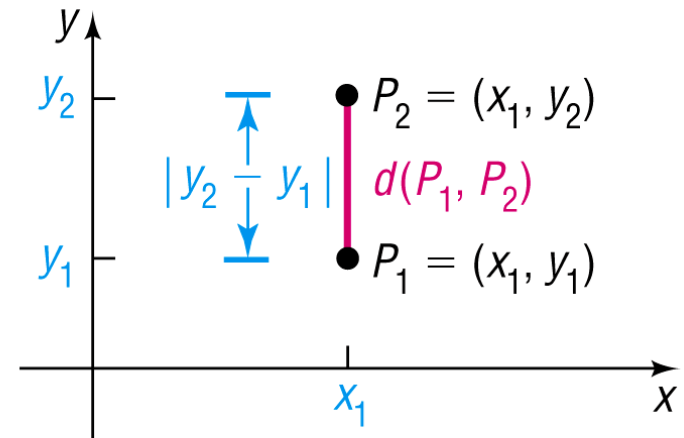


(b)

Proof continued



(a)



(b)

Example

Using the Distance Formula

Find the distance d between the points $(-4, 5)$ and $(3, 2)$.

Solution

Using the distance formula, equation (1), reveals that the distance d is

$$\begin{aligned} d &= \sqrt{[3 - (-4)]^2 + (2 - 5)^2} = \sqrt{7^2 + (-3)^2} \\ &= \sqrt{49 + 9} = \sqrt{58} \approx 7.62 \end{aligned}$$

Example

Using Algebra to Solve Geometry Problems

Consider the three points $A = (-2, 1)$, $B = (2, 3)$, and $C = (3, 1)$.

- (a) Plot each point and form the triangle ABC .
- (b) Find the length of each side of the triangle.
- (c) Show that the triangle is a right triangle.
- (d) Find the area of the triangle.

Solution

- (a) Figure 8 shows the points A, B, C and the triangle ABC .
- (b) To find the length of each side of the triangle, use the distance formula, equation (1).

$$d(A, B) = \sqrt{[2 - (-2)]^2 + (3 - 1)^2} = \sqrt{16 + 4} = \sqrt{20} = 2\sqrt{5}$$

$$d(B, C) = \sqrt{(3 - 2)^2 + (1 - 3)^2} = \sqrt{1 + 4} = \sqrt{5}$$

$$d(A, C) = \sqrt{[3 - (-2)]^2 + (1 - 1)^2} = \sqrt{25 + 0} = 5$$

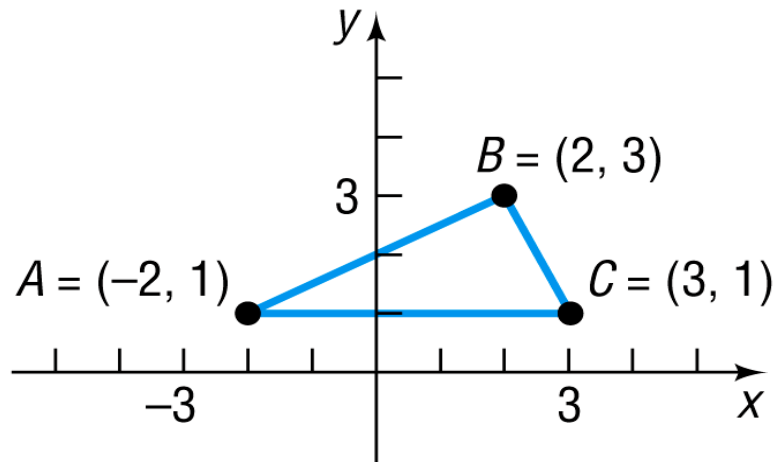


Figure 8

Solution continued

- (c) If the sum of the squares of the lengths of two of the sides equals the square of the length of the third side, then the triangle is a right triangle. Looking at Figure 8, it seems reasonable to conjecture that the angle at vertex B might be a right angle. We shall check to see whether

$$[d(A, B)]^2 + [d(B, C)]^2 = [d(A, C)]^2$$

Using the results in part (b) yields

$$\begin{aligned}[d(A, B)]^2 + [d(B, C)]^2 &= (2\sqrt{5})^2 + (\sqrt{5})^2 \\ &= 20 + 5 = 25 = [d(A, C)]^2\end{aligned}$$

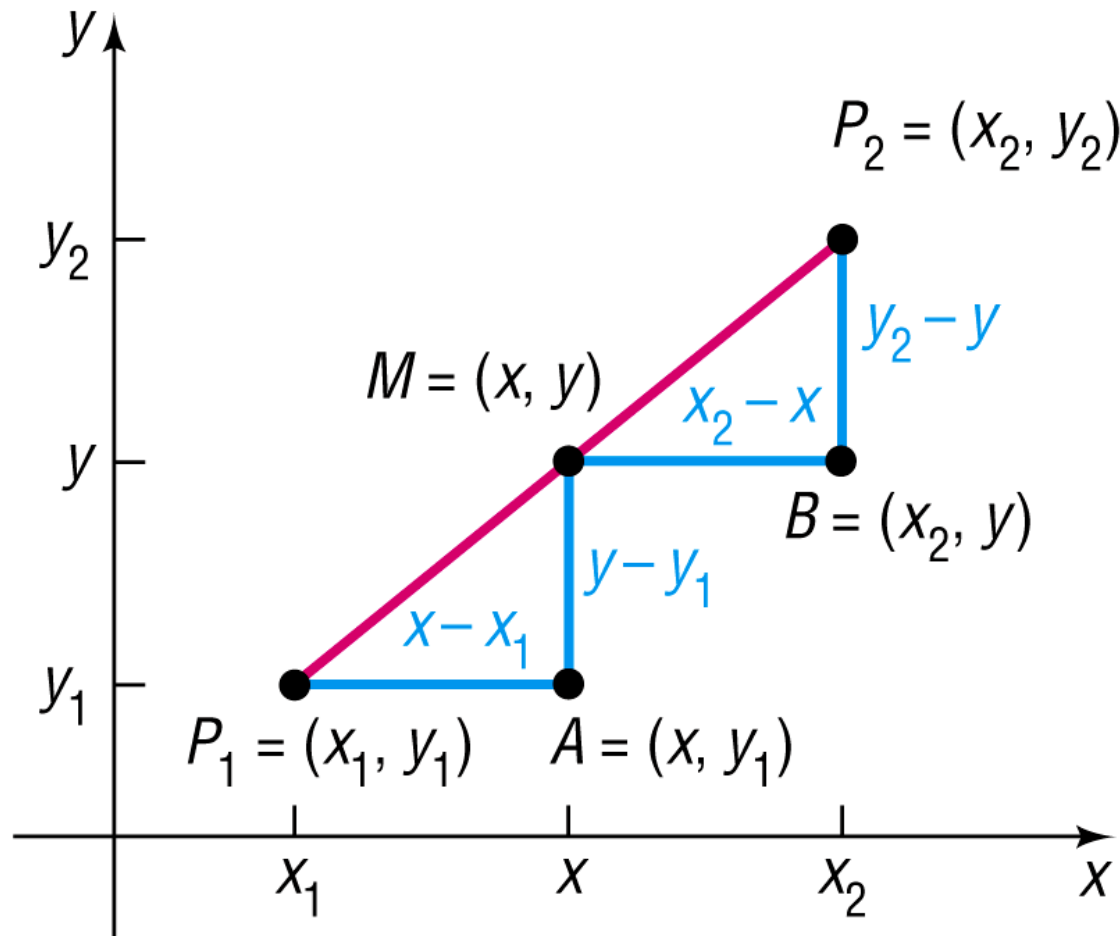
It follows from the converse of the Pythagorean Theorem that triangle ABC is a right triangle.

- (d) Because the right angle is at vertex B , the sides AB and BC form the base and height of the triangle. Its area is

$$\text{Area} = \frac{1}{2} (\text{Base}) (\text{Height}) = \frac{1}{2} (2\sqrt{5}) (\sqrt{5}) = 5 \text{ square units}$$

Use the Midpoint Formula

Derivation of Midpoint Formula



Theorem

Midpoint Formula

The midpoint $M = (x, y)$ of the line segment from $P_1 = (x_1, y_1)$ to $P_2 = (x_2, y_2)$ is

$$M = (x, y) = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right) \quad (2)$$

Example

Finding the Midpoint of a Line Segment

Find the midpoint of the line segment from $P_1 = (-5, 5)$ to $P_2 = (3, 1)$. Plot the points P_1 and P_2 and their midpoint.

Solution

Apply the midpoint formula (2) using $x_1 = -5$, $y_1 = 5$, $x_2 = 3$, and $y_2 = 1$. Then the coordinates (x, y) of the midpoint M are

$$x = \frac{x_1 + x_2}{2} = \frac{-5 + 3}{2} = -1 \quad \text{and} \quad y = \frac{y_1 + y_2}{2} = \frac{5 + 1}{2} = 3$$

That is, $M = (-1, 3)$. See Figure 10.

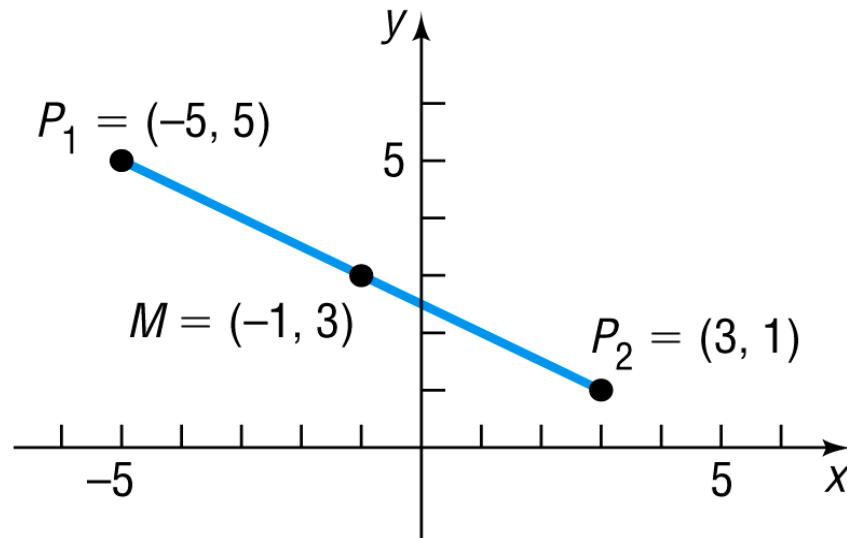


Figure 10