

# The Real Numbers and Their Representation



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# Chapter 6: The Real Numbers and Their Representation

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- 6.1 Real Numbers, Order, and Absolute Value
- 6.2 Operations, Properties, and Applications of Real Numbers
- 6.3 Rational Numbers and Decimal Representation
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- 6.5 Applications of Decimals and Percents

# **Section 6-3**

## **Rational Numbers and Decimal Representation**

# Rational Numbers and Decimal Representation

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- Define and identify rational numbers.
- Write a rational number in lowest terms.
- Add, subtract, multiply, and divide rational numbers in fraction form.
- Solve carpentry problem using operations with fractions.
- Apply the density property and find arithmetic mean.

# Rational Numbers and Decimal Representation

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- Convert a rational number in fraction form to a decimal number.
- Convert a terminating or repeating decimal to a rational number in fraction form.

# Definition: Rational Numbers

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**Rational Numbers =**

$\{x \mid x \text{ is a quotient of two integers, with denominator not equal to } 0\}$

# Lowest Terms

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A rational number is said to be in **lowest terms** if the greatest common factor of the numerator (top number) and the denominator (bottom number) is 1. Rational numbers are written in lowest terms by using the *fundamental property of rational numbers* (see next slide).

# Fundamental Property of Rational Numbers

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If  $a$ ,  $b$ , and  $k$  are integers with  $b \neq 0$  and  $k \neq 0$ , then

$$\frac{a \cdot k}{b \cdot k} = \frac{a}{b}.$$

# Example: Writing a Fraction in Lowest Terms

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Write  $\frac{24}{27}$  in lowest terms.

**Solution**

$$\frac{24}{27} = \frac{8 \cdot 3}{9 \cdot 3} = \frac{8}{9}.$$

# Cross-Product Test for Equality of Rational Numbers

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For rational numbers  $\frac{a}{b}$  and  $\frac{c}{d}$ ,  $b \neq 0$ ,  $d \neq 0$ ,

$$\frac{a}{b} = \frac{c}{d} \text{ if and only if } a \cdot d = b \cdot c.$$

$a$  and  $d$  are called the “**extremes.**”

$b$  and  $c$  are called the “**means.**”

# Adding and Subtracting Rational Numbers

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If  $\frac{a}{b}$  and  $\frac{c}{d}$  are rational numbers, then

$$\frac{a}{b} + \frac{c}{d} = \frac{ad + bc}{bd}$$

and

$$\frac{a}{b} - \frac{c}{d} = \frac{ad - bc}{bd}.$$

# Adding and Subtracting Rational Numbers

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In practical problems involving addition and subtraction of rational numbers, fractions are usually rewritten with the least common multiple of their denominators, called the **least common denominator**.

# Example: Adding and Subtracting Rational Numbers

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Perform each operation.

$$\text{a) } \frac{4}{9} + \frac{2}{15}$$

$$\text{b) } \frac{4}{9} - \frac{2}{15}$$

**Solution**

$$\text{a) } \frac{4}{9} + \frac{2}{15} = \frac{20}{45} + \frac{6}{45} = \frac{26}{45}$$

$$\text{b) } \frac{4}{9} - \frac{2}{15} = \frac{20}{45} - \frac{6}{45} = \frac{14}{45}$$

# Multiplying Rational Numbers

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If  $\frac{a}{b}$  and  $\frac{c}{d}$  are rational numbers, then

$$\frac{a}{b} \cdot \frac{c}{d} = \frac{a \cdot c}{b \cdot d}.$$

# Example: Multiplying Rational Numbers

Find the product  $\frac{5}{9} \cdot \frac{3}{10}$ .

**Solution**

$$\frac{5}{9} \cdot \frac{3}{10} = \frac{15}{90} = \frac{1 \cdot 15}{6 \cdot 15} = \frac{1}{6}$$

# Definition of Division

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If  $a$  and  $b$  are real numbers with  $b \neq 0$ , then

$$\frac{a}{b} = a \cdot \frac{1}{b}.$$

# Dividing Rational Numbers

If  $\frac{a}{b}$  and  $\frac{c}{d}$  are rational numbers  $\left(\frac{c}{d} \neq 0\right)$ , then

$$\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \cdot \frac{d}{c} = \frac{ad}{bc}.$$

# Example: Dividing Rational Numbers

Find the quotient  $\frac{2}{9} \div \frac{5}{6}$ .

## Solution

$$\frac{2}{9} \div \frac{5}{6} = \frac{2}{9} \cdot \frac{6}{5} = \frac{12}{45} = \frac{4 \cdot 3}{15 \cdot 3} = \frac{4}{15}$$

# Example: Applying Operations with Fractions to Carpentry

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A carpenter has a board 72 inches long that he must cut unto 10 pieces of equal length. This will require 9 cuts.

a) If each cut causes a waste of  $\frac{3}{16}$  inch, how many inches of actual board will remain after the cuts?

b) What will the length of each of the resulting pieces be?

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# Example: Applying Operations with Fractions to Carpentry

## Solution

a) We start with 72 inches and must subtract  $\frac{3}{16}$  nine times. We represent this by  $9\left(\frac{3}{16}\right)$ .

$$\begin{aligned} 72 - 9\left(\frac{3}{16}\right) &= 72 - \left(\frac{9}{1} \square \frac{3}{16}\right) \\ &= 72 - \frac{27}{16} = \frac{1125}{16} = 70\frac{5}{16} \end{aligned}$$

Thus,  $70\frac{5}{16}$  inches of actual board will remain.

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# Example: Applying Operations with Fractions to Carpentry

## Solution

b) The  $70\frac{5}{16}$  or  $\frac{1125}{16}$  inches of board from part (a) will be divided into 10 pieces of equal length.

$$\frac{1125}{16} \div 10 = \frac{1125}{16} \cdot \frac{1}{10}$$

$$= \frac{225}{32} = 7\frac{1}{32}$$

Each piece will measure  $7\frac{1}{32}$  inches.

# Density Property of Rational Numbers

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If  $r$  and  $t$  are distinct rational numbers, with  $r < t$ , then there exists a rational number  $s$  such that

$$r < s < t.$$

This leads to the conclusion that there are *infinitely many* rational numbers between two distinct rational numbers.

# Arithmetic Mean



To find the **arithmetic mean**, or **average**, of  $n$  numbers, we add the numbers and then divide the sum by  $n$ . For two numbers, the number that lies halfway between them is their average.

# Example: Finding an Arithmetic Mean (Average)

Find the average of  $\frac{3}{5}$  and  $\frac{2}{3}$ .

## Solution

$$\frac{3}{5} + \frac{2}{3} = \frac{9}{15} + \frac{10}{15} = \frac{19}{15}$$

Add the fractions.

$$\frac{19}{15} \div 2 = \frac{19}{15} \cdot \frac{1}{2} = \frac{19}{30}$$

Divide the sum by 2 to get the answer.

# Converting a Rational Number in Fraction Form to a Decimal Number

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A rational number in the form  $a/b$  can be expressed as a decimal most easily by entering it into a calculator.

For example, to write  $\frac{7}{8}$  as a decimal, enter 7, enter the operation of division, enter 8, and then press the equals key to find the following equivalence:

$$\frac{7}{8} = 0.875$$

# Decimal Form of Rational Numbers

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Any rational number can be expressed as either a *terminating* (stops) decimal or a *repeating* (reoccurring block of numbers) decimal.

# Criteria for Terminating and Repeating Decimals

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A rational number, in lowest terms, results in a **terminating decimal** if the only prime factor of the denominator is 2 or 5 (or both).

A rational number, in lowest terms, results in a **repeating decimal** if a prime other than 2 or 5 appears in the prime factorization of the denominator.

# Example: Determining Whether a Decimal Terminates or Repeats

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Without dividing, determine whether the decimal form of the given rational number terminates or repeats.

a)  $\frac{7}{15}$

b)  $\frac{15}{16}$

## Solution

- a) Repeats; there is a factor of 3 in the denominator.
- b) Terminates; the denominator is  $2^4$ .

# Example: Writing Decimals as Quotients of Integers

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Write each decimal as a quotient of integers.

a) 0.759

b) 7.8

**Solution**

$$\text{a) } 0.759 = \frac{759}{1000}$$

$$\text{b) } 7.8 = 7 + \frac{8}{10} = \frac{78}{10} = \frac{39}{5}$$