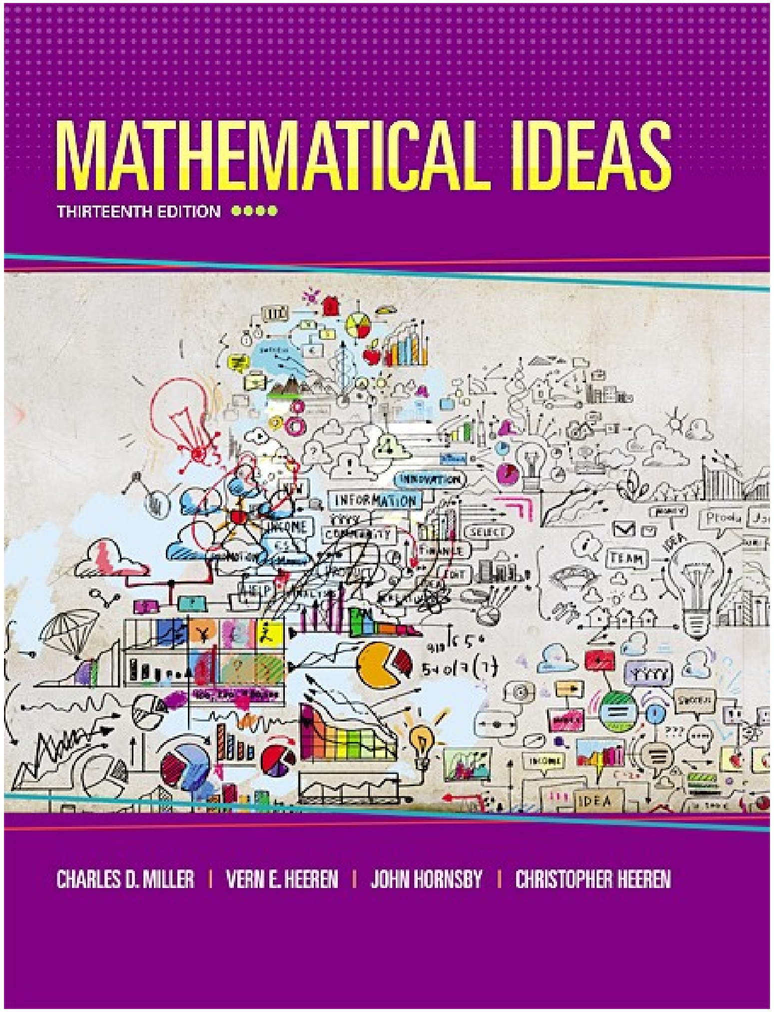
Chapter 5

Number Theory



Chapter 5: Number Theory

Section 5-3

Selected Topics from Number Theory

Selected Topics from Number Theory

- Understand and identify perfect numbers.
- Understand and identify deficient and abundant numbers.
- Understand amicable (friendly) numbers.
- State and evaluate Goldbach's conjecture.
- Understand and identify twin primes.
- State and evaluate Fermat's Last Theorem.

Perfect Numbers

A natural number is said to be **perfect** if it is equal to the sum of its proper divisors.

6 is perfect because $6 = 1 + 2 + 3$.

8 is not because $8 \neq 1 + 2 + 4$.

Deficient and Abundant Numbers

A natural number is **deficient** if it is greater than the sum of its proper divisors. It is **abundant** if it is less than the sum of its proper divisors.

Example: Identifying Deficient and Abundant Numbers

Decide whether 12 is deficient or abundant.

Solution

The proper divisors of 12 are 1, 2, 3, 4, and 6. Their sum is 16. Because 16 > 12, the number 12 is abundant.

Amicable (Friendly) Numbers

The natural numbers *a* and *b* are **amicable**, or **friendly**, if the sum of the proper divisors of *a* is *b*, and the sum of the proper divisors of *b* is *a*.

The smallest pair of amicable numbers is 220 and 284.

Goldbach's Conjecture (Not Proved)

Every even number greater than 2 can be written as the sum of two prime numbers.

Example: Expressing Numbers as Sums of Primes

Write each even number as the sum of two primes.

a) 12 b) 40

Solution

a) $12 = 5 + 7$

b) $40 = 17 + 23$

Twin Primes

Twin primes are prime numbers that differ by 2.

Examples: 3 and 5, 11 and 13

Twin Primes Conjecture (Not Proved)

There are infinitely many pairs of twin primes.

Fermat's Last Theorem

For *any* natural number $n \geq 3$, there are no triples (a, b, c) that satisfy the equation:

$$a^n + b^n = c^n.$$

Example: Using a Theorem Proved by Fermat

Every odd prime can be expressed as the difference of two squares in one and only one way.

Express 7 as the difference of two squares.

Solution

$$7 = 16 - 9 = 4^2 - 3^2$$