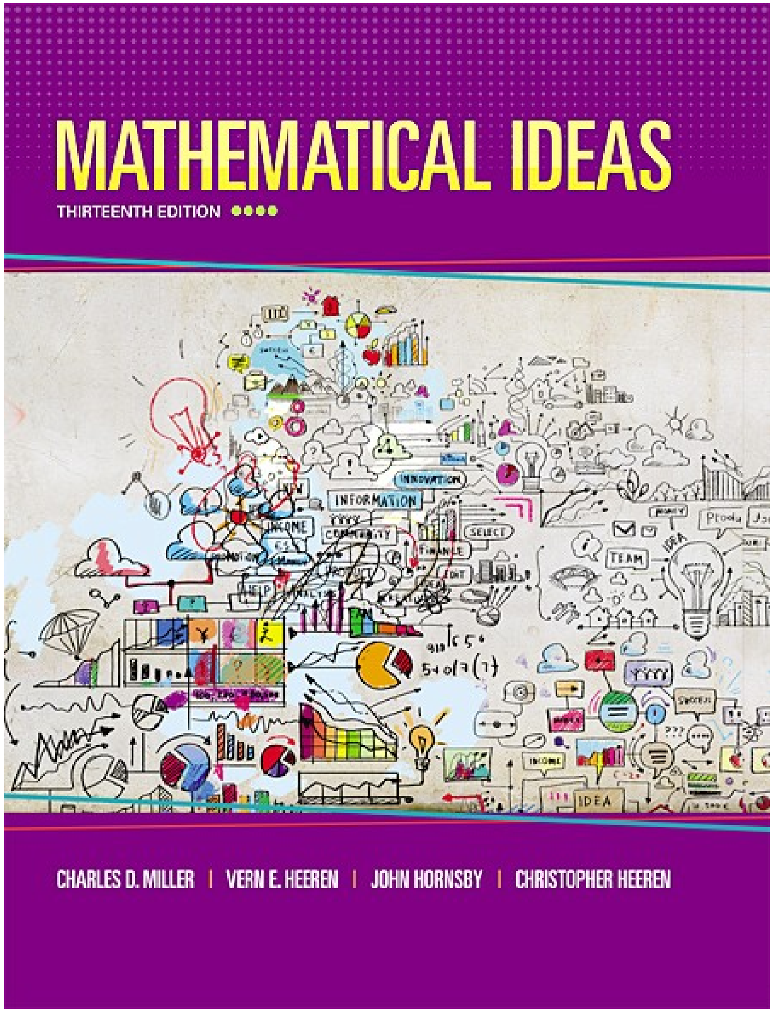
Chapter 6

The Real Numbers and Their Representation



Chapter 6: The Real Numbers and Their Representation

6.1 Real Numbers, Order, and Absolute Value

6.2 Operations, Properties, and Applications of Real Numbers

6.3 Rational Numbers and Decimal Representation

6.4 Irrational Numbers and Decimal Representation

6.5 Applications of Decimals and Percents

Section 6-3

Rational Numbers and Decimal Representation

Rational Numbers and Decimal Representation

- Define and identify rational numbers.
- Write a rational number in lowest terms.
- Add, subtract, multiply, and divide rational numbers in fraction form.
- Solve carpentry problem using operations with fractions.
- Apply the density property and find arithmetic mean.

Rational Numbers and Decimal Representation

- Convert a rational number in fraction form to a decimal number.
- Convert a terminating or repeating decimal to a rational number in fraction form.

Definition: Rational Numbers

Rational Numbers =

{x | x is a quotient of two integers, with denominator not equal to 0}

Lowest Terms

A rational number is said to be in **lowest terms** if the greatest common factor of the numerator (top number) and the denominator (bottom number) is 1. Rational numbers are written in lowest terms by using the *fundamental property of rational numbers* (see next slide).

Fundamental Property of Rational Numbers

If a, b, and k are integers with $b \neq 0$ and $k \neq 0$, then

$$\frac{a \cdot k}{b \cdot k} = \frac{a}{b}.$$

Example: Writing a Fraction in Lowest Terms

Write $\frac{24}{27}$ in lowest terms.

Solution

$$\frac{24}{27} = \frac{8 \cdot 3}{9 \cdot 3} = \frac{8}{9}.$$

Cross-Product Test for Equality of Rational Numbers

For rational numbers $\frac{a}{b}$ and $\frac{c}{d}$, $b \neq 0$, $d \neq 0$,

$$\frac{a}{b} = \frac{c}{d} \text{ if and only if } a \cdot d = b \cdot c.$$

a and *d* are called the "**extremes**."

b and *c* are called the "**means**."

Adding and Subtracting Rational Numbers

If $\dfrac{a}{b}$ and $\dfrac{c}{d}$ are rational numbers, then

$$\frac{a}{b} + \frac{c}{d} = \frac{ad + bc}{bd}$$

and

$$\frac{a}{b} - \frac{c}{d} = \frac{ad - bc}{bd}.$$

Adding and Subtracting Rational Numbers

In practical problems involving addition and subtraction of rational numbers, fractions are usually rewritten with the least common multiple of their denominators, called the **least common denominator**.

Example: Adding and Subtracting Rational Numbers

Perform each operation.

a) $\frac{4}{9}+\frac{2}{15}$

b) $\frac{4}{9}-\frac{2}{15}$

Solution

a) $\frac{4}{9}+\frac{2}{15}=\frac{20}{45}+\frac{6}{45}=\frac{26}{45}$

b) $\frac{4}{9}-\frac{2}{15}=\frac{20}{45}-\frac{6}{45}=\frac{14}{45}$

Multiplying Rational Numbers

If $\frac{a}{b}$ and $\frac{c}{d}$ are rational numbers, then

$$\frac{a}{b} \cdot \frac{c}{d} = \frac{a \cdot c}{b \cdot d}.$$

Example: Multiplying Rational Numbers

Find the product $\frac{5}{9} \cdot \frac{3}{10}$.

Solution

$$\frac{5}{9} \cdot \frac{3}{10} = \frac{15}{90} = \frac{1 \cdot 15}{6 \cdot 15} = \frac{1}{6}$$

Definition of Division

If a and b are real numbers with $b \neq 0,$ then

$$\frac{a}{b} = a \cdot \frac{1}{b}.$$

Dividing Rational Numbers

If $\dfrac{a}{b}$ and $\dfrac{c}{d}$ are rational numbers $\left(\dfrac{c}{d} \neq 0\right)$, then

$$\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \cdot \frac{d}{c} = \frac{ad}{bc}.$$

Example: Dividing Rational Numbers

Find the quotient $\frac{2}{9} \div \frac{5}{6}$.

Solution

$$\frac{2}{9} \div \frac{5}{6} = \frac{2}{9} \cdot \frac{6}{5} = \frac{12}{45} = \frac{4 \cdot 3}{15 \cdot 3} = \frac{4}{15}$$

Example: Applying Operations with Fractions to Carpentry

A carpenter has a board 72 inches long that he must cut unto 10 pieces of equal length. This will require 9 cuts.

a) If each cut causes a waste of $\frac{3}{16}$ inch, how many inches of actual board will remain after the cuts?

b) What will the length of each of the resulting pieces be?

Continued

Example: Applying Operations with Fractions to Carpentry

Solution

a) We start with 72 inches and must subtract $\frac{3}{16}$ nine times. We represent this by $9\left(\frac{3}{16}\right)$.

$$72 - 9\left(\frac{3}{16}\right) = 72 - \left(\frac{9}{1} \cdot \frac{3}{16}\right)$$

$$= 72 - \frac{27}{16} = \frac{1125}{16} = 70\frac{5}{16}$$

Thus, $70\frac{5}{16}$ inches of actual board will remain.

Continued

Example: Applying Operations with Fractions to Carpentry

Solution

b) The $70\frac{5}{16}$ or $\frac{1125}{16}$ inches of board from part (a) will be divided into 10 pieces of equal length.

$$\frac{1125}{16} \div 10 = \frac{1125}{16} \cdot \frac{1}{10}$$

$$= \frac{225}{32} = 7\frac{1}{32}$$

Each piece will measure $7\frac{1}{32}$ inches.

Density Property of Rational Numbers

If r and t are distinct rational numbers, with $r < t$, then there exists a rational number s such that

$$\boldsymbol{r < s < t}.$$

This leads to the conclusion that there are *infinitely many* rational numbers between two distinct rational numbers.

Arithmetic Mean

To find the **arithmetic mean**, or **average**, of *n* numbers, we add the numbers and then divide the sum by *n*. For two numbers, the number that lies halfway between them is their average.

Example: Finding an Arithmetic Mean (Average)

Find the average of $\frac{3}{5}$ and $\frac{2}{3}$.

Solution

$$\frac{3}{5}+\frac{2}{3}=\frac{9}{15}+\frac{10}{15}=\frac{19}{15}$$

Add the fractions.

$$\frac{19}{15}\div 2=\frac{19}{15}\cdot\frac{1}{2}=\frac{19}{30}$$

Divide the sum by 2 to get the answer.

Converting a Rational Number in Fraction Form to a Decimal Number

A rational number in the form a/b can be expressed as a decimal most easily by entering it into a calculator.

For example, to write $\frac{7}{8}$ as a decimal, enter 7, enter the operation of division, enter 8, and then press the equals key to find the following equivalence:

$$\frac{7}{8} = 0.875$$

Decimal Form of Rational Numbers

Any rational number can be expressed as either a *terminating* (stops) decimal or a *repeating* (reoccurring block of numbers) decimal.

Criteria for Terminating and Repeating Decimals

A rational number, in lowest terms, results in a **terminating decimal** if the only prime factor of the denominator is 2 or 5 (or both).

A rational number, in lowest terms, results in a **repeating decimal** if a prime other than 2 or 5 appears in the prime factorization of the denominator.

Example: Determining Whether a Decimal Terminates or Repeats

Without dividing, determine whether the decimal form of the given rational number terminates or repeats.

a) $\dfrac{7}{15}$ b) $\dfrac{15}{16}$

Solution

a) Repeats; there is a factor of 3 in the denominator.

b) Terminates; the denominator is 2^4.

Example: Writing Decimals as Quotients of Integers

Write each decimal as a quotient of integers.

a) 0.759

b) 7.8

Solution

a) $0.759 = \frac{759}{1000}$

b) $7.8 = 7 + \frac{8}{10} = \frac{78}{10} = \frac{39}{5}$