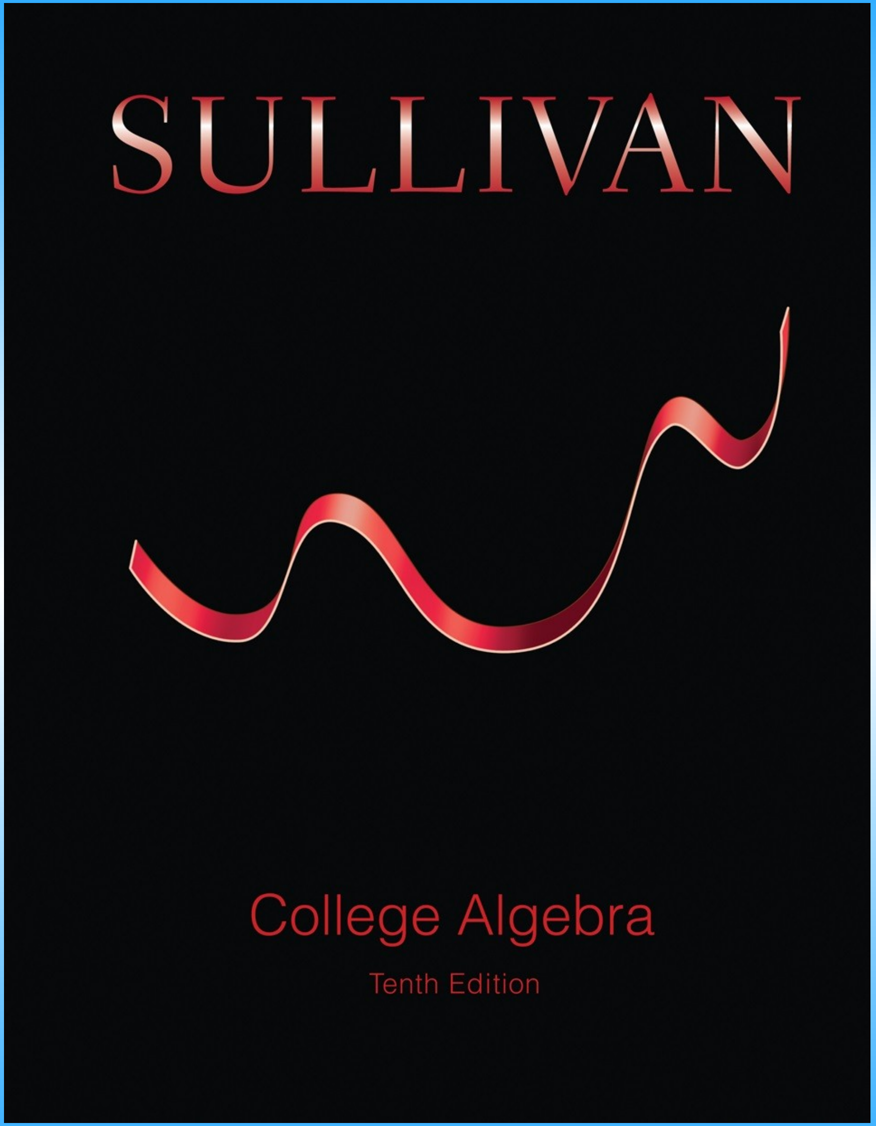
Chapter R

Section 5



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R.5 Factoring Polynomials

OBJECTIVES

1. Factor the Difference of Two Squares and the Sum and Difference of Two Cubes (p. 50)
2. Factor Perfect Squares (p. 51)
3. Factor a Second-Degree Polynomial: $x^2 + Bx + C$ (p. 52)
4. Factor by Grouping (p. 53)
5. Factor a Second-Degree Polynomial: $Ax^2 + Bx + C,\ A \neq 1$ (p. 54)
6. Complete the Square (p. 56)

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Factor the Difference of Two Squares and the Sum and Difference of Two Cubes

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Example

Factoring the Difference of Two Squares

Factor completely: $x^2 - 4$

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Solution

Note that $x^2 - 4$ is the difference of two squares, x^2 and 2^2.

$$x^2 - 4 = (x - 2)(x + 2)$$

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Example

Factoring the Difference of Two Cubes

Factor completely: $x^3 - 1$

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Solution

Because $x^3 - 1$ is the difference of two cubes, x^3 and 1^3,

$$x^3 - 1 = (x - 1)(x^2 + x + 1)$$

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Example

Factoring the Sum of Two Cubes

Factor completely: $x^3 + 8$

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Solution

Because $x^3 + 8$ is the sum of two cubes, x^3 and 2^3,

$$x^3 + 8 = (x + 2)(x^2 - 2x + 4)$$

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Example

Factoring the Difference of Two Squares

Factor completely: $x^4 - 16$

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Solution

Because $x^4 - 16$ is the difference of two squares, $x^4 = (x^2)^2$ and $16 = 4^2$,

$$x^4 - 16 = (x^2 - 4)(x^2 + 4)$$

But $x^2 - 4$ is also the difference of two squares. Then,

$$x^4 - 16 = (x^2 - 4)(x^2 + 4) = (x - 2)(x + 2)(x^2 + 4)$$

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Factor Perfect Squares

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Example

Factoring a Perfect Square

Factor completely: $x^2 + 6x + 9$

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Solution

The first term, x^2, and the third term, $9 = 3^2$, are perfect squares. Because the middle term, $6x$, is twice the product of x and 3, we have a perfect square.

$$x^2 + 6x + 9 = (x + 3)^2$$

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Example

Factoring a Perfect Square

Factor completely: $9x^2 - 6x + 1$

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Solution

The first term, $9x^2 = (3x)^2$, and the third term, $1 = 1^2$, are perfect squares. Because the middle term, $-6x$, is -2 times the product of $3x$ and 1, we have a perfect square.

$$9x^2 - 6x + 1 = (3x - 1)^2$$

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Example

Factoring a Perfect Square

Factor completely: $25x^2 + 30x + 9$

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Solution

The first term, $25x^2 = (5x)^2$, and the third term, $9 = 3^2$, are perfect squares. Because the middle term, $30x$, is twice the product of $5x$ and 3, we have a perfect square.

$$25x^2 + 30x + 9 = (5x + 3)^2$$

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Factor a Second-Degree Polynomial: $x^2 + Bx + C$

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To factor a second-degree polynomial $x^2 + Bx + C$, find integers whose product is C and whose sum is B. That is, if there are numbers a, b, where $ab = C$ and $a + b = B$, then

$$x^2 + Bx + C = (x + a)(x + b)$$

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Example

Factoring a Trinomial

Factor completely: $x^2 - x - 12$

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Solution

First determine all pairs of integers whose product is -12, and then compute each sum.

Integers whose product is -12	$1, -12$	$-1, 12$	$2, -6$	$-2, 6$	$3, -4$	$-3, 4$
Sum	-11	11	-4	4	-1	1

Since -1 is the coefficient of the middle term,

$$x^2 - x - 12 = (x + 3)(x - 4)$$

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Example

Identifying a Prime Polynomial

Show that $x^2 + 9$ is prime.

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Solution

First list the pairs of integers whose product is 9, and then compute their sums.

Integers whose product is 9	1, 9	−1, −9	3, 3	−3, −3
Sum	10	−10	6	−6

Since the coefficient of the middle term in $x^2 + 9 = x^2 + 0x + 9$ is 0 and none of the sums equals 0, we conclude that $x^2 + 9$ is prime.

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Theorem

Any polynomial of the form $x^2 + a^2$, a real, is prime.

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Factor by Grouping

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Example

Factoring by Grouping

Factor completely by grouping: $3(x-1)^2(x+2)^4 + 4(x-1)^3(x+2)^3$

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Solution

Here, $(x-1)^2(x+2)^3$ is a common factor of both $3(x-1)^2(x+2)^4$ and $4(x-1)^3(x+2)^3$. As a result,

$$\begin{aligned} 3(x-1)^2(x+2)^4 + 4(x-1)^3(x+2)^3 &= (x-1)^2(x+2)^3[3(x+2) + 4(x-1)] \\ &= (x-1)^2(x+2)^3[3x + 6 + 4x - 4] \\ &= (x-1)^2(x+2)^3(7x+2) \end{aligned}$$

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Example

Factoring by Grouping

Factor completely by grouping: $x^3 - 4x^2 + 2x - 8$

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Solution

To see whether factoring by grouping will work, group the first two terms and the last two terms. Then look for a common factor in each group. In this example, factor x^2 from $x^3 - 4x^2$ and 2 from $2x - 8$. The remaining factor in each case is the same, $x - 4$. This means that factoring by grouping will work, as follows:

$$\begin{aligned} x^3 - 4x^2 + 2x - 8 &= (x^3 - 4x^2) + (2x - 8) \\ &= x^2(x - 4) + 2(x - 4) \\ &= (x - 4)(x^2 + 2) \end{aligned}$$

Since $x^2 + 2$ and $x - 4$ are prime, the factorization is complete.

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Factoring a Second-Degree Polynomial: $Ax^2+Bx+C,\ A\neq 1$

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Steps for Factoring $Ax^2 + Bx + C$, When $A \neq 1$ and A, B, and C Have No Common Factors

STEP 1: Find the value of AC.

STEP 2: Find a pair of integers whose product is AC and that add up to B. That is, find a and b such that $ab = AC$ and $a + b = B$.

STEP 3: Write $Ax^2 + Bx + C = Ax^2 + ax + bx + C$.

STEP 4: Factor this last expression by grouping.

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Example

Factoring a Trinomial

Factor completely: $2x^2 - x - 6$

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Solution

Comparing $2x^2 - x - 6$ to $Ax^2 + Bx + C$, we find that $A = 2, B = -1$, and $C = -6$.

Step 1: The value of AC is $2 \cdot (-6) = -12$.
Step 2: Determine the pairs of integers whose product is $AC = -12$ and compute their sums.

Integers whose product is −12	1, −12	−1, 12	2, −6	−2, 6	3, −4	−3, 4
Sum	−11	11	−4	4	−1	1

Step 3: The integers whose product is -12 that add up to $B = -1$ are -4 and 3.

$$2x^2 - x - 6 = 2x^2 - 4x + 3x - 6$$

Step 4: Factor by grouping.

$$\begin{aligned} 2x^2 - 4x + 3x - 6 &= (2x^2 - 4x) + (3x - 6) \\ &= 2x(x - 2) + 3(x - 2) \\ &= (x - 2)(2x + 3) \end{aligned}$$

As a result,

$$2x^2 - x - 6 = (x - 2)(2x + 3)$$

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Complete the Square

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Completing the Square of $x^2 + bx$

Identify the coefficient of the first-degree term. Multiply this coefficient by $\frac{1}{2}$ and then square the result. That is, determine the value of b in $x^2 + bx$ and compute $\left(\frac{1}{2}b\right)^2$.

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Example

Completing the Square

Determine the number that must be added to each expression to complete the square. Then factor the expression.

Start	Add	Result	Factored Form
$y^2 + 8y$	$\left(\frac{1}{2} \cdot 8\right)^2 = 16$	$y^2 + 8y + 16$	$(y + 4)^2$
$x^2 + 12x$	$\left(\frac{1}{2} \cdot 12\right)^2 = 36$	$x^2 + 12x + 36$	$(x + 6)^2$
$a^2 - 20a$	$\left(\frac{1}{2} \cdot (-20)\right)^2 = 100$	$a^2 - 20a + 100$	$(a - 10)^2$
$p^2 - 5p$	$\left(\frac{1}{2} \cdot (-5)\right)^2 = \frac{25}{4}$	$p^2 - 5p + \frac{25}{4}$	$\left(p - \frac{5}{2}\right)^2$

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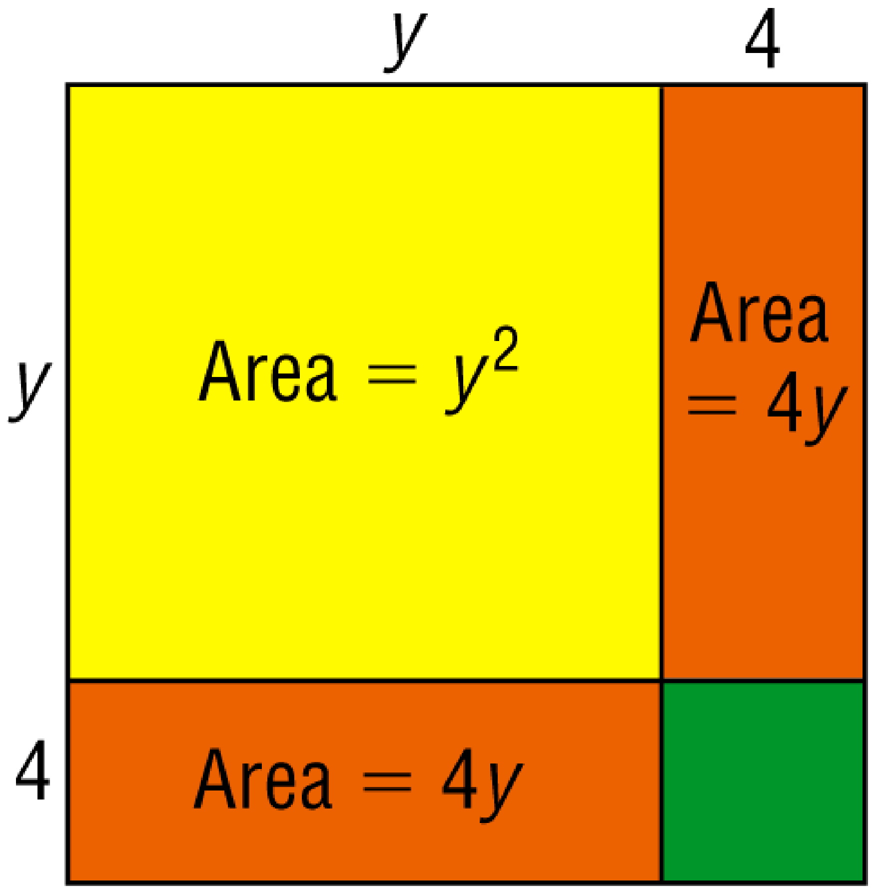
Figure: Completing the Square



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