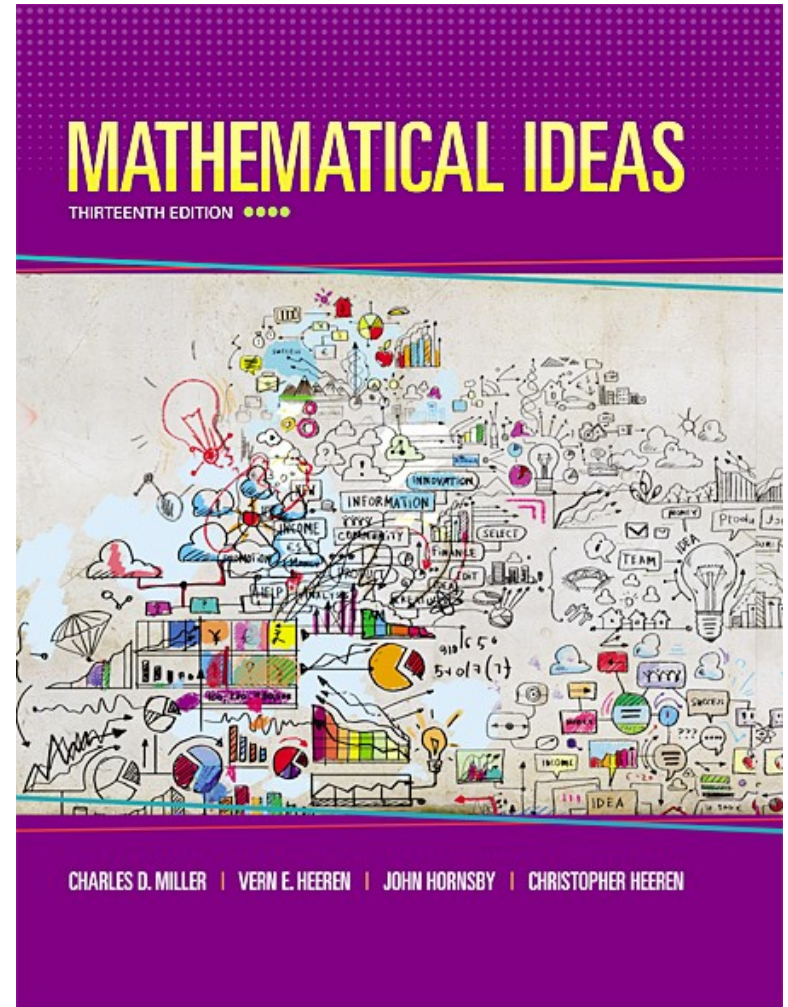


Chapter 5

Number Theory



Chapter 5: Number Theory

- 5.1 Prime and Composite Numbers
- 5.2 Large Prime Numbers
- 5.3 Selected Topics From Number Theory
- 5.4 Greatest Common Factor and Least Common Multiple
- 5.5 The Fibonacci Sequence and the Golden Ratio

Section 5-3

Selected Topics from Number Theory

Selected Topics from Number Theory

- Understand and identify perfect numbers.
- Understand and identify deficient and abundant numbers.
- Understand amicable (friendly) numbers.
- State and evaluate Goldbach's conjecture.
- Understand and identify twin primes.
- State and evaluate Fermat's Last Theorem.

Perfect Numbers

A natural number is said to be **perfect** if it is equal to the sum of its proper divisors.

6 is perfect because $6 = 1 + 2 + 3$.

8 is not because $8 \neq 1 + 2 + 4$.

Deficient and Abundant Numbers

A natural number is **deficient** if it is greater than the sum of its proper divisors. It is **abundant** if it is less than the sum of its proper divisors.

Example: Identifying Deficient and Abundant Numbers

Decide whether 12 is deficient or abundant.

Solution

The proper divisors of 12 are 1, 2, 3, 4, and 6. Their sum is 16. Because $16 > 12$, the number 12 is abundant.

Amicable (Friendly) Numbers

The natural numbers a and b are **amicable**, or **friendly**, if the sum of the proper divisors of a is b , and the sum of the proper divisors of b is a .

The smallest pair of amicable numbers is 220 and 284.

Goldbach's Conjecture (Not Proved)

Every even number greater than 2 can be written as the sum of two prime numbers.

Example: Expressing Numbers as Sums of Primes

Write each even number as the sum of two primes.

a) 12

b) 40

Solution

a) $12 = 5 + 7$

b) $40 = 17 + 23$

Twin Primes



Twin primes are prime numbers that differ by 2.

Examples: 3 and 5, 11 and 13

Twin Primes Conjecture (Not Proved)

There are infinitely many pairs of twin primes.

Fermat's Last Theorem

For *any* natural number $n \geq 3$, there are no triples (a, b, c) that satisfy the equation:

$$a^n + b^n = c^n.$$

Example: Using a Theorem Proved by Fermat

Every odd prime can be expressed as the difference of two squares in one and only one way.

Express 7 as the difference of two squares.

Solution

$$7 = 16 - 9 = 4^2 - 3^2$$