

AS1 (Assignment 1, Unit 3): Z-scores and Probability

Please type your answers in red 😊

1. What information is provided by the following z-score: -2.3?
The information that is provided by this z-score is that it is negative which means it is below the Mean and the numerical value of 2.3 shows that its 2.3 standard deviation units away from the Mean.
2. What information is provided by the numerical value of a z-score?
The numerical value of the z-score is amount of distance in standard deviation units the X-score is away from the mean.
3. A distribution has a standard deviation of $\sigma = 15$. Find the z-score for each of the following locations in the distribution.
 - a. Above the mean by 10 points.
Answer: +0.67
 - b. Above the mean by 30 points.
Answer: +2.00
 - c. Below the mean by 10 points.
Answer: -0.67
 - d. Below the mean by 5 points.
Answer: -0.33
4. For a distribution with a standard deviation of $\sigma = 16$, describe the location of each of the following z-scores in terms of its position relative to the mean. For example, $z = +1.00$ is a location that is 16 points above the mean.
 - a. $z = +2.00$ Answer: 32
 - b. $z = +.50$ Answer: 8
 - c. $z = -1.00$ Answer: -16
 - d. $z = -0.25$ Answer: -4

5. For a population with $\mu = 60$ and $\sigma = 6$,

- a. Find the z-score for each of the following X values. (Note: You should be able to find these values using the definition of a z-score. You should not need to use a formula or do any serious calculations.)

$$X = 66: z = \underline{+1.00} \quad X = 75: z = \underline{+12.50} \quad X = 56: z = \underline{+9.33}$$

$$X = 72: z = \underline{+12.00} \quad X = 48: z = \underline{+8.00} \quad X = 54: z = \underline{+9.00}$$

- b. Find the score (X value) that corresponds to each of the following z-scores. (Again, you should not need a formula or any serious calculations.)

$$z = 1.00: X = \underline{66} \quad z = 0.50: X = \underline{63} \quad z = 1.50: X = \underline{69}$$

$$z = -0.50: X = \underline{57} \quad z = -2.00: X = \underline{48} \quad z = -1.50: X = \underline{51}$$

6. For a population with a mean of $\mu = 40$ and $\sigma = 8$, find the z-score for each of the following X values. (Note: You probably will need to use a formula and a calculator to find these values)

$$X = 45: z = \underline{+0.63} \quad X = 52: z = \underline{+1.50} \quad X = 41: z = \underline{+0.13}$$

$$X = 30: z = \underline{-1.25} \quad X = 25: z = \underline{-1.88} \quad X = 38: z = \underline{-0.25}$$

7. For a population with a mean of $\mu = 100$ and a standard deviation of $\sigma = 12$,

- a. Find the z-score for each of the following X values.

$$X = 108: z = \underline{+0.67} \quad X = 115: z = \underline{+1.25} \quad X = 130: z = \underline{+2.50}$$

$$X = 90: z = \underline{-0.83} \quad X = 88: z = \underline{-1.00} \quad X = 95: z = \underline{-0.42}$$

- b. Find the score (X value) that corresponds to each of the following z-scores.

$$z = -0.40: X = \underline{95.2} \quad z = -0.50: X = \underline{94} \quad z = 1.80: X = \underline{121.6}$$

$$z = 0.75: X = \underline{109} \quad z = 1.50: X = \underline{118} \quad z = -1.25: X = \underline{115}$$

8. A population has a mean of $\mu = 70$ and a standard deviation of $\sigma = 10$.

a. For the population, find the z-score for each of the following X values.

$$X = 69: z = \underline{-0.10}$$

$$X = 84: z = \underline{+1.40}$$

$$X = 63: z = \underline{-0.70}$$

$$X = 54: z = \underline{-1.60}$$

$$X = 48: z = \underline{-2.20}$$

$$X = 45: z = \underline{-2.50}$$

b. For the same population, find the score (X value) that corresponds to each of the following z-scores.

$$z = 0.50: X = \underline{75}$$

$$z = 1.50: X = \underline{85}$$

$$z = -2.50: X = \underline{45}$$

$$z = -0.25: X = \underline{67.5}$$

$$z = -0.50: X = \underline{65}$$

$$z = 1.25: X = \underline{82.5}$$

9. A sample has a mean of $M = 30$ and a standard deviation of $s = 7$. Find the z-score of for each of the following X values from this sample.

$$X = 32: z = \underline{+0.29}$$

$$X = 34: z = \underline{+0.57}$$

$$X = 36: z = \underline{+0.86}$$

$$X = 28: z = \underline{-0.29}$$

$$X = 20: z = \underline{-1.43}$$

$$X = 18: z = \underline{-1.71}$$

10. A sample has a mean of $M = 40$ and a standard deviation of $s = 5$. For this sample, find the X value corresponding to each of the following z-scores.

$$z = 0.40: X = \underline{42}$$

$$z = 1.20: X = \underline{46}$$

$$z = 2.00: X = \underline{50}$$

$$z = -0.80: X = \underline{36}$$

$$z = -0.60: X = \underline{37}$$

$$z = -1.40: X = \underline{33}$$

11. Find the z-score corresponding to a score of $X = 50$ of the following distributions.

a. $\mu = 40$ and $\sigma = 20$

$$\text{Answer: } \underline{+0.50}$$

b. $\mu = 40$ and $\sigma = 10$

$$\text{Answer: } \underline{+1.00}$$

c. $\mu = 40$ and $\sigma = 5$

$$\text{Answer: } \underline{+2.00}$$

d. $\mu = 40$ and $\sigma = 2$

$$\text{Answer: } \underline{+5.00}$$

Find the X value corresponding to $z = 0.40$ for each of the following distributions.

e. $\mu = 40$ and $\sigma = 4$Answer: 41.6

f. $\mu = 40$ and $\sigma = 8$Answer: 43.2

g. $\mu = 40$ and $\sigma = 16$Answer: 46.4

h. $\mu = 40$ and $\sigma = 32$Answer: 52.8

12. For each of the following populations, would a score of $X = 55$ be considered a central score (near the middle of the distribution) or an extreme score (far out in the tail of the distribution)?

a. $\mu = 45$ and $\sigma = 10$Answer: +1.00 central

b. $\mu = 45$ and $\sigma = 2$Answer: +5.00 extreme

c. $\mu = 90$ and $\sigma = 20$Answer: -1.75 central

d. $\mu = 60$ and $\sigma = 20$Answer: -0.25 central

13. A distribution of exam scores has a mean of $\mu = 78$.

a. If your score is $X = 70$, which standard deviation would give you a lesser standing: $\sigma = 4$ or $\sigma = 8$?

Answer: $\sigma = 4$

b. If your score is $X = 80$, which standard deviation would give you a better grade: $\sigma = 4$ or $\sigma = 8$?

Answer: $\sigma = 4$

14. For each of the following, identify the exam score that should lead to the lower grade.

a. A score of $X = 74$ on an exam with $M = 82$ and $\sigma = 8$; or a score of $X = 40$ on an exam with $\mu = 50$ and $\sigma = 20$.

Answer: $X=74$

b. A score of $X = 51$ on an exam with $\mu = 45$ and $\sigma = 2$; or a score of $X = 90$ on an exam with $\mu = 70$ and $\sigma = 20$.

Answer: $X=90$

c. A score of $X = 62$ on an exam with $\mu = 50$ and $\sigma = 8$; or a score of $X = 23$ on an exam with $\mu = 20$ and $\sigma = 2$ Answer: $X=62$ & $X= 23$, Both scores are equal

15. A population consists of the following $N = 5$ scores: 5, 5, 4, 4, and 12.

a. Compute μ and σ for the population

Mean = 6 Standard deviation = 3.03

b. Find the z-score for each score in the population

$X=5, z = -0.33$ $X=4, z = -0.66$ $X=12, z = +0.66$

15. A sample consists of the following $n = 6$ scores: 6, 4, 5, 3, 4, 2

a. Compute the mean and standard deviation for the sample.

Mean = 4 Standard deviation = 1.41

b. Find the z-score for each score in the sample.

$X=6, z = +1.42$ $X=4, z = 0$ $X=5, z = +0.71$ $X=3, z = -.71$ $X=2, z = -1.41$

16. There are 60 black marbles, 20 blue marbles, and 14 red marbles in a jar.

a. What is the probability of selecting one red marble? 15%

b. What is the probability of selecting one black marble? 64%

c. What is the probability of selecting one blue marble? 21.3%

d. Which has the highest probability of being selected? The black marbles

e. Which has the lowest probability of being selected? The red marbles

17. (This question is independent of #16):

Probability values range from 0 (0%) to 1 (100%).

18. Please describe at least three characteristics of the NORMAL DISTRIBUTION

1. It is symmetrical in shape

2. All measures of central tendency are located dead center and all have the same value

3. There are more scores located near the mean (the center of the distribution...that "average" range) than there are scores as we move away from the mean

19. In a normal population of scores, what percent of people have “average” scores?

Answer 68.13%

20. In a normal distribution, what percentage of people would be located at or below 2 standard deviations higher than the mean?

Answer 47.72%

21. Answer the following questions based on a distribution with a $\mu = 55$ and $\sigma = 5$:

- a. What range of scores is considered “average”? 50 to 60
- b. What percentage of people has an average score? 68%
- c. What percentage of people has extremely high or extremely low scores? 2.28%
- d. What *range of scores* (requires numbers to be noted in the blank spaces) have the highest probability of being selected? 54 to 56