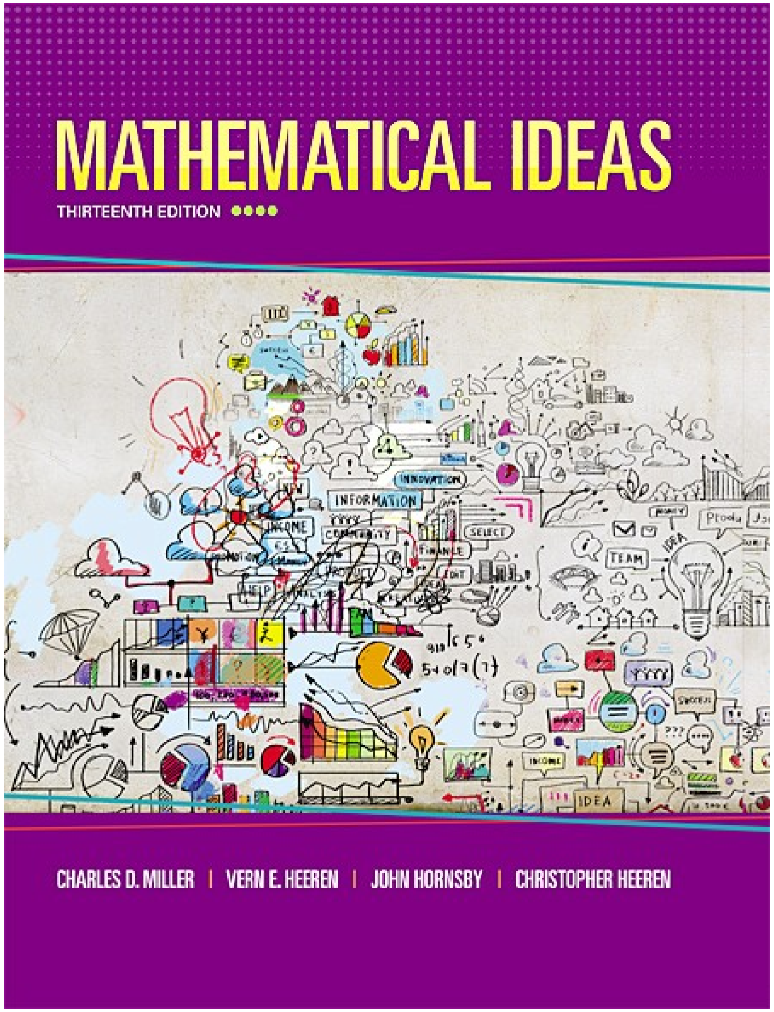
Chapter 7

The Basic Concepts of Algebra



Chapter 7: The Basic Concepts of Algebra

Section 7-5

Properties of Exponents and Scientific Notation

Properties of Exponents and Scientific Notation

- Identify the base and the exponent in an exponential expression, and evaluate exponential expressions with natural number exponents.
- Interpret the use of zero and negative integer exponents.
- Apply the product, quotient, power, and special rules for exponents.

Properties of Exponents and Scientific Notation

- Convert a number in standard notation to scientific notation and a number in scientific notation to standard notation.
- Compute with numbers in scientific notation.

Exponential Expression

If a is a real number and n is a natural number, then the exponential expression a^n is defined as

$$a^n = \underbrace{a \cdot a \cdot a \cdot \ldots \cdot a}_{n \text{ factors of } a}.$$

The number a is the *base* and n is the *exponent*.

Example: Evaluating Exponential Expressions

Evaluate each expression.

a) 5^2 b) $(–2)^4$ c) $–2^4$

Solution

a) $5^2 = 5 \cdot 5 = 25$

b) $(-2)^4 = (-2) \cdot (-2) \cdot (-2) \cdot (-2) = 16$

c) $-2^4 = -2 \cdot 2 \cdot 2 \cdot 2 = -16$

The Product Rule for Exponents

If *m* and *n* are natural numbers and *a* is any real number, then

$$a^m \cdot a^n = a^{m+n}.$$

Example: Applying the Product Rule

Apply the product rule in each case.

a) $5^2 \cdot 5^4$ b) $x^2 \cdot x^4 \cdot x^3$ c) $(4m^2n^3)(-2m^4n^5)$

Solution

a) $5^2 \cdot 5^4 = 5^{2+4} = 5^6$

b) $x^2 \cdot x^4 \cdot x^3 = x^9$

c) $(4m^2n^3)(-2m^4n^5) = -8m^6n^8$

Zero Exponent

If a is any nonzero real number, then

$$a^0 = 1.$$

Example: Zero Exponent

Evaluate each expression.

a) 5^0 b) $-x^0 \quad (x \neq 0)$ c) $3^0 + 11^0$

Solution

a) $5^0 = 1$

b) $-x^0 = -1$

c) $3^0 + 11^0 = 1 + 1 = 2$

Negative Exponent

For any natural number *n* and any nonzero real number *a*,

$$a^{-n} = \frac{1}{a^n}.$$

Example: Applying the Definition of Negative Exponents

Write the following expressions with only positive exponents and simplify.

a) 2^{-3} b) $(5z^2)^{-3}$ c) $5z^{-3}$

Solution

a) $\dfrac{1}{2^3} = \dfrac{1}{8}$ b) $\dfrac{1}{(5z^2)^3}$ c) $\dfrac{5}{z^3}$

Special Rules for Negative Exponents

If $a \neq 0$ and $b \neq 0$, then

$$\frac{1}{a^{-n}} = a^n \text{ and } \frac{a^{-n}}{b^{-m}} = \frac{b^m}{a^n}.$$

Example:

Evaluate each expression.

a) $\dfrac{1}{3^{-2}}$ b) $\dfrac{4^{-3}}{5^{-2}}$

Solution

a) $3^2 = 9$ b) $\dfrac{5^2}{4^3} = \dfrac{25}{64}$

The Quotient Rule for Exponents

If m and n are integers and a is any nonzero real number, then

$$\frac{a^m}{a^n} = a^{m-n}.$$

Example: Applying the Quotient Rule

Apply the quotient rule for exponents.

a) $\dfrac{3^8}{3^2}$ b) $\dfrac{x^5}{x^8}$ c) $\dfrac{z^{-3}}{z^{-7}}$

Solution

a) $3^{8-2} = 3^6$ b) $x^{-3} = \dfrac{1}{x^3}$ c) $z^{-3-(-7)} = z^4$

Power Rules for Exponents

If *a* and *b* are real numbers, and *m* and *n* are integers, then

$$\left(a^m\right)^n = a^{mn},$$

$$(ab)^m = a^m b^m,$$

and $$\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m} \left(b \neq 0\right).$$

Special Rules for Negative Exponents

If $a \neq 0$ and $b \neq 0$ and n is an integer, then

$$a^{-n} = \left(\frac{1}{a}\right)^{n} \quad \text{and} \quad \left(\frac{a}{b}\right)^{-n} = \left(\frac{b}{a}\right)^{n}.$$

Example: Applying Special Rules for Negative Exponents

Write the following expression with only positive exponents, and then evaluate.

$$\left(\frac{3}{5}\right)^{-3}$$

Solution

$$\left(\frac{3}{5}\right)^{-3} = \left(\frac{5}{3}\right)^{3} = \frac{125}{27}$$

Scientific Notation

A number is written in **scientific notation** when it is expressed in the form

$$a \times 10^n,$$

where $1 \le |a| < 10,$ and n is an integer.

Example: $8000 = 8 \cdot 1000 = 8 \times 10^3.$

Converting to Scientific Notation

Step 1 **Position the decimal point.** Place a caret, ^, to the right of the first nonzero digit, where the decimal point will be placed.

Step 2 **Determine the numeral for the exponent.** Count the number of digits from the decimal to the caret. This number gives the absolute value of the exponent on 10.

Converting to Scientific Notation

Step 3 **Determine the sign for the exponent.** Decide whether multiplying by 10^n should make the result of Step 1 larger or smaller. The exponent should be positive to make the result larger; it should be negative to make the result smaller.

Example: Converting to Scientific Notation

Convert each number from standard notation to scientific notation.

a) 4,500,000 b) 0.000034

Solution

a) $4_{\wedge}500{,}000 \quad = 4.5 \times 10^{6}$

b) $0.00003_{\wedge}4 \quad = 3.4 \times 10^{-5}$

Converting from Scientific Notation to Standard Notation

Multiplying a number by a positive power of 10 makes the number larger, so move the decimal point to the right if n is positive in 10^n.

Multiplying a number by a negative power of 10 makes the number smaller, so move the decimal point to the left if n is negative in 10^n.

If n is zero, leave the decimal point where it is.

Example: Converting from Scientific Notation

Convert each number from scientific notation to standard notation.

a) 1.97×10^{5} b) 3.8×10^{-3} c) -4.5×10^{0}

Solution

a) 197,000

b) 0.0038

c) –4.5

Example: Using Scientific Notation in Computation

Evaluate using scientific notation.

$$\frac{1,920,000 \times 0.0015}{0.000032 \times 45,000} = \frac{1.92 \times 10^{6} \times 1.5 \times 10^{-3}}{3.2 \times 10^{-5} \times 4.5 \times 10^{4}}$$

$$= \frac{1.92 \times 1.5 \times 10^{6} \times 10^{-3}}{3.2 \times 4.5 \times 10^{-5} \times 10^{4}}$$

$$= \frac{1.92 \times 1.5 \times 10^{3}}{3.2 \times 4.5 \times 10^{-1}}$$

$$= \frac{1.92 \times 1.5}{3.2 \times 4.5} \times 10^{4}$$

$$= 0.2 \times 10^{4}$$

$$= 2.0 \times 10^{-1} \times 10^{4}$$

$$= 2.0 \times 10^{3} \text{ or } 2000$$