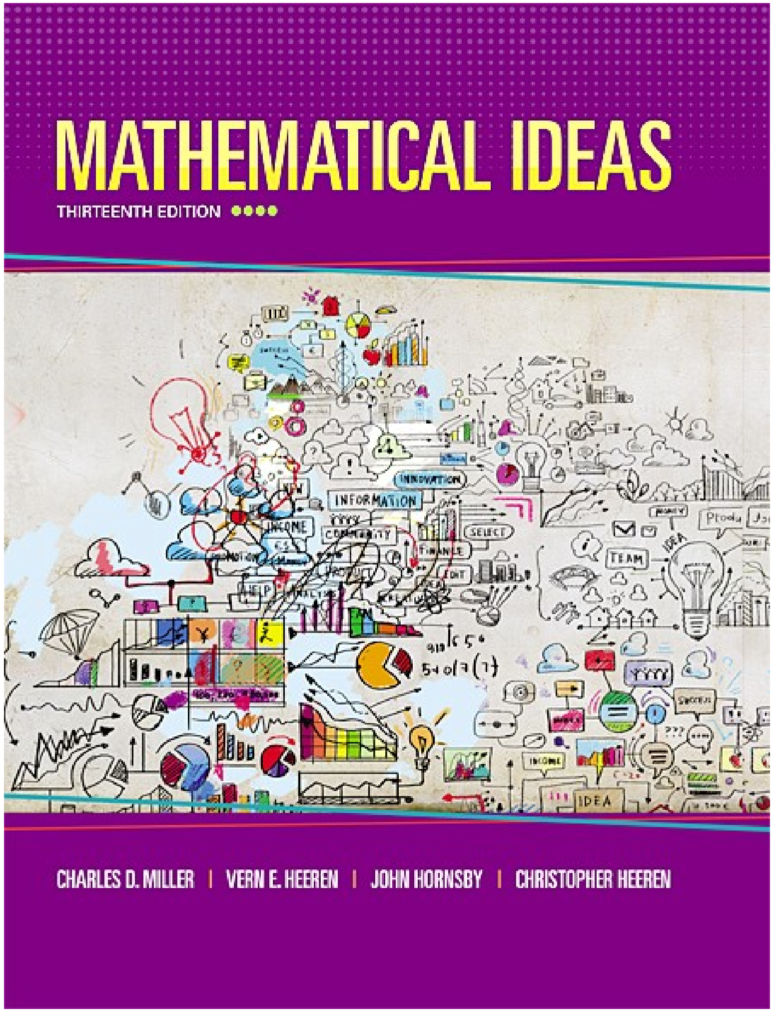
Chapter 8

Graphs, Functions and Systems of Equations and Inequalities



Chapter 8: Graphs, Functions, and Systems of Equations and Inequalities

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8.8 Applications of Linear Systems

8.9 Linear Inequalities, Systems, and Linear Programming

Section 8-1

The Rectangular Coordinate System and Circles

The Rectangular Coordinate System and Circles

- Plot ordered pairs in a rectangular coordinate system.
- Find the distance between two points using the distance formula.
- Find the midpoint of a segment using the midpoint formula.
- Find the equation of a circle given the coordinates of the center and the radius.

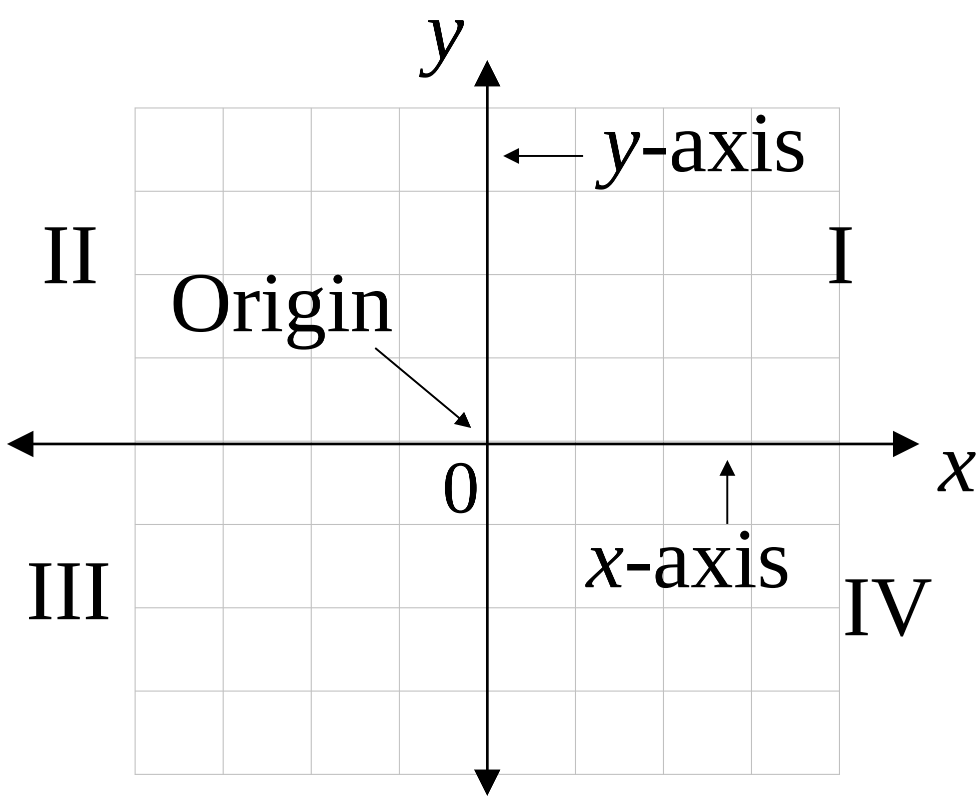
The Rectangular Coordinate System and Circles

- Find the center and radius of a circle given its equation.
- Apply the midpoint formula and the definition of a circle.

Rectangular Coordinates

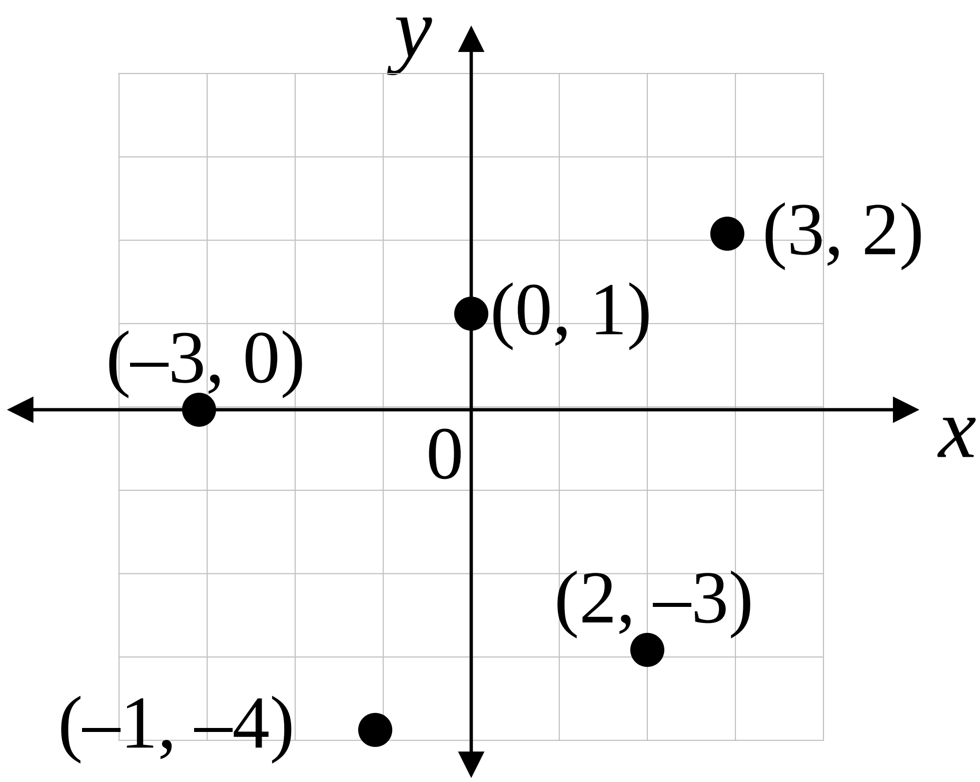
(3, 2) is an example of an **ordered pair** – a pair of numbers written within parentheses in which the order of the numbers is important. The two numbers are the **components** of the ordered pair. An ordered pair is graphed using two number lines that intersect at right angles at the zero points as shown on the next slide.

Rectangular (Cartesian) Coordinate System



The **quadrants** are numbered I, II, III, IV.

Plot Points



Distance Formula

The distance between the points (x_1, y_1) and (x_2, y_2) is

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}.$$

This result is called the **distance formula**.

Example: Finding the Distance between Two Points

Find the distance between (1, –2) and (5, 7).

Solution

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$d = \sqrt{(5 - 1)^2 + (7 - (-2))^2}$$

$$d = \sqrt{4^2 + 9^2} = \sqrt{16 + 81} = \sqrt{97}$$

Midpoint

The **midpoint** of a line segment is the point on the segment that is equidistant from both endpoints.

Midpoint Formula

The coordinates of the midpoint of the segment with endpoints (x_1, y_1) and (x_2, y_2) are

$$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right).$$

Example: Finding the Midpoint of a Segment

Find the coordinates of the midpoint of the line segment with endpoints (1, –2) and (5, 7).

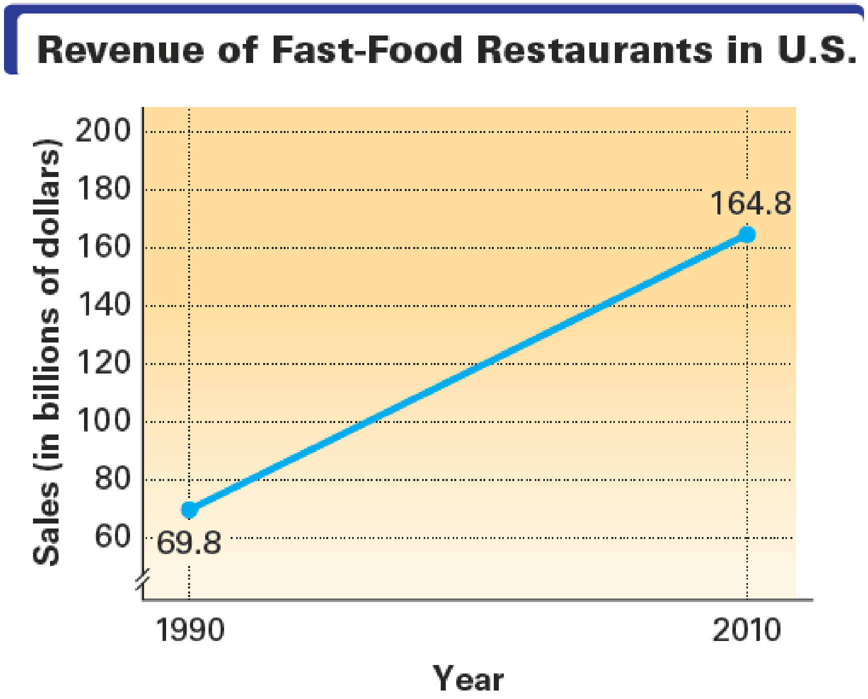
Solution

$$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

$$\left(\frac{1+5}{2}, \frac{-2+7}{2}\right) = \left(3, \frac{5}{2}\right)$$

Example: Applying the Midpoint Formula to Data

The figure shows how a graph can indicate the increase in the revenue generated by fast-food restaurants in the United States from $69.8 billion in 1990 to $164.8 billion in 2010. Use the midpoint formula and the two given points to estimate the revenue from fast-food restaurants in 2000, and compare it to the actual figure of $107.1 billion.



Source: National Restaurant Association

Example: Applying the Midpoint Formula to Data

Solution

The year 2000 lies halfway between 1990 and 2010, so we must find the coordinates of the midpoint *M* of the segment that has endpoints

$$(1990, 69.8) \text{ and } (2010, 164.8)$$

$$M = \left(\frac{1990 + 2010}{2}, \frac{69.8 + 164.8}{2} \right) = (2000, 117.3)$$

Our estimate is $117.3 billion, which is greater than the actual figure of $107.1 billion.

Circles

A **circle** is the set of all points in a plane that lie a fixed distance from a fixed point. The fixed point is called the **center** and the fixed distance is called the **radius**.

Equation of a Circle

The equation of a circle of radius r with center (h, k) is

$$(x - h)^2 + (y - k)^2 = r^2.$$

In particular, a circle of radius r with center at the origin has equation

$$x^2 + y^2 = r^2.$$

Example: Finding an Equation of a Circle

Find an equation of the circle with center at (2, –5) and radius 4.

Solution

$$(x - h)^2 + (y - k)^2 = r^2$$

$$(x - 2)^2 + (y - (-5))^2 = 4^2$$

$$(x - 2)^2 + (y + 5)^2 = 16$$

Example: Completing the Square and Graphing a Circle

Graph $x^2 + y^2 + 2x - 4y - 4 = 0.$

Solution

$$(x^2 + 2x \quad\;) + (y^2 - 4y \quad\;) = 4$$

$$(x^2 + 2x + 1) + (y^2 - 4y + 4) = 4 + 1 + 4$$

$$(x + 1)^2 + (y - 2)^2 = 9$$

The circle has center (–1, 2) and radius 3. It is graphed on the next slide.

Example: Completing the Square and Graphing a Circle