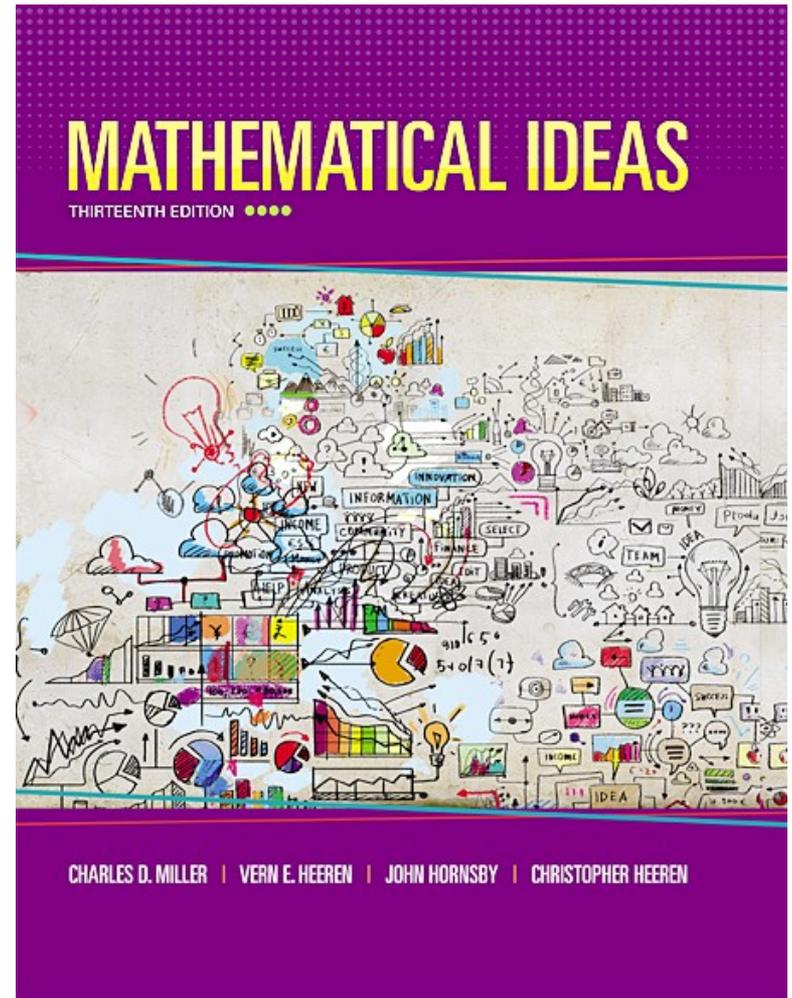


Chapter 8

Graphs, Functions and Systems of Equations and Inequalities



Chapter 8: Graphs, Functions, and Systems of Equations and Inequalities

8.1 The Rectangular Coordinate System and Circles

8.2 Lines, Slope, and Average Rate of Change

8.3 Equations of Lines

8.4 Linear Functions, Graphs, and Models

8.5 Quadratic Functions, Graphs and Models

8.6 Exponential and Logarithmic Functions, Graphs, and Models

8.7 Systems of Linear Equations

Chapter 8: Graphs, Functions, and Systems of Equations and Inequalities

8.8 Applications of Linear Systems

8.9 Linear Inequalities, Systems, and Linear Programming

Section 8-3

Equations of Lines

Equations of Lines

- Find an equation of a line using the point-slope form.
- Find an equation of a line using the slope-intercept form.
- Use the slope-intercept form to sketch the graph of a line from the equation.
- Find an equation of a line parallel or perpendicular to a given line through a given point.

Equations of Lines

- Know how the formulas for the Fahrenheit and Celsius temperature scales are derived.

Point-Slope Form

The equation of the line through (x_1, y_1) with slope m is written in **point-slope form** as

$$y - y_1 = m(x - x_1).$$

Example: Finding an Equation Given the Slope and a Point

Find the standard form of an equation of the line with slope $\frac{1}{3}$, passing through the point $(-3, 2)$.

Solution

$$y - y_1 = m(x - x_1)$$

$$y - 2 = \frac{1}{3}(x - (-3))$$

$$3y - 6 = x + 3$$

Multiply by 3

$$x - 3y = -9$$

Standard form

Example: Finding an Equation Given Two Points

Find the standard form of an equation of the line passing through the points $(2, 1)$ and $(-1, 3)$.

Solution

$$m = \frac{3 - 1}{-1 - 2} = -\frac{2}{3}$$

Find the slope.

$$y - 1 = -\frac{2}{3}(x - 2)$$

Use either point in the form.

$$2x + 3y = 7$$

Standard form

Slope-Intercept Form

The equation of the line with slope m and y -intercept $(0, b)$ is written in **slope-intercept form** as

$$y = mx + b.$$

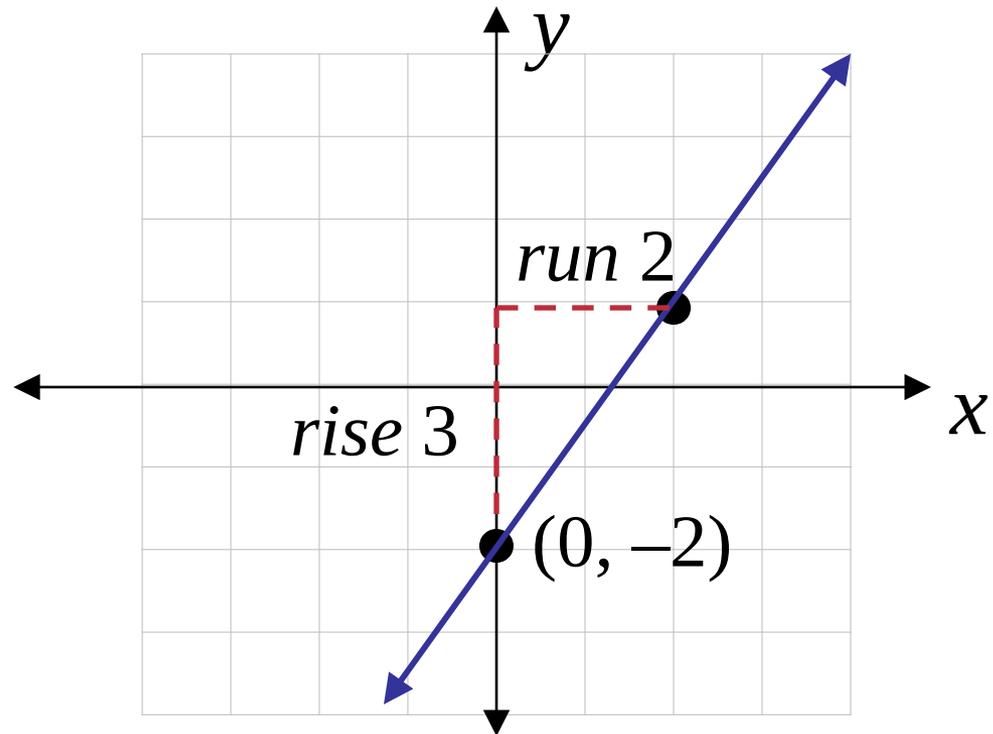
↑ ↑
Slope y -intercept

Example: Graphing a Line Using Slope and the y -Intercept

Graph the line with equation $y = \frac{3}{2}x - 2$.

Solution

Plot the intercept $(0, -2)$ and use the slope: *rise 3*, *run 2*.



Summary of Forms of Linear Equations

$$Ax + By = C$$

Standard form

$$x = a$$

Vertical line

$$y = b$$

Horizontal line

$$y = mx + b$$

Slope-intercept form

$$y - y_1 = m(x - x_1)$$

Point-Slope form

Example: Using the Slope Relationship (Perpendicular Lines)

Find the slope-intercept form of the equation of the line perpendicular to the graph of $2x + 3y = 6$, passing through the point $(-4, 5)$.

Solution

A line perpendicular to this line must have a slope that is the negative reciprocal of $-\frac{2}{3}$, which is $\frac{3}{2}$.

Example: Using the Slope Relationship (Perpendicular Lines)

Use the point $(-4, 5)$ and the slope $\frac{3}{2}$.

$$y - 5 = \frac{3}{2} [x - (-4)]$$

$$y - 5 = \frac{3}{2} (x + 4)$$

$$y - 5 = \frac{3}{2} x + 6$$

$$y = \frac{3}{2} x + 11$$

Example: Deriving the Formula for Converting Celsius to Fahrenheit

In the Fahrenheit Scale, water freezes at 32 degrees and boils at 212 degrees. In the Celsius scale, water freezes at 0 degrees and boils at 100 degrees. Use this information to derive the formula for converting Celsius to Fahrenheit.

Solution

Let x represent Celsius and y represent Fahrenheit. The two ordered pairs are:

$(0, 32)$ and $(100, 212)$

Example: Deriving the Formula for Converting Celsius to Fahrenheit

Find the equation of the line containing these points by first finding the slope.

$$m = \frac{212 - 32}{100 - 0}$$

$$m = \frac{180}{100}$$

$$m = \frac{9}{5}$$

Example: Deriving the Formula for Converting Celsius to Fahrenheit

Now, substitute the y -intercept into the slope-intercept form to find the equation of the line.

$$y = mx + b$$

$$y = \frac{9}{5}x + 32$$

Replace x with C and y with F to represent the two scales.

$$F = \frac{9}{5}C + 32$$