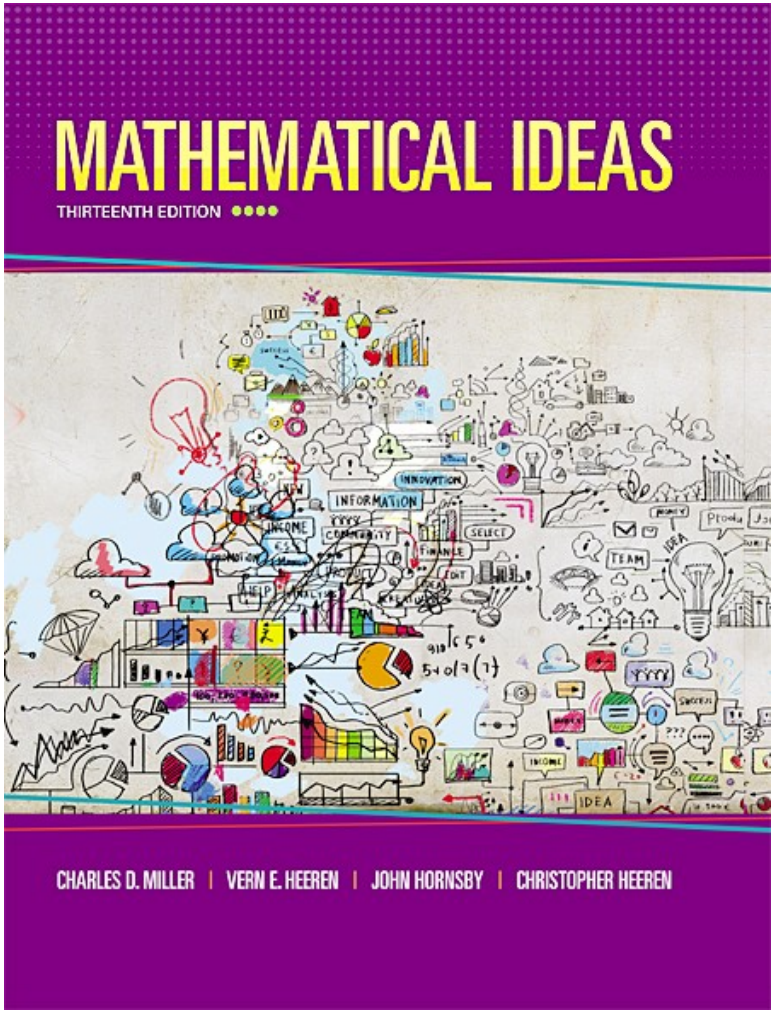
Chapter 8

Graphs, Functions and Systems of Equations and Inequalities



Chapter 8: Graphs, Functions, and Systems of Equations and Inequalities

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Section 8-2

Line, Slope, and Average Rate of Change

Line, Slope, and Average Rate of Change

- Determine ordered pairs and graph a linear equation in two variables.
- Find the x-and y-intercepts of the graph of a linear equation.
- Find the slope of a nonvertical line given two points on the line.
- Graph a line given its slope and a point on the line.

Line, Slope, and Average Rate of Change

- Determine whether two lines are either parallel or perpendicular to each other by comparing their slopes.
- Interpret slope as average rate of change.

Linear Equation in Two Variables

An equation that can be written in the form

$$\boldsymbol{Ax + By = C} \text{ (where } A \text{ and } B \text{ are not both 0)}$$

is a **linear equation in two variables**. This form is called **standard form**.

Linear Equation in Two Variables

A linear equation in two variables will have solutions written as ordered pairs. In general, these equations will have an infinite number of solutions.

All first-degree equations with two variables have straight-line graphs.

Intercepts

The ***x*-intercept** is the point (if any) where the line crosses the x-axis, and the ***y*-intercept** is the point (if any) where the line crosses the y-axis.

To find the x-intercept of the graph of a linear equation, let $y = 0$ and solve for x.

To find the y-intercept, let $x = 0$ and solve for y.

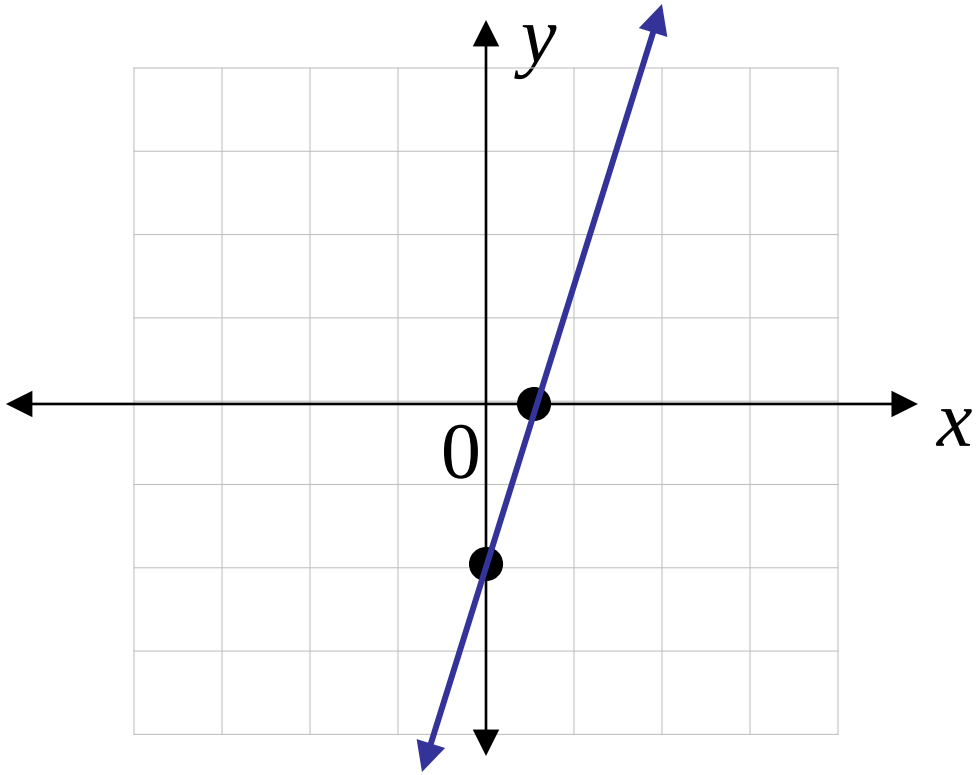
Example: Graphing an Equation Using Intercepts

Find the x- and y-intercepts of $4x - y = 2$ and graph the equation.

Solution

$(0, -2)$

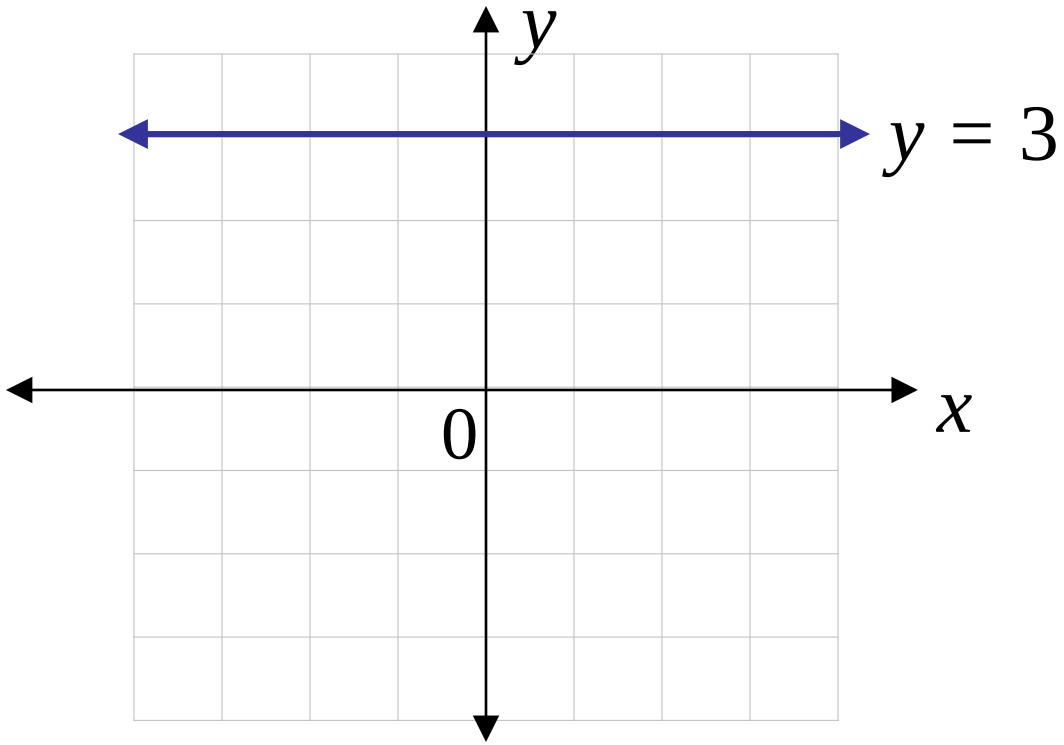
$\left(\frac{1}{2}, 0\right)$



Example: Graphing with a Single Intercept

Graph $y = 3$.

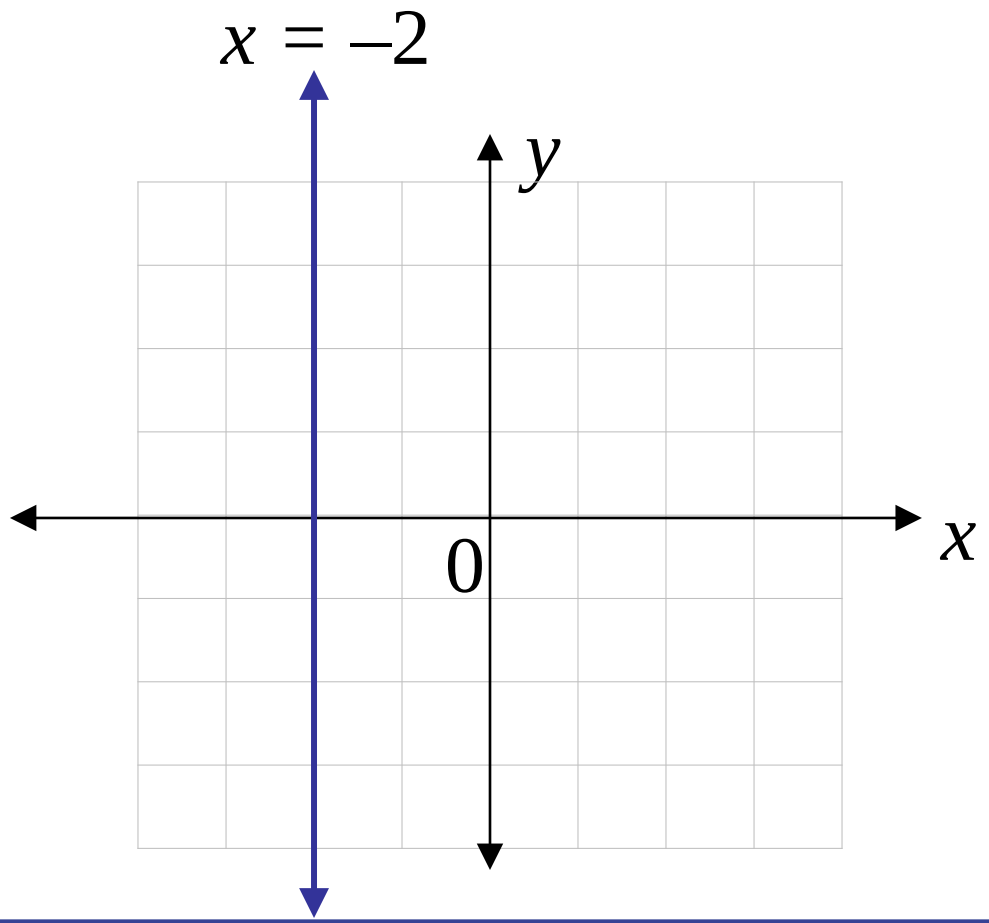
Solution



Example: Graphing with a Single Intercept

Graph $x = -2$.

Solution



Slope

The measure of the steepness of a line is called the *slope* of the line. One way to get a measure of the steepness of a line is to compare the vertical change in the line (the *rise*) to the horizontal change (the *run*) while moving from one fixed point to another.

Slope

If $x_1 \neq x_2$, the slope of the line through the distinct points (x_1, y_1) and (x_2, y_2) is

$$m = \frac{\text{rise}}{\text{run}} = \frac{\text{change in } y}{\text{change in } x} = \frac{y_2 - y_1}{x_2 - x_1}.$$

Example: Using the Slope Formula

Find the slope of the line that passes through the points (3, 5) and (–2, 1).

Solution

$$\frac{y_2 - y_1}{x_2 - x_1} = \frac{1 - 5}{-2 - 3}$$

$$= \frac{4}{5}$$

Vertical and Horizontal Lines

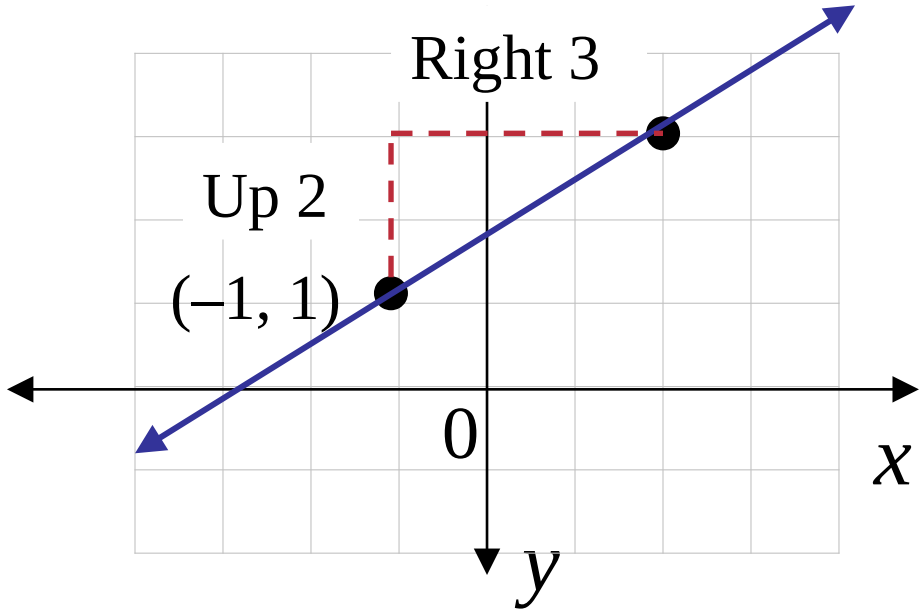
A vertical line has an equation of the form $x = a$, where a is a real number, and its slope is undefined.

A horizontal line has an equation of the form $y = b$, where b is a real number, and its slope is 0.

Example: Graph a Line Using Slope and a Point

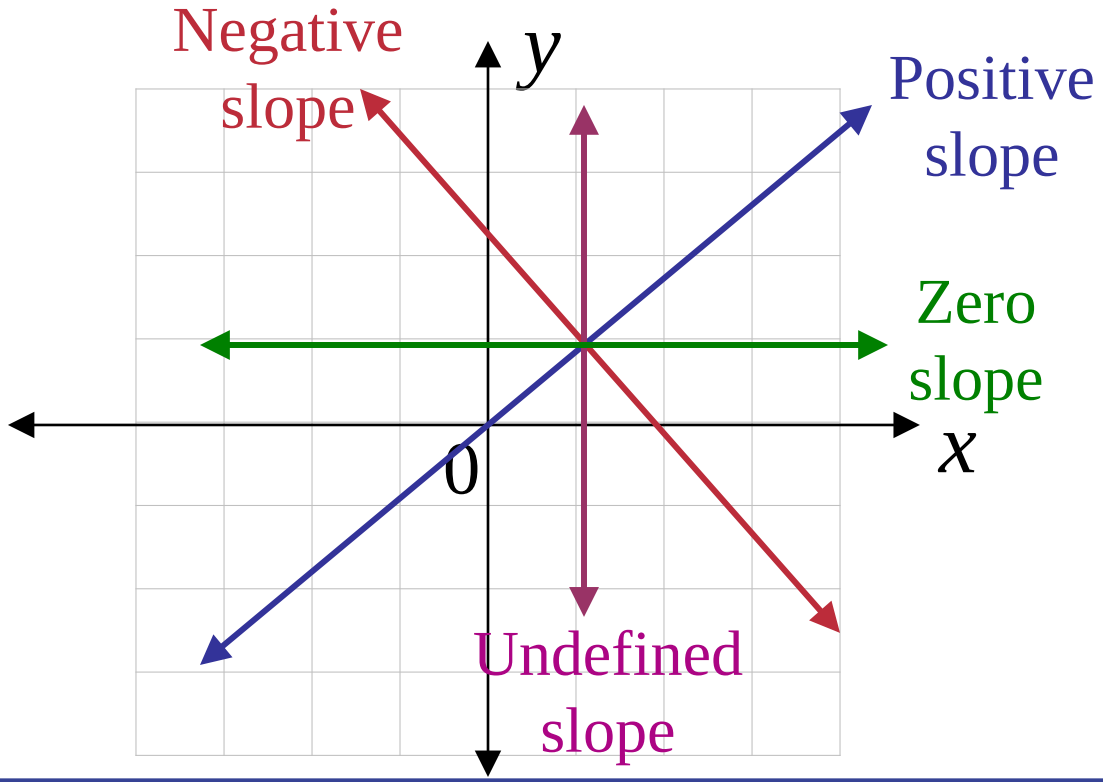
Graph the line that has slope $\frac{2}{3}$ and passes through (–1, 1).

Solution



Positive and Negative Slopes

A line with a positive slope rises from left to right, while a line with a negative slope falls from left to right.



Parallel and Perpendicular Lines

Two nonvertical lines with the same slope are parallel. Two nonvertical parallel lines have the same slope. Furthermore, any two vertical lines are parallel.

If neither is vertical, two perpendicular lines have slopes that are negative reciprocals – that is, their product is –1. Two lines with slopes that are negative reciprocals are perpendicular. Every vertical line is perpendicular to every horizontal line.

Parallel and Perpendicular Lines

Given lines L_1 with slope 2/3, L_2 with slope 2/3, and L_3 with slope –3/2 we have the following:

L_1 and L_2 are parallel,

and

L_3 is perpendicular to both L_1 and L_2.

Average Rate of Change

The slope of a line is the ratio of the change in *y* (vertical change) to the change in *x* (horizontal change). This idea can be extended to real-life situations as follows: the slope gives an average rate of change of *y* per unit change in *x*, where the value of *y is dependent on the value of x*.

Example: Finding the Average Rate of Change

If a computer was worth $2000 in the year 2004 and only $800 in 2007, find the average rate of change in the value of the computer over the 3 years.

Solution

Determine the slope of the line segment between the points (2004, 2000) and (2007, 800).

$$\frac{y_2 - y_1}{x_2 - x_1} = \frac{2000 - 800}{2004 - 2007} = -400$$

The value *dropped* by about $400 per year.