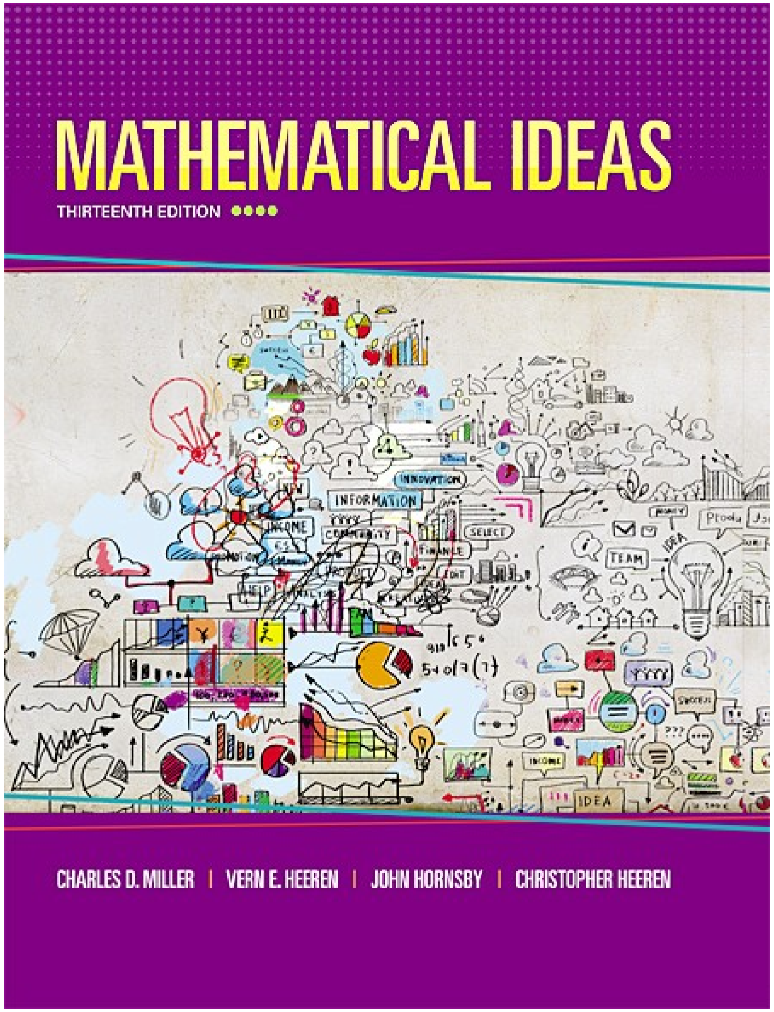
Chapter 8

Graphs, Functions and Systems of Equations and Inequalities



Chapter 8: Graphs, Functions, and Systems of Equations and Inequalities

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8.8 Applications of Linear Systems

8.9 Linear Inequalities, Systems, and Linear Programming

Section 8-7

Systems of Linear Equations

Systems of Linear Equations

- Know the meaning of *systems of linear equations*.
- Identify the solution of a system of linear equations from a graph, and classify the system.
- Solve a system of two linear equations in two variables using the elimination method.
- Solve a system of two linear equations in two variables using the substitution method.

Linear System in Two Variables

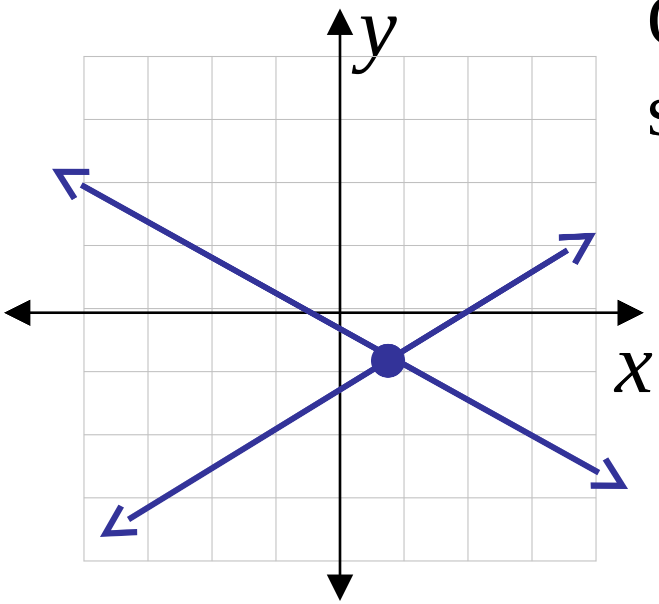
When multiple equations are considered together the set of equations is called a **system of equations**. For example

$$y = 3x + 4$$
$$y = -2x + 2$$

The point where the graphs intersect is a solution of each of the individual equations. It is also the solution of the system of equations.

Graphs of a Linear System in Two Variables (Three Possibilities)

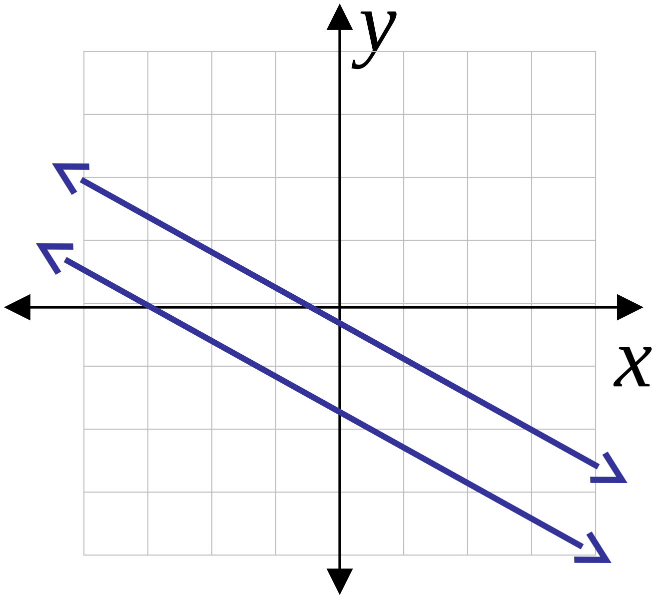
1. The two graphs intersect in a single point. The system is **consistent** and the equations are **independent.**



One solution

Graphs of a Linear System in Two Variables (Three Possibilities)

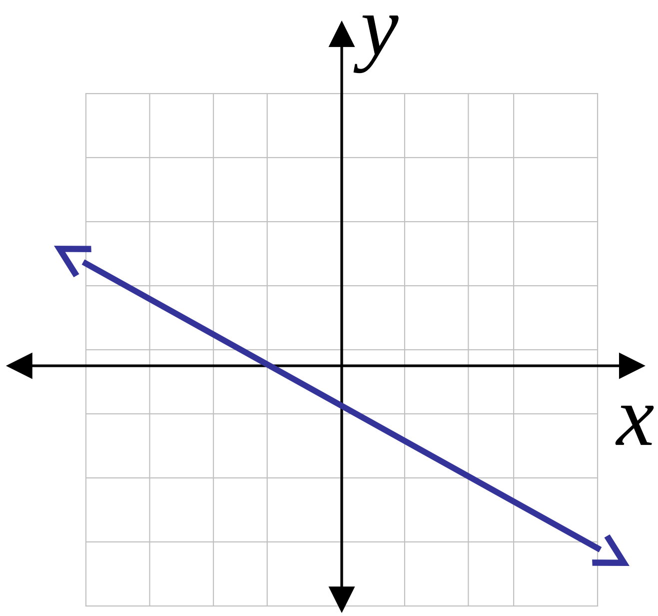
2. The graphs are parallel lines. In this case, the system is **inconsistent** and the equations are **independent**. There is no solution common to both equations.



No solution

Graphs of a Linear System in Two Variables (Three Possibilities)

3. The graphs are the same line. The system is **consistent** and the equations are **dependent**, because any solution of one equation is also a solution of the other. The solution set is an infinite set of ordered pairs.



Infinite number of solutions

Elimination Method

We can use algebraic methods to solve systems. One method is called the **elimination method**. The elimination method involves combining the two equations of the system so that one variable is eliminated. We use the fact that

If $a = b$ and $c = d$ then $a + c = b + d$.

Solving Linear Systems by Elimination

Step 1 Write both equations in standard form $Ax + By = C$.

Step 2 Make the coefficients of one pair of variable terms opposites. Multiply one or both equations by appropriate numbers so that the sum of the coefficients of either x or y is zero.

Step 3 Add the new equations to eliminate a variable. The sum should be an equation with just one variable.

Solving Linear Systems by Elimination

Step 4 Solve the equation from Step 3.

Step 5 Find the other value. Substitute the result of Step 4 into either of the given equations and solve for the other variable.

Step 6 Find the solution set. Check the solution in both of the given equations. Then write the solution set.

Example: Solving a System by Elimination

Solve the system.

$$3x - 2y = 4 \quad (1)$$
$$2x + y = 5 \quad (2)$$

Solution

Step 1 Both equations are in standard form.

Step 2 Multiply equation (2) by 2 to get opposite coefficients on *y*.

$$3x - 2y = 4$$
$$4x + 2y = 10$$

Example: Solving a System by Elimination

Solution (continued)

Step 3 Add the equations to eliminate *y*.

$$7x = 14$$

Step 4 Solve to find $x = 2$.

Step 5 Substitute in one of the original equations to find *y*.

$$2(2) + y = 5$$
$$y = 1$$

Step 6 The solution (2, 1) checks in both original equations.

Substitution Method

Linear systems can also be solved by the **substitution method**. This method is useful for solving linear systems in which one variable has coefficient 1 or –1. It is also the best choice for solving many *nonlinear* systems in advanced algebra courses.

Solving Linear Systems by Substitution

Step 1 **Solve for one variable in terms of the other.** Solve one of the equations for either variable.

Step 2 **Substitute** for that variable in the other equation. The result should be an equation with just one variable.

Step 3 **Solve** the equation from Step 2.

Solving Linear Systems by Substitution

Step 4 **Find the other value.** Substitute the result of Step 3 into the equation from Step 1 to find the value of the other variable.

Step 5 **Find the solution set.** Check the solution in both of the given equations. Then write the solution set.

Example: Solving a System by Substitution

Solve the system. $3x - 2y = 4$ (1)

$2x + y = 5$ (2)

Solution

Step 1 Solve equation (2) for y.

$$y = -2x + 5$$

Step 2 Substitute $-2x + 5$ for y in equation (1).

$$3x - 2(-2x + 5) = 4$$

Example: Solving a System by Substitution

Solution (continued)

Step 3 Solve the equation

$$3x + 4x - 10 = 4$$

$$7x = 14$$

$$x = 2$$

Step 4 Substitute to find *y*.

$$y = -2(2) + 5 = 1$$

Step 5 The solution (2, 1) checks in both original equations.

Special Cases – Solving a System

Solving the system

$$3x - 2y = 4$$
$$-6x + 4y = 7$$

leads to a false statement such as 0 = 15. This indicates that the two equations have no solutions in common. The system is inconsistent with the empty set as the solution set.

Special Cases – Solving a System

Solving the system $\begin{aligned} 3x - 2y &= 4 \\ -6x + 4y &= -8 \end{aligned}$

leads to a true statement such as 0 = 0. This indicates that the two equations are dependent. Choose one equation, solve for *x* and the solution set can be written as

$$\left\{\left(\frac{2y+4}{3}, y\right)\right\}.$$