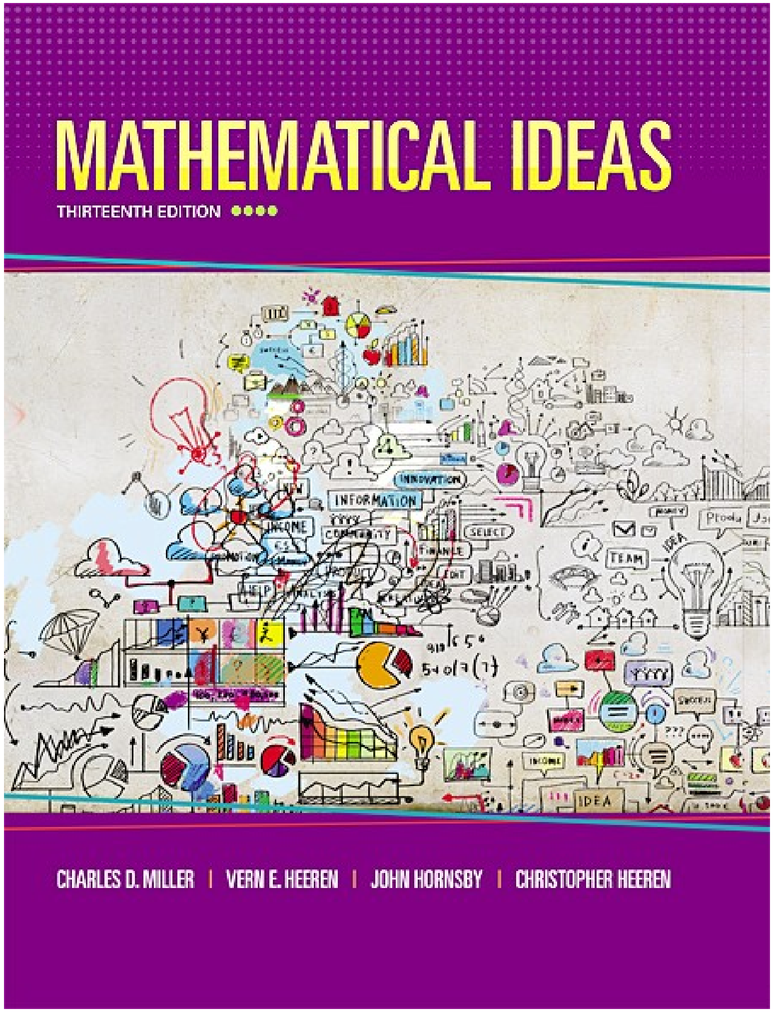
Chapter 7

The Basic Concepts of Algebra



Chapter 7: The Basic Concepts of Algebra

Section 7-6

Polynomials and Factoring

Polynomials and Factoring

- Know the basic terminology of polynomials.
- Add and subtract polynomials.
- Multiply polynomials.
- Multiply two binomials using the FOIL method.
- Find special products of binomials.
- Factor the greatest common factor from a polynomial.

Polynomials and Factoring

- Factor a trinomial using the FOIL method in reverse.
- Factor a perfect square trinomial and a difference of squares.

Basic Terminology

A **term**, or **monomial**, is defined to be a number, a variable, or a product of numbers and variables. A **polynomial** is a term or a finite sum or difference of terms, with only nonnegative integer exponents permitted on the variables. If the terms of a polynomial contain only the variable *x*, then the polynomial is called a **polynomial in *x***.

Examples include $3x^5 + 4x - 1$, 7, and $8r$.

Basic Terminology

The greatest exponent in a polynomial in one variable is the **degree** of the polynomial. A nonzero constant is said to have degree 0. (The polynomial 0 has no degree.)

A term containing more than one variable has degree equal to the sum of all the exponents appearing on the variables in the term. For example, $4x^2y^3z$ has degree 6.

Basic Terminology

A polynomial containing exactly three terms is called a **trinomial** and one containing exactly two terms is a **binomial**.

Addition and Subtraction

All the properties of the real numbers hold for polynomials.

Like terms are terms that have the exact same variable factors. Polynomials are added by adding coefficients of like terms; polynomials are subtracted by subtracting coefficients of like terms.

Example: Adding and Subtracting Polynomials

Add or subtract as indicated.

a) $(2x^3 + 3x^2 - 7x + 1) + (6x^3 - x^2 + 2x + 5)$

b) $(4m^4 - m^2 - 7m + 9) - (3m^4 - m^2 + m - 3)$

Solution

a) $8x^3 + 2x^2 - 5x + 6$

b) $m^4 - 8m + 12$

Multiplication of Polynomials

The associative and distributive properties, together with the properties of exponents, can also be used to find the product of two polynomials.

Example: Multiplication of Polynomials

Multiply $(3x^2 + x - 2)(5x^3 + x + 2)$.

Solution

$$
\begin{array}{r}
5x^3 + x + 2 \\
3x^2 + x - 2 \\
\hline
-10x^3 \qquad - 2x - 4 \\
5x^4 \qquad + x^2 + 2x \\
15x^5 \qquad + 3x^3 + 6x^2 \\
\hline
15x^5 + 5x^4 - 7x^3 + 7x^2 - 4
\end{array}
$$

FOIL Method

The FOIL method is a convenient way to find the product of two binomials. The memory aid FOIL (for First, Outside, Inside, Last) gives the pairs of terms to be multiplied to get the product. This is demonstrated on the next slide.

Example: FOIL Method

Multiply $(3m - 2)(5m + 7)$.

Solution

F O I L

$$=(3m)(5m)+(3m)(7)+(-2)(5m)+(-2)(7)$$

$$=15m^2+21m-10m-14$$

$$=15m^2+11m-14$$

Special Products - the Sum and Difference of Two Terms

$$(x+y)(x-y)=x^2-y^2$$

The product $x^2 - y^2$ is called the **difference of two squares.**

Example: Multiplying (*x* + *y*)(*x* – *y*)

Multiply $(4p - 9)(4p + 9)$.

Solution

$$= (4p)^2 - 9^2$$

$$= 16p^2 - 81$$

Special Products – Squares of Binomials

$$(x+y)^2 = x^2 + 2xy + y^2$$

$$(x-y)^2 = x^2 - 2xy + y^2$$

Example: Squaring Binomials

Multiply $(2x+5)^2$

Solution

$$=(2x)^2+2(2x)(5)+(5)^2$$

$$=4x^2+20x+25$$

Example: Squaring Binomials

Multiply $\left(2x+y^2\right)^2$.

Solution

$$=\left(2x\right)^2+2(2x)(y^2)+\left(y^2\right)^2$$

$$=4x^2+4xy^2+y^4$$

Factoring

The process of finding polynomials whose product equals a given polynomial is called **factoring**. For example, since

$$3x + 9 = 3(x + 3),$$

both 3 and $x + 3$ are called **factors** of $3x + 9$. $3(x + 3)$ is called a **factored form** of $3x + 9$.

Factoring

A polynomial that cannot be written as a product of two polynomials with integer coefficients is a **prime polynomial**. A polynomial is **factored completely** when it is written as a product of prime polynomials with integer coefficients.

Factoring Out the Greatest Common Factor

Some polynomials are factored using the distributive property. We look for a monomial that is the greatest common factor (GCF) of all the terms of the polynomial. This technique is demonstrated on the next slide.

Example: Greatest Common Factor

Factor out the greatest common factor from each polynomial.

a) $8x^3 + 12x^2$

b) $m^3(m+2) - 3m^2(m+2) + m(m+2)$

Solution

a) $4x^2 \cdot 2x + 4x^2 \cdot 3 = 4x^2(2x+3)$

b) $m(m+2)(m^2 - 3m + 1)$

Factoring Trinomials

When a polynomial has more than three terms, it can sometimes be factored by a method called **factoring by grouping**. This technique is demonstrated on the next slide.

Example: Factoring by Grouping

Factor by grouping.

$$mp^2 + 4m + 3p^2 + 12$$

Solution

$$\underbrace{mp^2 + 4m}_{\text{factor of } m} + \underbrace{3p^2 + 12}_{\text{factor of } 3}$$

$$m(p^2 + 4) + 3(p^2 + 4)$$

$$(p^2 + 4)(m + 3)$$

Factoring Trinomials

Factoring is the inverse of multiplying. Since the product of two binomials is usually a trinomial, we can expect factorable trinomials (with GCF of the terms equal to 1) to have two binomial factors. Thus, factoring trinomials requires using FOIL in an inverse manner.

Example: Factoring Trinomials

Factor $6x^2 - 7x - 5$.

Solution

To factor this, we must find integers *a*, *b*, *c*, and *d* such that

$$6x^2 - 7x - 5 = (ax + b)(cx + d)$$

After a few attempts, using FOIL to check, we find

$$6x^2 - 7x - 5 = (3x - 5)(2x + 1).$$

Perfect Square Trinomials

$$x^2 + 2xy + y^2 = (x + y)^2$$

$$x^2 - 2xy + y^2 = (x - y)^2$$

Example: Perfect Square Trinomial

Factor $169x^2 + 104x + 16$

Solution

$$= (13x)^2 + 2(13x)(4) + (4)^2$$

$$= (13x + 4)^2$$

Example: Perfect Square Trinomial

Factor $16x^2 + 24xy + 9y^2$.

Solution

$$=(4x)^2 + 2(4x)(3y) + (3y)^2$$

$$=(4x + 3y)^2$$

Factoring Special Binomials – Difference of Squares

$$x^2 - y^2 = (x + y)(x - y)$$

Example: Difference of Squares

Factor $25k^2 - 4m^2$.

Solution

$$=(5k)^2 - (2m)^2$$

$$=(5k+2m)(5k-2m)$$

Example: Difference of Squares

Factor $16x^4 - 1.$

Solution

$$=\left(4x^2+1\right)\left(4x^2-1\right)$$

$$=\left(4x^2+1\right)\left(2x-1\right)\left(2x+1\right)$$