

The Basic Concepts of Algebra



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Chapter 7: The Basic Concepts of Algebra

- 7.1 Linear Equations
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Section 7-1

Linear Equations

Linear Equations

- Solve a linear equation.
- Identify a linear equation as a conditional equation, an identity, or a contradiction.
- Solve a literal equation or a formula for a specified variable.
- Use a linear model in an application.

Algebraic Expression

An **algebraic expression** involves only the basic operations of addition, subtraction, multiplication, or division (except by 0), or raising to powers or taking roots on any collection of variables and numbers.

$$4x - 7, \quad \sqrt{m} + 3, \quad \text{and} \quad \frac{xy^3z}{k}$$

Algebraic expressions

Equation

An **equation** is a statement that two algebraic expressions are equal. A *linear equation in one variable* involves only real numbers and one variable.

$$4x - 7 = 8 \quad \text{and} \quad 5k + 3 = k + 1$$

Linear equations

Linear Equation in One Variable

An equation in the variable x is a **linear equation** if it can be written in the form

$$Ax + B = C$$

where A , B , and C are real numbers, with $A \neq 0$.

Terminology

A linear equation in one variable is also called a **first-degree equation**.

If the variable in an equation is replaced by a real number that makes the statement of the equation true, then that number is a **solution** of the equation. An equation is **solved** by finding its **solution set**, the set of all answers.

Equivalent equations are equations with the same solution set.

Addition Property of Equality

For all real numbers A , B , and C , the equations

$$A = B \quad \text{and} \quad A + C = B + C$$

are equivalent. (The same number may be added to both sides of an equation without changing the solution set.)

Multiplication Property of Equality

For all real numbers A , B , and C , where $C \neq 0$, the equations

$$A = B \quad \text{and} \quad AC = BC$$

are equivalent. (Both sides of an equation may be multiplied by the same nonzero number without changing the solution set.)

Solving a Linear Equation in One Variable

- Step 1 Clear fractions.** Eliminate any fractions by multiplying both sides of the equation by a common denominator.
- Step 2 Simplify each side separately.** Use the distributive property to clear parentheses, and combine like terms as needed.
- Step 3 Isolate the variable terms on one side.** Use the addition property of equality.

Solving a Linear Equation in One Variable

Step 4 Transform so that the coefficient of the variable is 1. Use the multiplication property of equality.

Step 5 **Check.** Substitute the solution into the original equation.

Example: Using the Distributive Property to Solve a Linear Equation

Solve $2(x - 5) + 3x = x + 6$.

Solution

$$2x - 10 + 3x = x + 6$$

Distributive property.

$$5x - 10 = x + 6$$

Combine like terms.

$$5x - 10 + 10 = x + 6 + 10$$

Add 10.

$$5x = x + 16$$

Combine like terms.

Example: Using the Distributive Property to Solve a Linear Equation

Solution (continued)

$$5x = x + 16$$

$$5x - x = x + 16 - x \quad \text{Subtract } x.$$

$$4x = 16$$

Combine like terms.

$$\frac{4x}{4} = \frac{16}{4}$$

Divide by 4.

$$x = 4$$

Simplify.

Check that the solution set is $\{4\}$ by substituting 4 for x in the original equation.

Example: Solving a Linear Equation with Fractions

Solve $\frac{x+7}{6} + \frac{2x-8}{2} = -4$.

Solution

$$6\left(\frac{x+7}{6} + \frac{2x-8}{2}\right) = 6(-4)$$

Multiply by LCD, 6.

$$6\left(\frac{x+7}{6}\right) + 6\left(\frac{2x-8}{2}\right) = 6(-4)$$

Distributive property.

$$x + 7 + 3(2x - 8) = -24$$

Multiply.

Example: Solving a Linear Equation with Fractions

Solution (continued)

$$x + 7 + 6x - 24 = -24$$

$$7x - 17 = -24$$

$$7x - 17 + 17 = -24 + 17$$

$$7x = -7$$

$$\frac{7x}{7} = \frac{-7}{7}$$

$$x = -1$$

Now check.

Distributive property.

Combine like terms.

Add 17.

Combine like terms.

Divide by 7.

Types of Linear Equations

Type	Number of Solutions	Final Line When Solving
Conditional	One	$x = \text{number}$
Identity	Infinite; solution set {all real numbers}	True statement, such as $0 = 0$.
Contradiction	None; solution set $\emptyset$	False statement, such as $0 = 1$.

Example: Contradiction

Solve $3k + 4 - 2k = k$.

Solution

$$k + 4 = k$$

Combine like terms.

$$k - k + 4 = k - k$$

Subtract k .

$$4 = 0$$

Combine like terms.

Because the result is *false*, the equation has no solution. The solution set is $\emptyset$, so the original equation is a contradiction.

Example: Identity

Solve $3k + 4 - 2k = k + 4$.

Solution

$$k + 4 = k + 4$$

Combine like terms.

$$k - k + 4 = k - k$$

Subtract k .

$$4 = 4$$

Combine like terms.

Because the result is *true*, the solution set is {all real numbers} and the original equation is an identity.

Literal Equations and Formulas

An equation involving *variables* (or letters), such as $y = mx + b$ is called a **literal equation**.

Formulas are literal equations in which more than one letter is used to express a relationship.

It may be necessary to solve for one of the variables in a formula. This process is called **solving for a specified variable**.

Solving for a Specified Variable

- Step 1** If the equation contains fractions, multiply both sides by the LCD to clear the fractions.
- Step 2** Transform so that all terms with the specified variable are on one side and all terms without that variable are on the other side.
- Step 3** Divide each side by the factor that is the coefficient of the specified variable.

Example: Solving for a Specified Variable

Solve the formula $P = 2L + 2W$ for L .

Solution

$$P - 2W = 2L + 2W - 2W \quad \text{Subtract } 2W.$$

$$P - 2W = 2L \quad \text{Combine like terms.}$$

$$\frac{P - 2W}{2} = \frac{2L}{2} \quad \text{Divide by 2.}$$

$$\frac{P - 2W}{2} = L \quad \text{or} \quad \frac{P}{2} - W = L.$$

Models



A **mathematical model** is an equation (or inequality) that describes the relationship between two quantities. A *linear model* is a linear equation.

Example: Temperature Conversion

The relationship between degrees Celsius (C) and degrees Fahrenheit (F) is modeled by the linear equation

$$F = \frac{9}{5}C + 32$$

Find the Fahrenheit temperature that corresponds to 30 degrees Celsius.

Example: Temperature Conversion

Solution

Because $C = 30$, the equation becomes

$$F = \frac{9}{5}C + 32$$

$$F = \frac{9}{5}(30) + 32$$

$$F = 54 + 32$$

$$F = 86$$

The Celsius temperature 30 degrees corresponds to 86 degrees Fahrenheit.