

## AS1 (Assignment 1, Unit 3): Z-scores and Probability

Please type your answers in red 😊

1. What information is provided by the sign (+/-) of a z-score?  
The sign gives the indication of whether or not something is above or below the mean. + refers to above and - refers to below.
2. What information is provided by the numerical value of the z-score?  
The number following the symbol indicates the distance from the mean by the measure of standard deviation.
3. A distribution has a standard deviation of  $\sigma = 10$ . Find the z-score for each of the following locations in the distribution.
  - a. Above the mean by 5 points.  
Answer: **.5** due to the standard deviation being 10 but the score being only 5 above the mean.  $10/2$  is 5 making the answer **.5**
  - b. Above the mean by 2 points.  
Answer: **.2** due to the standard deviation being 10 but the score being only 2 above the mean.  $.2$  of 10 is 2 making the answer **.2**
  - c. Below the mean by 20 points.  
Answer: **-2** due to being lower than the mean so you need to take away 10 two times.
  - d. Below the mean by 15 points.  
Answer: **-1.5** due to being lower than the mean so you need to take away 10 one time and then 5 one time.
4. For a distribution with a standard deviation of  $\sigma = 20$ , describe the location of each of the following z-scores in terms of its position relative to the mean. For example,  $+1.00$  is a location that is 20 points above the mean.
  - a.  $z = +2.00$   
Answer: **+2.00** is a location that is 40 points above the mean
  - b.  $z = +.50$   
Answer: **+.50** is a location that is 10 points above the mean
  - c.  $z = -1.00$   
Answer: **-1.00** is a location that is 20 points below the mean
  - $z = -0.25$   
Answer: **-0.25** is a location that is 5 points below the mean
5. For a population with  $\mu = 80$  and  $\sigma = 10$ ,

- a. Find the z-score for each of the following X values. (Note: You should be able to find these values using the definition of a z-score. You should not need to use a formula or do any serious calculations.)

$$X = 75: z = -0.5 \quad X = 100: z = +2.00 \quad X = 60: z = -2.00$$

$$X = 95: z = +1.5 \quad X = 50: z = -3.00 \quad X = 85: z = +0.5$$

- b. Find the score (X value) that corresponds to each of the following z-scores. (Again, you should not need a formula or any serious calculations.)

$$z = 1.00: X = 90 \quad z = 0.20: X = 82 \quad z = 1.50: X = 95$$

$$z = -0.50: X = 75 \quad z = -2.00: X = 60 \quad z = -1.50: X = 65$$

6. For a population with a mean of  $\mu = 40$  and  $\sigma = 11$ , find the z-score for each of the following X values. (Note: You probably will need to use a formula and a calculator to find these values)

$$X = 45: z = .36 \quad X = 52: z = 1.09 \quad X = 41: z = .09$$

$$X = 30: z = -0.82 \quad X = 25: z = -1.36 \quad X = 38: z = -.18$$

7. For a population with a mean of  $\mu = 100$  and a standard deviation of  $\sigma = 20$ ,

- a. Find the z-score for each of the following X values.

$$X = 108: z = .40 \quad X = 115: z = .75 \quad X = 130: z = 1.5$$

$$X = 90: z = -.5 \quad X = 88: z = -.6 \quad X = 95: z = -.25$$

- b. Find the score (X value) that corresponds to each of the following z-scores.

$$z = -0.40: X = 92 \quad z = -0.50: X = 90 \quad z = 1.80: X = 136$$

$$z = 0.75: X = 115 \quad z = 1.50: X = 130 \quad z = -1.25: X = 75$$

8. A population has a mean of  $\mu = 60$  and a standard deviation of  $\sigma = 12$ .
- a. For the population, find the z-score for each of the following X values.

$$X = 69: z = .75 \quad X = 84: z = 2.00 \quad X = 63: z = .25$$

$$X = 54: z = -.5 \quad X = 48: z = -1.00 \quad X = 45: z = -1.25$$

- b. For the same population, find the score (X value) that corresponds to each of the following z-scores.

$$z = 0.50: X = 66 \quad z = 1.50: X = 78 \quad z = -2.50: X = 30$$

$$z = -0.25: X = 57 \quad z = -0.50: X = 54 \quad z = 1.25: X = 75$$

9. A sample has a mean of  $M = 30$  and a standard deviation of  $s = 8$ . Find the z-score of for each of the following X values from this sample.

$$X = 32: z = .25 \quad X = 34: z = .50 \quad X = 36: z = .75$$

$$X = 28: z = -.25 \quad X = 20: z = -1.25 \quad X = 18: z = -1.50$$

10. A sample has a mean of  $M = 25$  and a standard deviation of  $s = 5$ . For this sample, find the X value corresponding to each of the following z-scores.

$$z = 0.40: X = 27 \quad z = 1.20: X = 31 \quad z = 2.00: X = 35$$

$$z = -0.80: X = 21 \quad z = -0.60: X = 22 \quad z = -1.40: X = 18$$

11. Find the z-score corresponding to a score of  $X = 45$  of the following distributions.

a.  $\mu = 40$  and  $\sigma = 20$

Answer: .25

b.  $\mu = 40$  and  $\sigma = 10$

Answer: .5

c.  $\mu = 40$  and  $\sigma = 5$

Answer: 1.00

d.  $\mu = 40$  and  $\sigma = 2$

Answer: 2.50

Find the X value corresponding to  $z = 0.25$  for each of the following distributions.

e.  $\mu = 40$  and  $\sigma = 4$ .....Answer: 41

f.  $\mu = 40$  and  $\sigma = 8$ .....Answer: 42

g.  $\mu = 40$  and  $\sigma = 16$ .....Answer: 44

h.  $\mu = 40$  and  $\sigma = 32$ .....Answer: 48

12. For each of the following populations, would a score of  $X = 50$  be considered a central score (near the middle of the distribution) or an extreme score (far out in the tail of the distribution)?

a.  $\mu = 45$  and  $\sigma = 10$ .....Answer: central score

b.  $\mu = 45$  and  $\sigma = 2$ .....Answer: extreme score

c.  $\mu = 90$  and  $\sigma = 20$ .....Answer: extreme score

d.  $\mu = 60$  and  $\sigma = 20$ .....Answer: central score

13. A distribution of exam scores has a mean of  $\mu = 78$ .

a. If your score is  $X = 70$ , which standard deviation would give you a better grade:  $\sigma = 4$  or  $\sigma = 8$ ?

Answer: 8

b. If your score is  $X = 80$ , which standard deviation would give you a better grade:  $\sigma = 4$  or  $\sigma = 8$ ?

Answer: 4

14. For each of the following, identify the exam score that should lead to the better grade.

a. A score of  $X = 74$  on an exam with  $M = 82$  and  $\sigma = 8$ ; or a score of  $X = 40$  on an exam with  $\mu = 50$  and  $\sigma = 20$ .

Answer: A score of  $X = 74$  on an exam with  $M = 82$  and  $\sigma = 8$

b. A score of  $X = 51$  on an exam with  $\mu = 45$  and  $\sigma = 2$ ; or a score of  $X = 90$  on an exam with  $\mu = 70$  and  $\sigma = 20$ .

Answer: A score of  $X = 51$  on an exam with  $\mu = 45$  and  $\sigma = 2$ ;

- c. A score of  $X = 62$  on an exam with  $\mu = 50$  and  $\sigma = 8$ ; or a score of  $X = 23$  on an exam with  $\mu = 20$  and  $\sigma = 2$      Answer: **Wouldnt it be the same? They both have a z score of 1.50**

15. A population consists of the following  $N = 5$  scores: 0, 6, 4, 3, and 12.

- a. Compute  $\mu$  and  $\sigma$  for the population

Mean = **5** Standard deviation = **4**

- b. Find the z-score for each score in the population

$X=0, z = -1.25$     $X=6, z = 0.25$     $X=4, z = -0.25$     $X=3, z = -.50$     $X=12, z = 1.75$

15. A sample consists of the following  $n = 7$  scores: 5, 0, 4, 5, 1, 2, and 4.

- a. Compute the mean and standard deviation for the sample.

Mean = **3** Standard deviation = **2**

Find the z-score for each score in the sample.

$X=5, z = 1.00$     $X=0, z = -1.50$     $X=4, z = 0.50$     $X=5, z = 1.00$     $X=1, z = -1.00$

$X=2, z = -.50$     $X=4, z = 0.50$

16. There are 20 black marbles, 40 blue marbles, and 5 red marbles in a jar.

- a. What is the probability of selecting one red marble? **.08**
- b. What is the probability of selecting one black marble? **0.30**
- c. What is the probability of selecting one blue marble? **0.62**
- d. Which has the highest probability of being selected? **Blue**
- e. Which has the lowest probability of being selected? **Red**

17. Probability values range from **-1.00** to **+1.00**

18. Please describe at least three characteristics of the NORMAL DISTRIBUTION

1. **The mean, median, and mode all equal the same**

2. Data is symmetrical on graphs

3. Average scores are in the middle of the graph

19. In a normal population of IQ scores, what percent of people have “average” IQ’s?

Answer 68.13

20. In a normal distribution, what percentage of people would be located at or below 2 standard deviations from the mean?

Answer 2.28

21. Answer the following questions based on a distribution with a  $\mu = 30$  and  $\sigma = 5$ :

a. What range of scores is considered “average”? 25 to 35

b. What percentage of people has an average score? 60

c. What percentage of people has extremely high or extremely low scores? 30

d. What *range of scores* (requires numbers to be noted in the blank spaces) have the highest probability of being selected? 20 to 40