Chapter 2

The Basic Concepts of Set Theory

MATHEMATICAL IDEAS

THIRTEENTH EDITION ●●●●

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Chapter 2: The Basic Concepts of Set Theory

2.1 Symbols and Terminology

2.2 Venn Diagrams and Subsets

2.3 Set Operations

2.4 Surveys and Cardinal Numbers

Section 2-3

Set Operations

Set Operations

- Determine intersections of sets

- Determine unions of sets.

- Determine the difference of two sets.

- Understand ordered pairs and their uses.

- Determine Cartesian products of sets.

- Analyze sets and set operations with Venn diagrams.

- Apply De Morgan's laws for sets.

Intersection of Sets

The **intersection** of sets A and B, written $A \cap B$, is the set of elements common to both A and B, or:

$$A \cap B = \{x \mid x \in A \text{ and } x \in B\}.$$

Example: Finding Intersections

Find each intersection.

    a) $\{1, 3, 5, 7, 9\} \cap \{1, 2, 3, 4, 5, 6\}$

    b) $\{2, 4, 6\} \cap \varnothing$

Solution

    a) $\{1, 3, 5\}$

    b) $\varnothing$

Union of Sets

The **union** of sets A and B, written $A \cup B$, is the set of elements belonging to either of the sets, or:

$$A \cup B = \{x \mid x \in A \text{ or } x \in B\}$$

Example: Finding Unions

Find each union.

    a) $\{1, 3, 5, 7, 9\} \cup \{1, 2, 3, 4, 5, 6\}$

    b) $\{2, 4, 6\} \cup \varnothing$

Solution

    a) $\{1, 2, 3, 4, 5, 6, 7, 9\}$

    b) $\{2, 4, 6\}$

Difference of Sets

The **difference** of sets A and B, written $A - B$, is the set of elements belonging to set A and not to set B, or:

$$A - B = \{x \mid x \in A \text{ and } x \notin B\}.$$

Example: Finding Set Differences

Let U = {a, b, c, d, e, f, g, h}, A = {a, b, c, e, h}, B = {c, e, g}, and C = {a, c, d, g, e}.

Find each set.

   a) $A - B$

   b) $(B - A) \cup C'$

Solution

   a)   {a, b, h}

   b)   {g} $\cup$ {b, f, h} = {b, f, g, h}

Ordered Pairs

In the **ordered pair** (a, b), a is called the **first component** and b is called the **second component**. In general $(a, b) \neq (b, a)$.

Two ordered pairs are **equal** provided that their first components are equal and their second components are equal.

Cartesian Product of Sets

The **Cartesian product** of sets A and B, written $A \times B$, is:

$$A \times B = \{(a,b) \mid a \in A \text{ and } b \in B\}.$$

Example: Finding Cartesian Products

Let $A = \{a, b\}$, $B = \{1, 2, 3\}$.

Find each set.

    a) $A \times B$

    b) $B \times B$

Solution

    a) $\{(a, 1), (a, 2), (a, 3), (b, 1), (b, 2), (b, 3)\}$

    b) $\{(1, 1), (1, 2), (1, 3), (2, 1), (2, 2), (2, 3),$
          $(3, 1), (3, 2), (3, 3)\}$

Cardinal Number of a Cartesian Product

If $n(A) = a$ and $n(B) = b$, then:

$$n(A \times B) = n(B \times A)$$

$$= n(A) \cdot n(B) = n(B) \cdot n(A)$$

$$= ab = ba$$

Example: Finding Cardinal Numbers of Cartesian Products

If $n(A) = 12$ and $n(B) = 7$, then find:
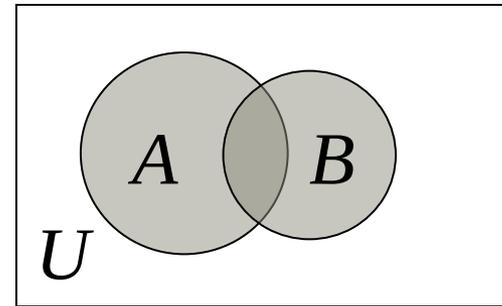
$n(A \times B)$ and $n(B \times A)$.

Solution

$n(A \times B) = n(B \times A)$

$\qquad = n(A) \cdot n(B) = n(B) \cdot n(A)$

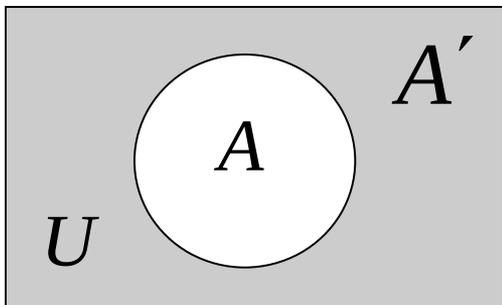$\qquad = 7 \cdot 12 = 84$
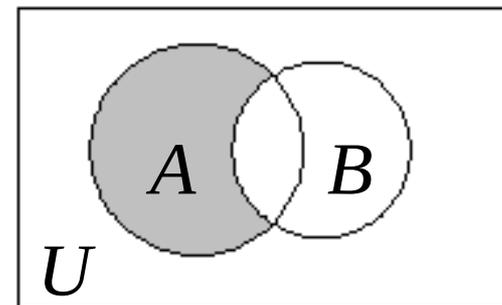
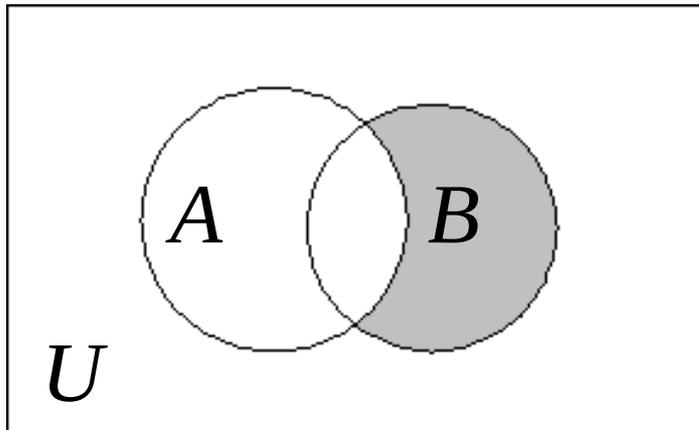# Venn Diagrams of Set Operations

$A \cap B$
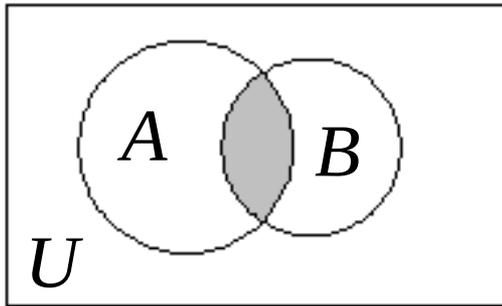
$A \cup B$

A'

$A - B$

Example: Shading Venn Diagrams to Represent Sets

Draw a Venn Diagram to represent the set $A' \cap B$.
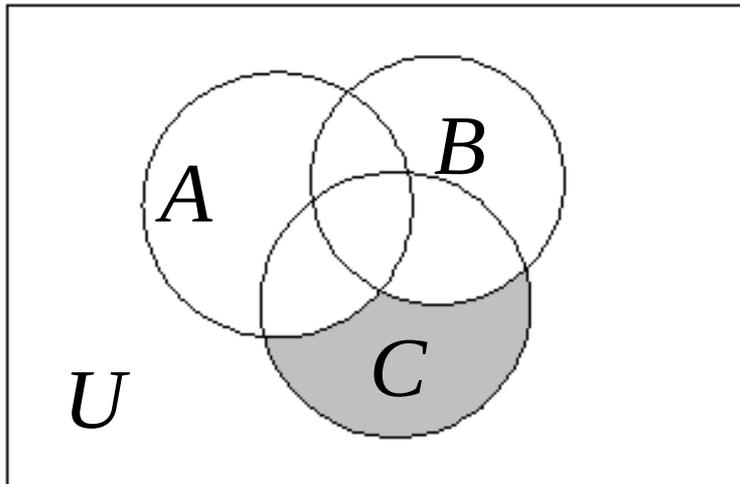
Solution

Example: Shading Venn Diagrams to Represent Sets

Draw a Venn Diagram to represent the set $(A' \cap B') \cap C.$

Solution

De Morgan's Laws

For any sets *A* and *B:*

$$(A \cap B)' = A' \cup B' \text{ and } (A \cup B)' = A' \cap B'.$$