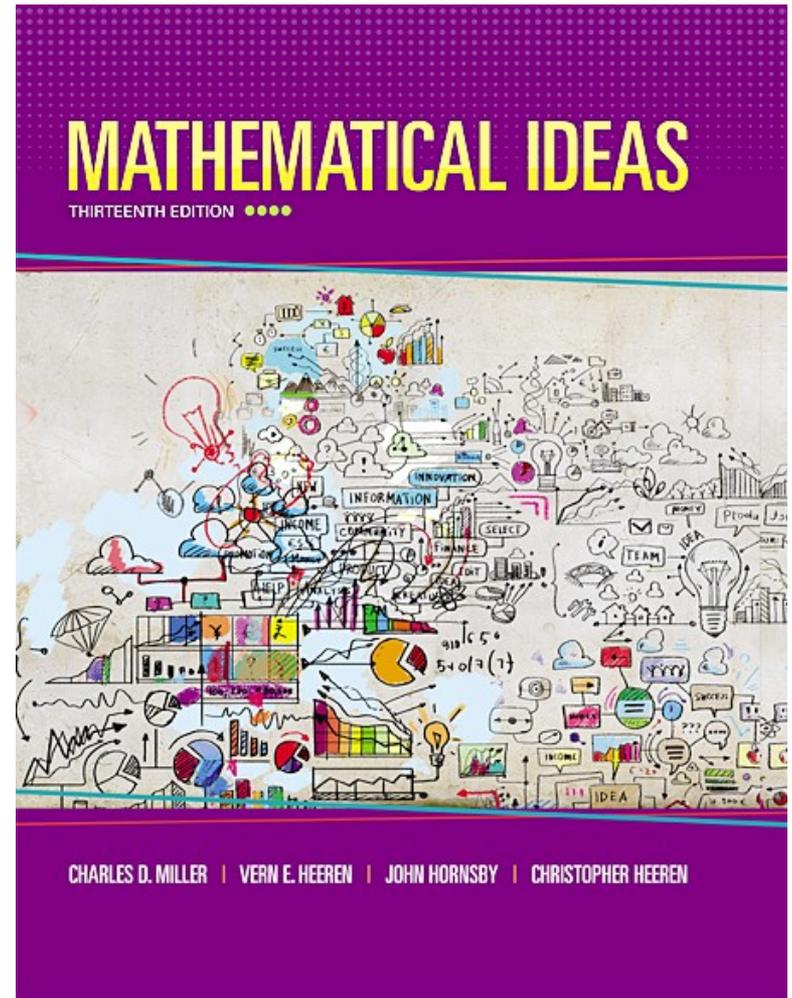


Chapter 2

The Basic Concepts of Set Theory



Chapter 2: The Basic Concepts of Set Theory

2.1 Symbols and Terminology

2.2 Venn Diagrams and Subsets

2.3 Set Operations

2.4 Surveys and Cardinal Numbers

Section 2-2

Venn Diagrams and Subsets

Venn Diagrams and Subsets

- Use Venn diagrams to depict set relationships.
- Determine the complement of a set within a universal set.
- Determine if one set is a subset of another.
- Understand the distinction between a subset and a proper subset.
- Determine the number of subsets of a given set.

Venn Diagrams

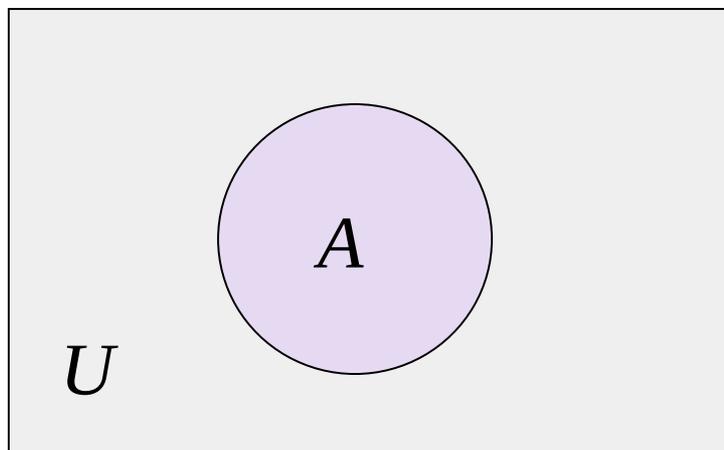


In set theory, the universe of discourse is called the **universal set**, typically designated with the letter *U*.

Venn Diagrams were developed by the logician John Venn (1834 – 1923). In these diagrams, the universal set is represented by a rectangle and other sets of interest within the universal set are depicted as circular regions.

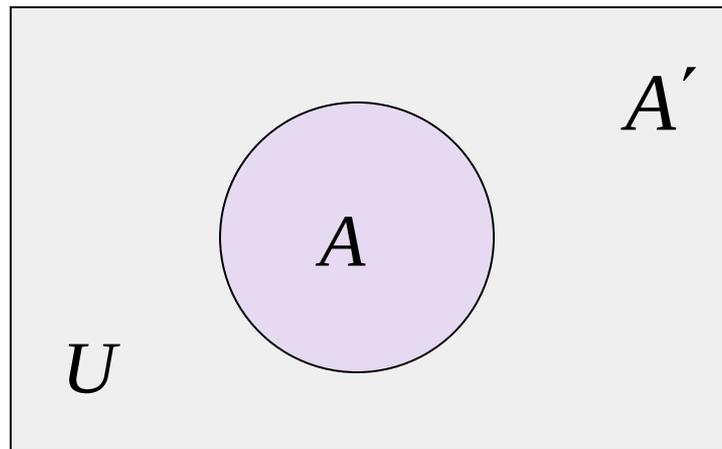
Venn Diagrams

The rectangle represents the universal set, U , while the portion bounded by the circle represents set A .



Complement of a Set

The shaded region inside U and outside the circle is labeled A' (read “A **prime**”). This set, called the **complement of A** , contains all elements that are contained in U , but not in A .



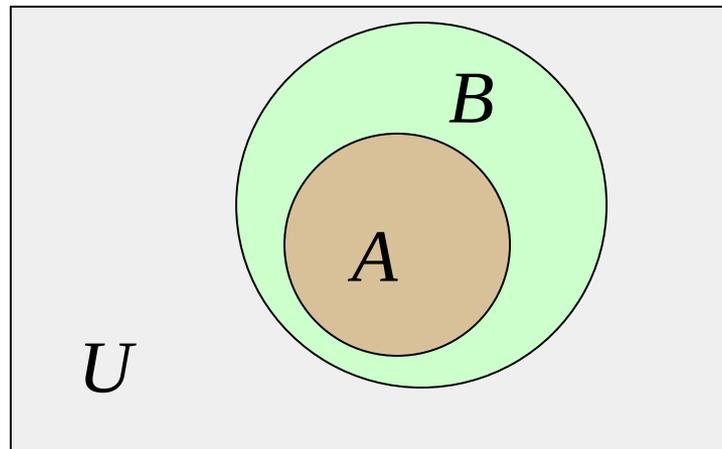
Complement of a Set

For any set A within the universal set U , the complement of A , written A' , is the set of all elements of U that are not elements of A . That is:

$$A' = \{x \mid x \in U \text{ and } x \notin A\}.$$

Subsets of a Set

Set A is a subset of set B if every element of A is also an element of B . In symbols this is written $A \subseteq B$.



Example: Determining If One Set Is a Subset of Another

Fill in the blank with \subseteq or $\not\subseteq$ to make a true statement.

a) $\{a, b, c\}$ ____ $\{a, c, d\}$

b) $\{1, 2, 3, 4\}$ ____ $\{1, 2, 3, 4\}$

Solution

a) $\{a, b, c\}$ $\not\subseteq$ $\{a, c, d\}$

b) $\{1, 2, 3, 4\}$ \subseteq $\{1, 2, 3, 4\}$

Set Equality (Alternative Definition)

Suppose that A and B are sets. Then $A = B$ if $A \subseteq B$ and $B \subseteq A$ are both true.

Proper Subset of a Set

Set A is a **proper subset** of set B if $A \subseteq B$ and $A \neq B$.

In symbols, this is written $A \subset B$.

Example: Determining Subsets and Proper Subsets

Decide whether \subseteq , \subset , or both could be placed in each blank to make a true statement.

a) $\{a, b, c\}$ ____ $\{a, b, c, d\}$

b) $\{1, 2, 3, 4\}$ ____ $\{1, 2, 3, 4\}$

Solution

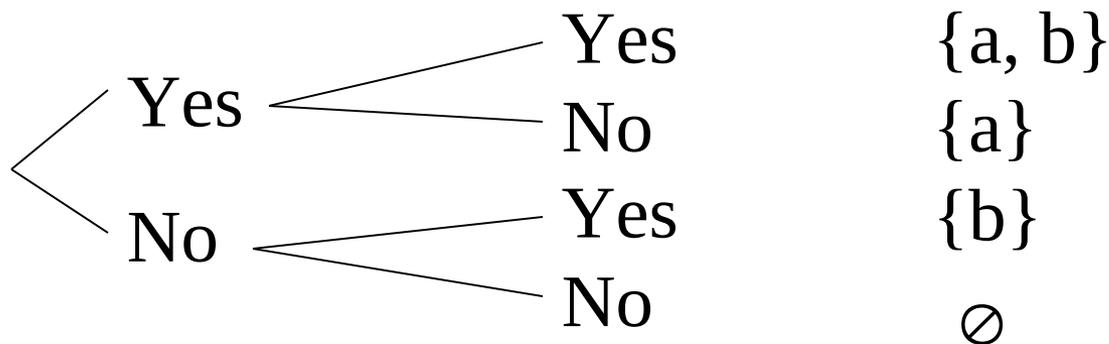
a) both

b) \subseteq

Counting Subsets

One method of counting subsets involves using a **tree diagram**. The figure below shows the use of a tree diagram to find the subsets of $\{a, b\}$.

$a \in$ subset? $b \in$ subset? 4 subsets



Number of Subsets



The number of subsets of a set with n elements is 2^n .

The number of proper subsets of a set with n elements is $2^n - 1$.

Example: Finding Numbers of Subsets and Proper Subsets

Find the number of subsets and the number of proper subsets of the set $\{m, a, t, h, y\}$.

Solution

Since there are 5 elements, the number of subsets is $2^5 = 32$.

The number of proper subsets is $32 - 1 = 31$.