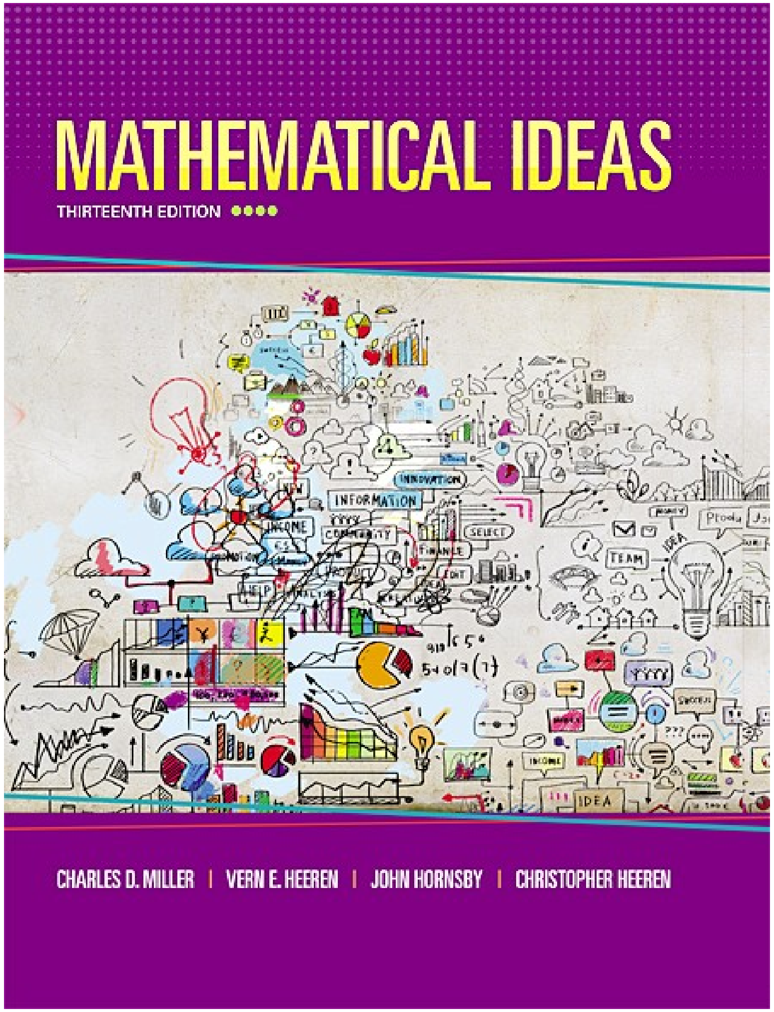
Chapter 2

The Basic Concepts of Set Theory



Chapter 2: The Basic Concepts of Set Theory

Section 2-1

Symbols and Terminology

Symbols and Terminology

- Use three methods to designate sets.
- Understand important categories of numbers, and determine cardinal numbers of sets.
- Distinguish between finite and infinite sets.
- Determine if two sets are equal.

Designating Sets

A **set** is a collection of objects. The objects belonging to the set are called the **elements**, or **members**, of the set.

Sets are designated using:

1) *word description*,
2) *the listing method*, and
3) *set-builder notation*.

Designating Sets

Word description

The set of even counting numbers less than 10

The listing method

{2, 4, 6, 8}

Set-builder notation

$\{x | x$ is an even counting number less than $10\}$

Designating Sets

Sets are commonly given names (capital letters).

$A = \{1, 2, 3, 4\}$

The set containing no elements is called the **empty set** (*null set*) and is denoted by { } or $\emptyset$.

To show 2 is an element of set *A* use the symbol $\in$.

$2 \in \{1, 2, 3, 4\}$

$a \notin \{1, 2, 3, 4\}$

Example: Listing Elements of Sets

Give a complete listing of all of the elements of the set $\{x|x$ is a natural number between 3 and 8$\}$.

Solution

$\{4, 5, 6, 7\}$

Sets of Numbers

Natural numbers (*counting*) {1, 2, 3, 4, …}

Whole numbers {0, 1, 2, 3, 4, …}

Integers {…,–3, –2, –1, 0, 1, 2, 3, …}

Rational numbers $\left\{ \frac{p}{q} \middle| p \text{ and } q \text{ are integers, with } q \neq 0 \right\}$

May be written as a terminating decimal, like 0.25, or a repeating decimal, like 0.333…

Irrational $\{x \mid x$ is not expressible as a quotient of integers$\}$

Decimal representations never terminate and never repeat.

Real numbers $\{x \mid x$ can be expressed as a decimal$\}$

Cardinality

The number of elements in a set is called the **cardinal number**, or **cardinality**, of the set.

The symbol $\boldsymbol{n(A)}$, read "***n*** of ***A***," represents the cardinal number of set A.

Example: Finding Cardinal Numbers

Find the cardinal number of each set.

a) $K = \{a, l, g, e, b, r\}$

b) $M = \{2\}$

c) $\emptyset$

Solution

a) $n(K) = 6$

b) $n(M) = 1$

c) $n(\emptyset) = 0$

Finite and Infinite Sets

If the cardinal number of a set is a particular whole number, we call that set a **finite set**.

Whenever a set is so large that its cardinal number is not found among the whole numbers, we call that set an **infinite set**.

Example: Designating an Infinite Set

Designate all odd counting numbers by the three counting methods of set notation.

Solution

Word description:

The set of all odd counting numbers

Listing method: {1, 3, 5, 7, 9, …}

Set-builder notation:

$$\{x|x \text{ is an odd counting number}\}$$

Equality of Sets

Set *A* is **equal** to set *B* provided the following two conditions are met:

1. Every element of *A* is an element of *B*, and
2. Every element of *B* is an element of *A*.

Example: Determining Whether Two Sets Are Equal

State whether the sets in each pair are equal.

a) {a, b, c, d} and {a, c, d, b}

b) {2, 4, 6} and $\{x | x$ is an even number}

Solution

a) Yes, order of elements does not matter.

b) No, {2, 4, 6} does not represent all the even numbers.