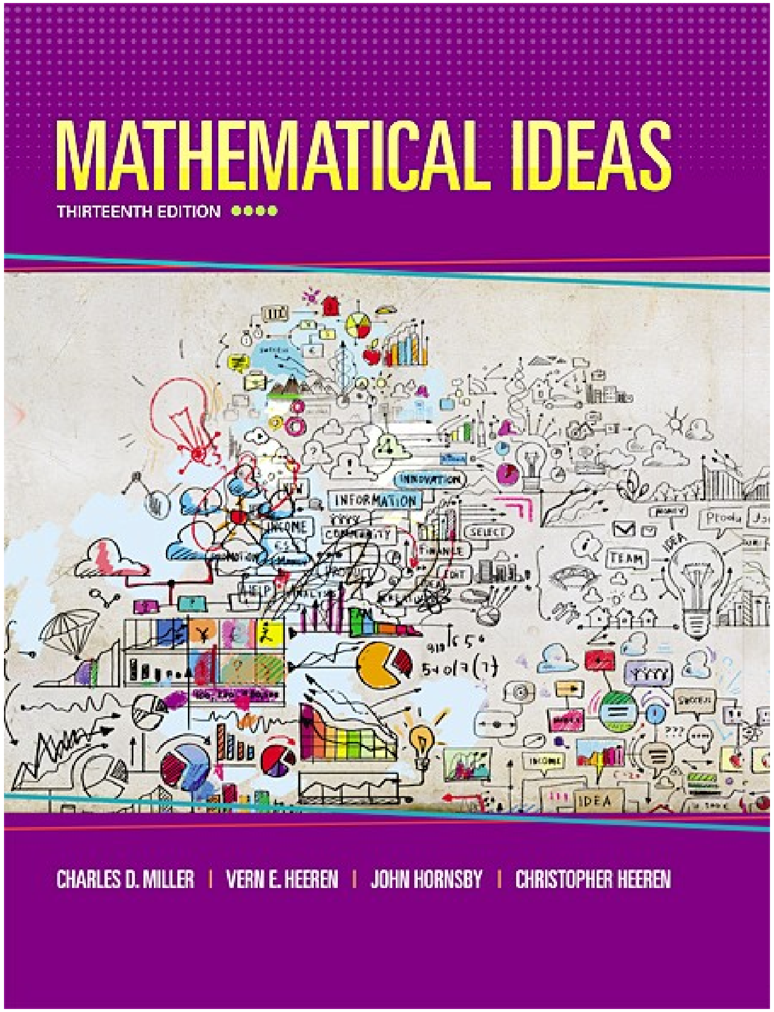
Chapter 6

The Real Numbers and Their Representation



Chapter 6: The Real Numbers and Their Representation

6.1 Real Numbers, Order, and Absolute Value

6.2 Operations, Properties, and Applications of Real Numbers

6.3 Rational Numbers and Decimal Representation

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Section 6-1

Real Numbers, Order, and Absolute Value

Real Numbers, Order, and Absolute Value

- Represent a number on a number line.
- Identify a number as positive, negative, or zero.
- Identify a number as belonging to one or more sets of numbers.
- Given two numbers, a and b, determine whether $a = b$, $a < b$, or $a > b$.
- Given a number a, determine its additive inverse, and absolute value.
- Interpret signed numbers in tables of economic and occupations data.

Sets of Real Numbers

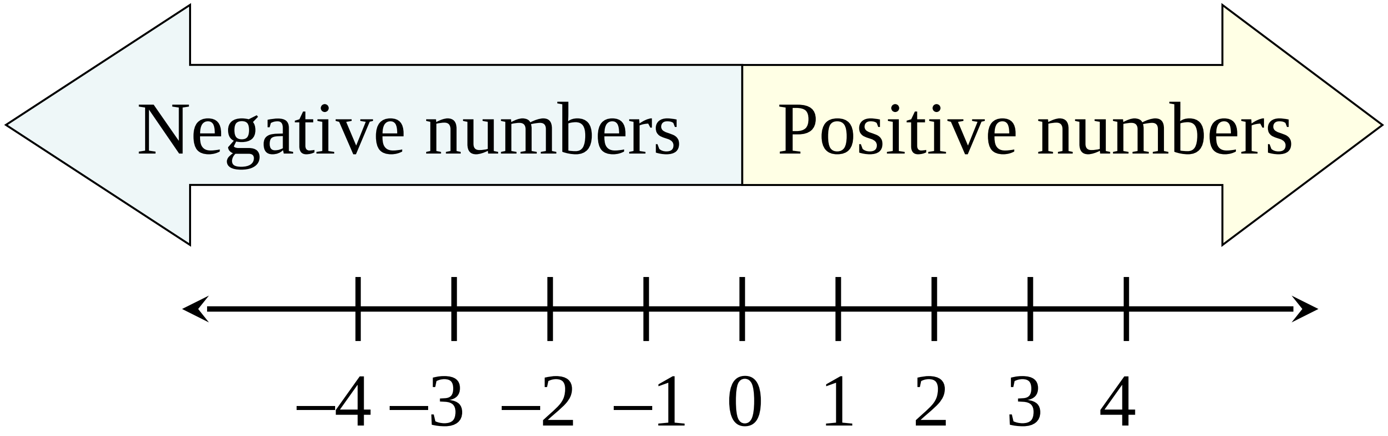
Natural Numbers

{1, 2, 3, 4, …} is the set of **natural numbers**.

Whole Numbers

{0, 1, 2, 3, 4, …} is the set of **whole numbers**.

Number Line



Positive and negative numbers are called **signed** numbers.

Sets of Real Numbers

The set of numbers marked on the number line on the previous slide, including positive and negative numbers and zero, is part of the set of *integers*.

Integers

$\{\ldots, -3, -2, -1, 0, 1, 2, 3, \ldots\}$ is the set of **integers**.

Sets of Real Numbers

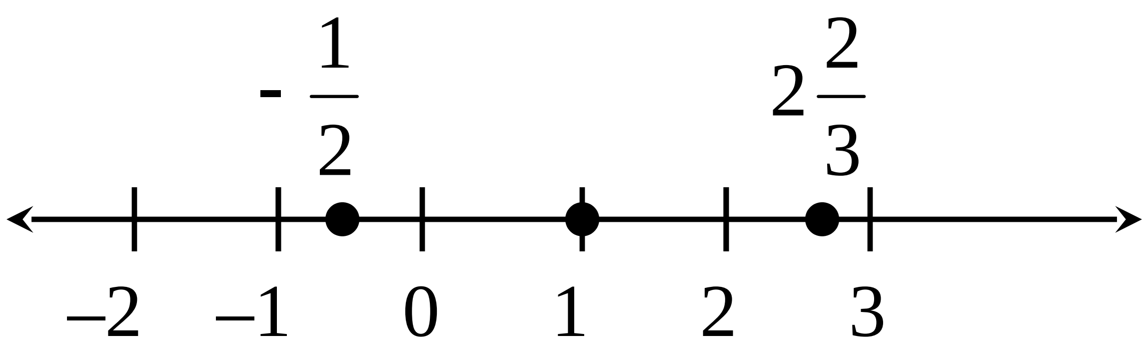
Numbers such as $\frac{1}{2}$ and $-1\frac{2}{3}$ are not integers, they are rational numbers.

Rational Numbers

$\{x \mid x$ is a quotient of two integers, with denominator not equal to $0\}$ is the set of **rational numbers**.

Sets of Real Numbers

The **graph** of a number is a point on the number line. A few numbers are *graphed* below.



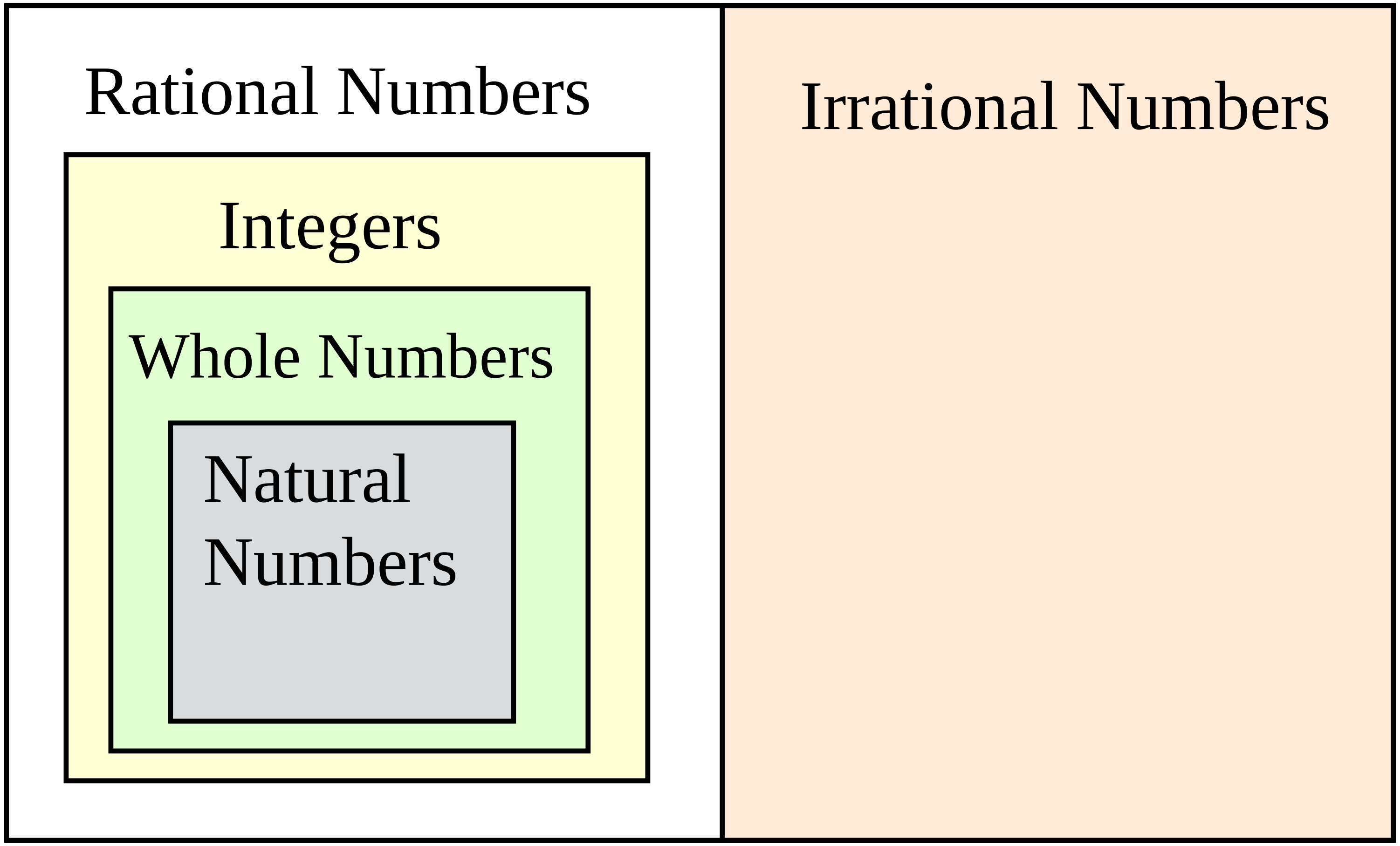
Sets of Real Numbers

Not all numbers are rational. For example, $\sqrt{2}$ can not be written as the quotient of two integers. It is called *irrational*.

Irrational Numbers

$\{x \mid x$ is a number on the number line that is not rational$\}$ is the set of **irrational numbers**.

The Real Numbers



Example: Identifying Elements of a Set of Numbers

List the numbers from the set $\left\{-3, -\dfrac{1}{2}, 0, 1, 1.8, \sqrt{7}\right\}$ that are:

a) natural numbers
b) rational numbers
c) real numbers

Solution

a) natural numbers: the only natural number is 1.
b) rational numbers: {–3, –1/2, 0, 1, 1.8}
c) real numbers: all the numbers are real numbers

Order in the Real Numbers

Two real numbers may be compared, or ordered, using ideas of equality and inequality.

Order in the Real Numbers

The **law of trichotomy** states that, for two numbers a and b, one and only one of the following is true.

$a = b$	a equals b
$a < b$	a is less than b
$a > b$	a is greater than b

Order in the Real Numbers

The symbol $\leq$ means "is less than or equal to."
The statement is true if the = part or < part is true.

$5 \leq 7$ is true, as is $5 \leq 5$.

The symbol $\geq$ means "is greater than or equal to."
The statement is true if the = part or > part is true.

Additive Inverses

For any real number x (except 0), there is exactly one number on the number line the same distance from 0 as x but on the opposite side of 0. For example, 3 and –3 are the same distance from 0 but on opposite sides. These numbers are **additive inverses**, **negatives**, or **opposites** of each other.

Double Negative Rule

For any real number x,

$$-(-x) = x.$$

Absolute Value

The **absolute value** of a real number can be defined as the distance between 0 and the number on the number line. The symbol for the absolute value of x is $|x|$, read "**the absolute value of *x***."

The absolute value of a number is ***never negative***.

Absolute Value

For any real number *x*,

$$|x| = \begin{cases} x & \text{if } x \geq 0 \\ -x & \text{if } x < 0. \end{cases}$$

Example: Using Absolute Value

Simplify by finding the absolute value.

a) $|4|$ b) $|-2|$ c) $-|-3|$ d) $|1 - 8|$

Solution

a) 4

b) 2

c) –3

d) 7

Applications

When looking at the amount of *change*, without regard to whether the change is an increase (positive) or decrease (negative), use the absolute value of the number.

Example: Interpreting Signed Numbers in a Table

Given the temperature readings of the days below, which day had the greatest amount of change?

	A.M. Temp.	P.M. Temp.	P.M. – A.M.
Day 1	23° F	42° F	19° F
Day 2	32° F	55° F	23° F
Day 3	40° F	13° F	–27° F

Example: Interpreting Signed Numbers in a Table

Day 1	23^{o} F	42^{o} F	19^{o} F
Day 2	32^{o} F	55^{o} F	23^{o} F
Day 3	40^{o} F	13^{o} F	-27^{o} F

Solution

Day 3 had the greatest change at $|-27| = 27$ degrees.