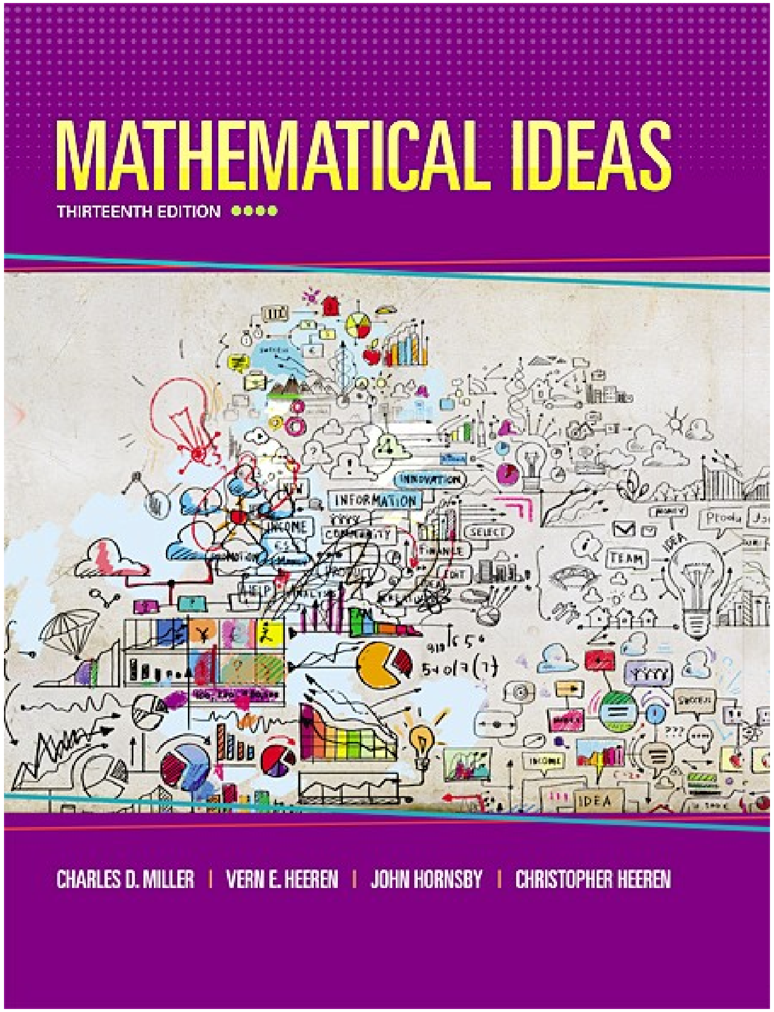
Chapter 1

The Art of Problem Solving



Chapter 1: The Art of Problem Solving

1.1 Solving Problems by Inductive Reasoning

1.2 An Application of Inductive Reasoning: Number Patterns

1.3 Strategies for Problem Solving

1.4 Numeracy in Today's World

Section 1-2

An Application of Inductive Reasoning: Number Patterns

An Application of Inductive Reasoning: Number Patterns

- Be able to recognize arithmetic and geometric sequences.
- Be able to apply the method of successive differences to predict the next term in a sequence.
- Be able to recognize number patterns.
- Be able to use sum formulas.
- Be able to recognize triangular, square, and pentagonal numbers.

Number Sequences

Number Sequence
A list of numbers having a first number, a second number, and so on, called the *terms* of the sequence.

Arithmetic Sequence
A sequence that has a common difference between successive terms.

Geometric Sequence
A sequence that has a common ratio between successive terms.

Example: Identifying Arithmetic or Geometric Sequences

For each sequence, determine if it is an *arithmetic sequence,* a *geometric sequence,* or *neither*. If it is either arithmetic or geometric, give the next term in the sequence.

a. 5, 10, 15, 20, 25, …

b. 3, 12, 48, 192, 768, …

Solution

a. Subtract the terms and find that the common difference is 5. 10 – 5 = 5, 20 – 15 = 5.

Arithmetic sequence; next term: 25 + 5 = 30

Example: Identifying Arithmetic or Geometric Sequences

b. 3, 12, 48, 192, 768, …

Solution

a. If you divide any two successive terms, you find a common ratio of 4.

Geometric sequence

Next term: 768(4) = 3072

$$\frac{12}{3}=4,\ \frac{48}{12}=4,\ \frac{192}{48}=4,\ \frac{768}{192}=4$$

Successive Differences

Process to determine the next term of a sequence using subtraction to find a common difference.

Example: Successive Differences

Use the method of successive differences to find the next number in the sequence.

14, 22, 32, 44,...

14		22		32		44		58	
	8		10		12		14		Find differences
		2		2		2			Find differences

Build up to next term: 58

Number Patterns and Sum Formulas

Sum of the First *n* Odd Counting Numbers

If *n* is any counting number, then

$$1 + 3 + 5 + \cdots + (2n - 1) = n^2.$$

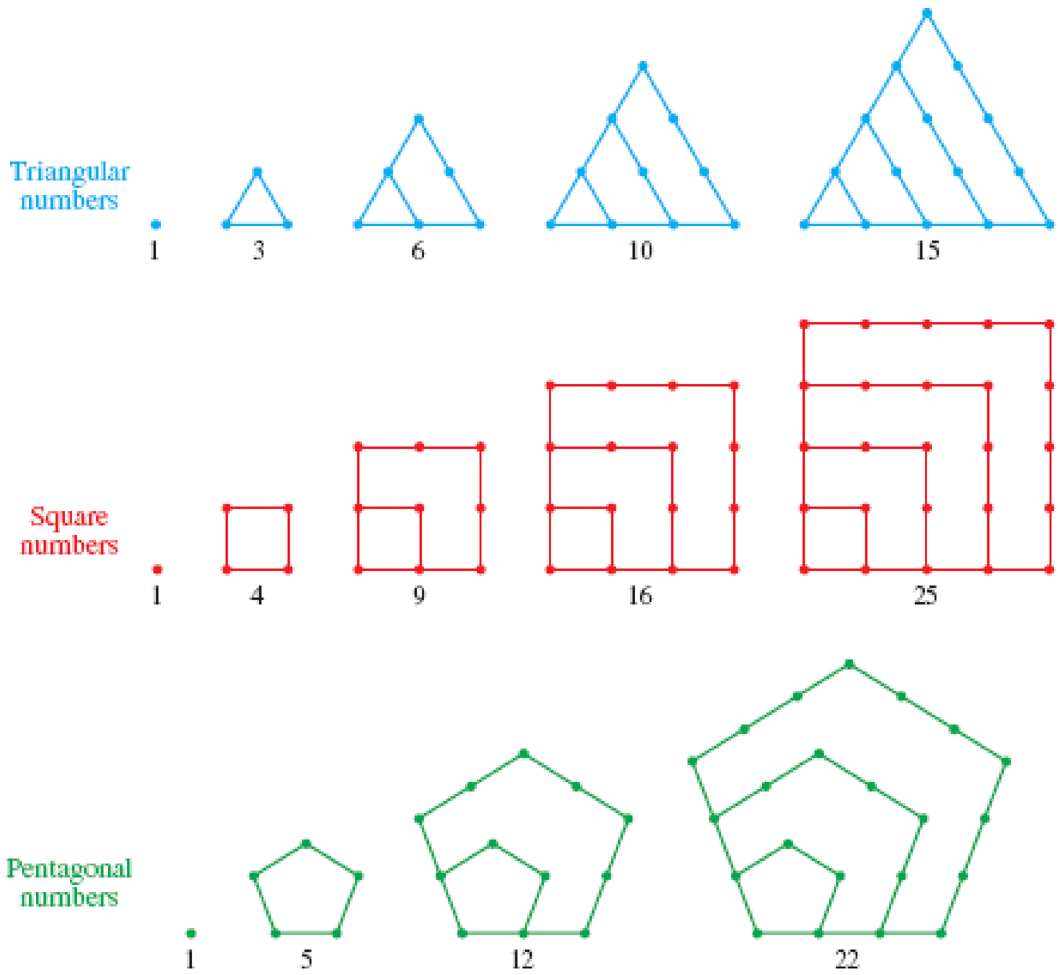
Special Sum Formulas

For any counting number *n*,

$$(1 + 2 + 3 + \cdots + n)^2 = 1^3 + 2^3 + \cdots + n^3$$

and $$1 + 2 + 3 + \cdots + n = \frac{n(n+1)}{2}.$$

Figurate Numbers



Formulas for Triangular, Square, and Pentagonal Numbers

For any natural number n,

the nth triangular number is given by $T_n = \frac{n(n+1)}{2}$,

the nth square number is given by $S_n = n^2$, and

the nth pentagonal number is given by $P_n = \frac{n(3n-1)}{2}$.

Example: Using the Formulas for Figurate Numbers

Use a formula to find the sixth pentagonal number.

Solution

Use the formula: $P_n = \dfrac{n(3n - 1)}{2}$

with $n = 6$:

$$P_6 = \frac{6[3(6) - 1]}{2} = 51$$